Flow Measurements

J. Łukasik[1,2] and W. Trautmann[1]

[1] GSI, Darmstadt, Germany ♦ [2] IFJ-PAN, Kraków, Poland

INDRA@GSI: Au+Au @ 40-150 (+15) AMeV

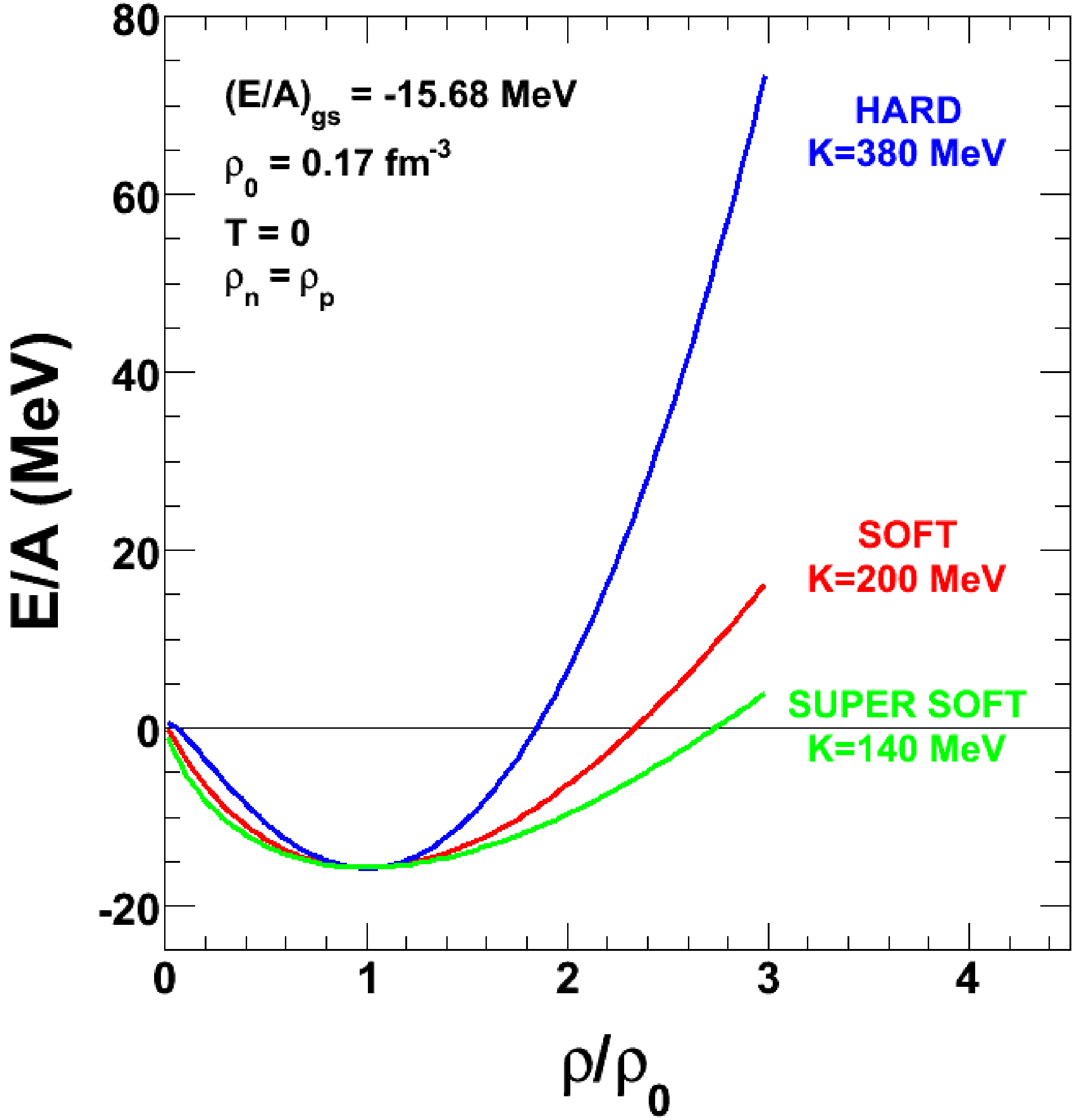
FOPI: Au+Au @ 90-1500 AMeV

- Flow, reaction plane and corrections
  - standard methods
  - new method
  - tests using CHIMERA-QMD simulation
    - reaction plane dispersion corr. (complete evts + unknown rpl.)
    - multi-hit loses corr. (filtered evts + known rpl.)
    - removing autocorrelations
    - filtered evts + unknown rpl. (all corrections)
- Flow systematics (INDRA+FOPI)
- ^{124,129}Xe+^{112,124}Sn @ 100 AMeV (INDRA)

Why collective flow?

- EOS of nuclear matter (including symmetry term)
- Momentum dependence of the meanfield
- In-medium modification of the σ_{NN}
- ...

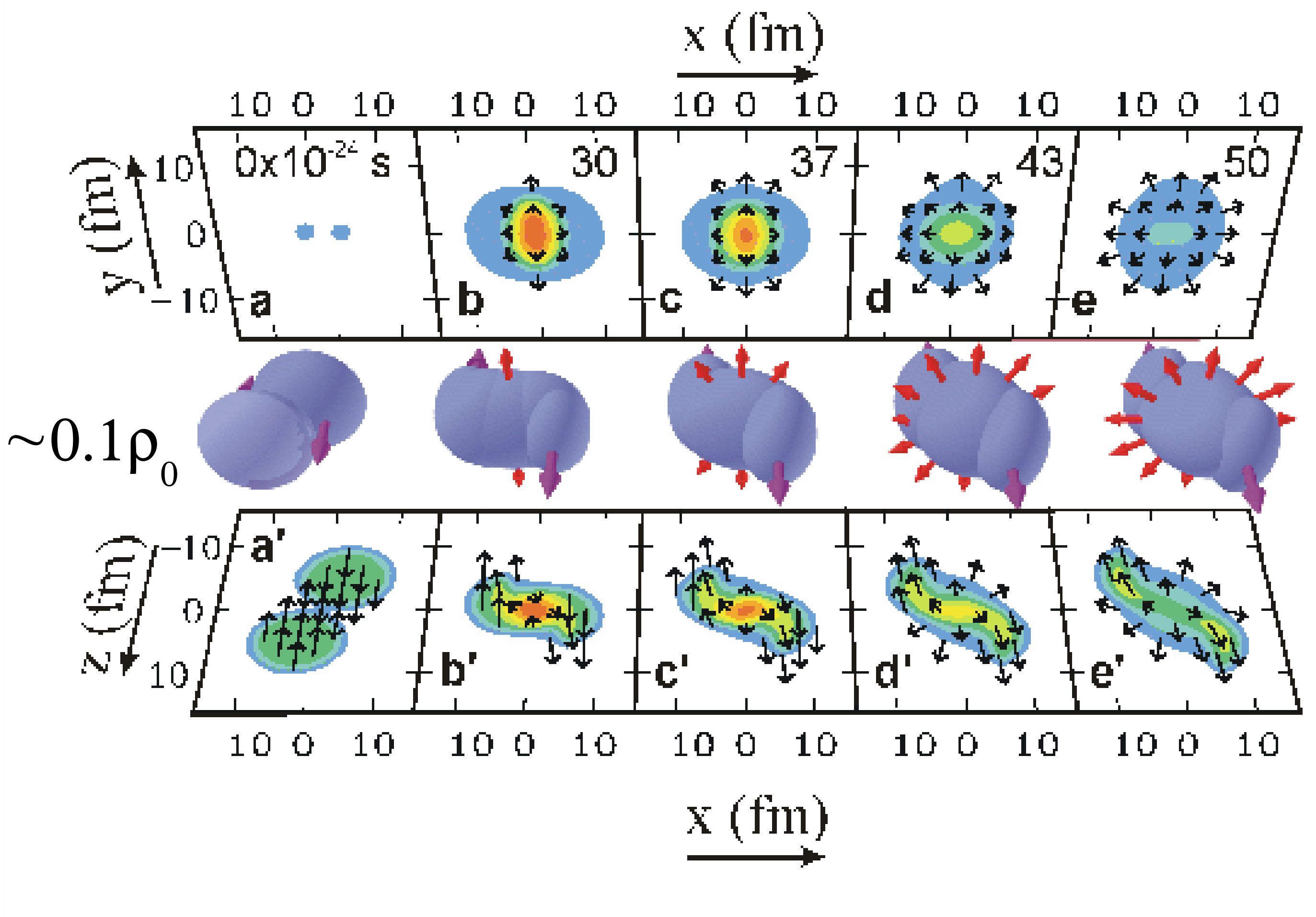
Equation of State

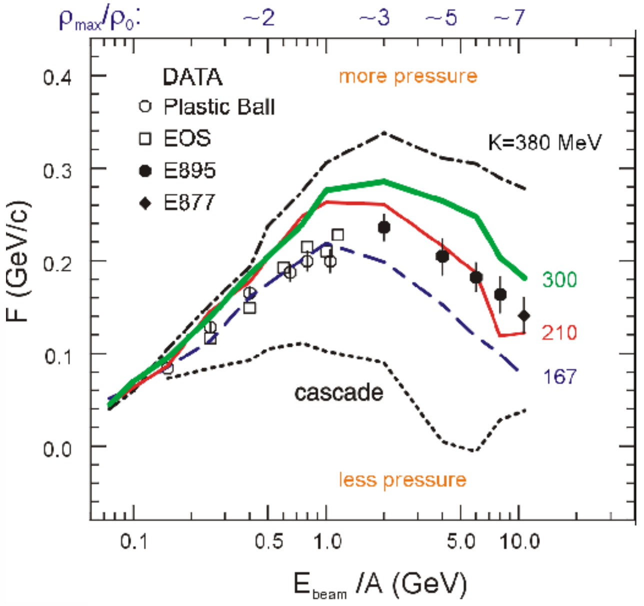


Directed flow

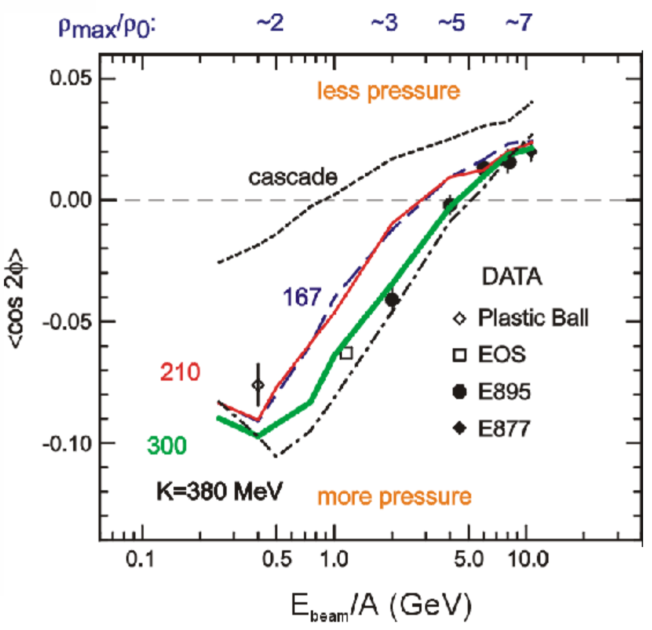
P. Danielewicz et al. Science 298(02)1592

Au+Au @ 2 AGeV, b=6 fm (BEM)

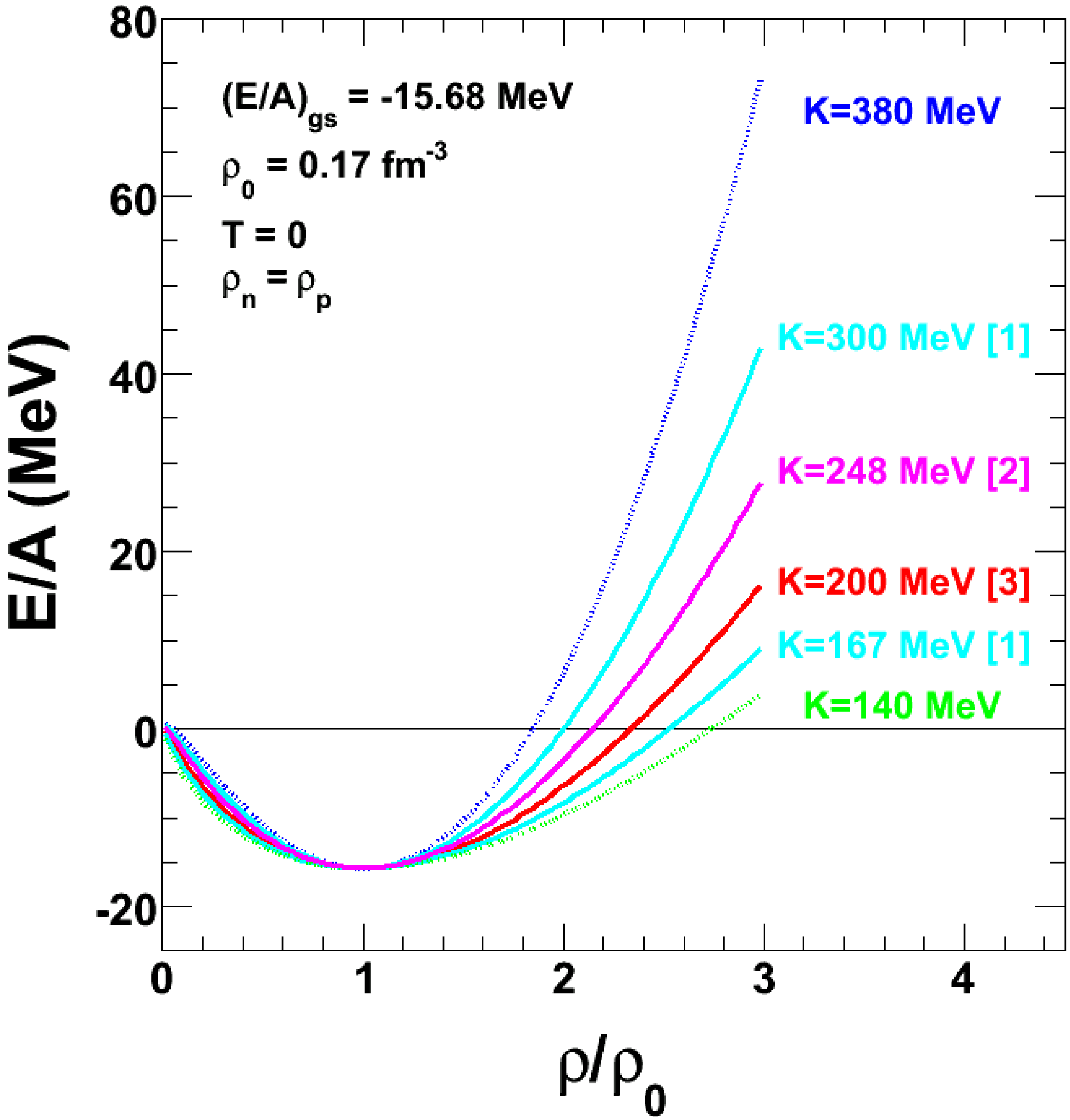




Elliptic flow



Equation of State



[1] Flow:
P. Danielewicz et al.,
Science 298 (02) 1592

[2] ISGMR:
J. Piekarewicz,
PRC 69 (04) 041301

[3] K^+:
Ch. Hartnack et al.,
PRL 96 (06) 012302

Fourier decomposition of the azimuthal distributions with respect to the **reaction plane** (ϕ_R):

$$\frac{dN}{d(\phi-\phi_R)} \propto 1 + 2\sum_{n\geq 1} v_n \cos n(\phi-\phi_R)$$

$$v_1 \equiv \langle \cos(\phi-\phi_R)\rangle \qquad \text{directed flow}$$

$$v_2 \equiv \langle \cos 2(\phi-\phi_R)\rangle \qquad \text{elliptic flow}$$

$$v_n = v_n(b, Z, A, y, p^{\perp})$$

Correction for the dispersion of the estimated reaction plane:

$$v_n^{meas} \equiv \langle \cos n(\phi-\phi_E)\rangle = v_n \langle \cos n\Delta\phi\rangle \qquad \text{where:} \qquad \langle \cos n\Delta\phi\rangle \equiv \langle \cos n(\phi_R-\phi_E)\rangle$$

'Q-vector' method (P. Danielewicz and G. Odyniec, Phys. Lett. B 157(1985)146):

$$\vec{Q} = \sum_{i=1}^{N} \omega_i \vec{p}_i^{\,\perp}, \qquad \omega_i = \text{sign}(y_{cm})$$

Standard correction obtained using **random sub-events** (with sub-Q-vectors $\vec{Q}_1 + \vec{Q}_2 = \vec{Q}$, $\Delta\phi_{12} = \sphericalangle(\vec{Q}_1, \vec{Q}_2)$), assuming applicability of central limit theorem and using small angle expansion:

$$\langle \cos n\Delta\phi\rangle_{Dan} = \langle \cos n\frac{\Delta\phi_{12}}{2}\rangle$$

Extension: Gaussian model by J.-Y. Ollitrault

'Gaussian model' (based on 'central limit' assumption)
(J.-Y. Ollitrault, nucl-ex/9711003):

$$\frac{d^2N}{dQd\Delta\phi} = \frac{Q}{\pi\sigma^2} e^{-\frac{(\vec{Q}-\vec{Q}_0)^2}{\sigma^2}}$$

$$\langle \cos n\Delta\phi \rangle_{Oll} = \frac{\sqrt{\pi}}{2} \chi e^{-\chi^2/2} \left[I_{\frac{n-1}{2}}(\frac{\chi^2}{2}) + I_{\frac{n+1}{2}}(\frac{\chi^2}{2}) \right]$$

The *resolution parameter* $\boxed{\chi \equiv Q_0/\sigma}$ can be obtained from sub-events, assuming they are **independent**, **isotropic**, equivalent and also normally distributed:

$$\frac{d^4N}{d\vec{Q}_1 d\vec{Q}_2} = \frac{1}{\pi^2\sigma^4_{sub}} \exp\left(-\frac{(\vec{Q}_1 - \vec{Q}_{0sub})^2}{\sigma^2_{sub}}\right) \exp\left(-\frac{(\vec{Q}_2 - \vec{Q}_{0sub})^2}{\sigma^2_{sub}}\right)$$

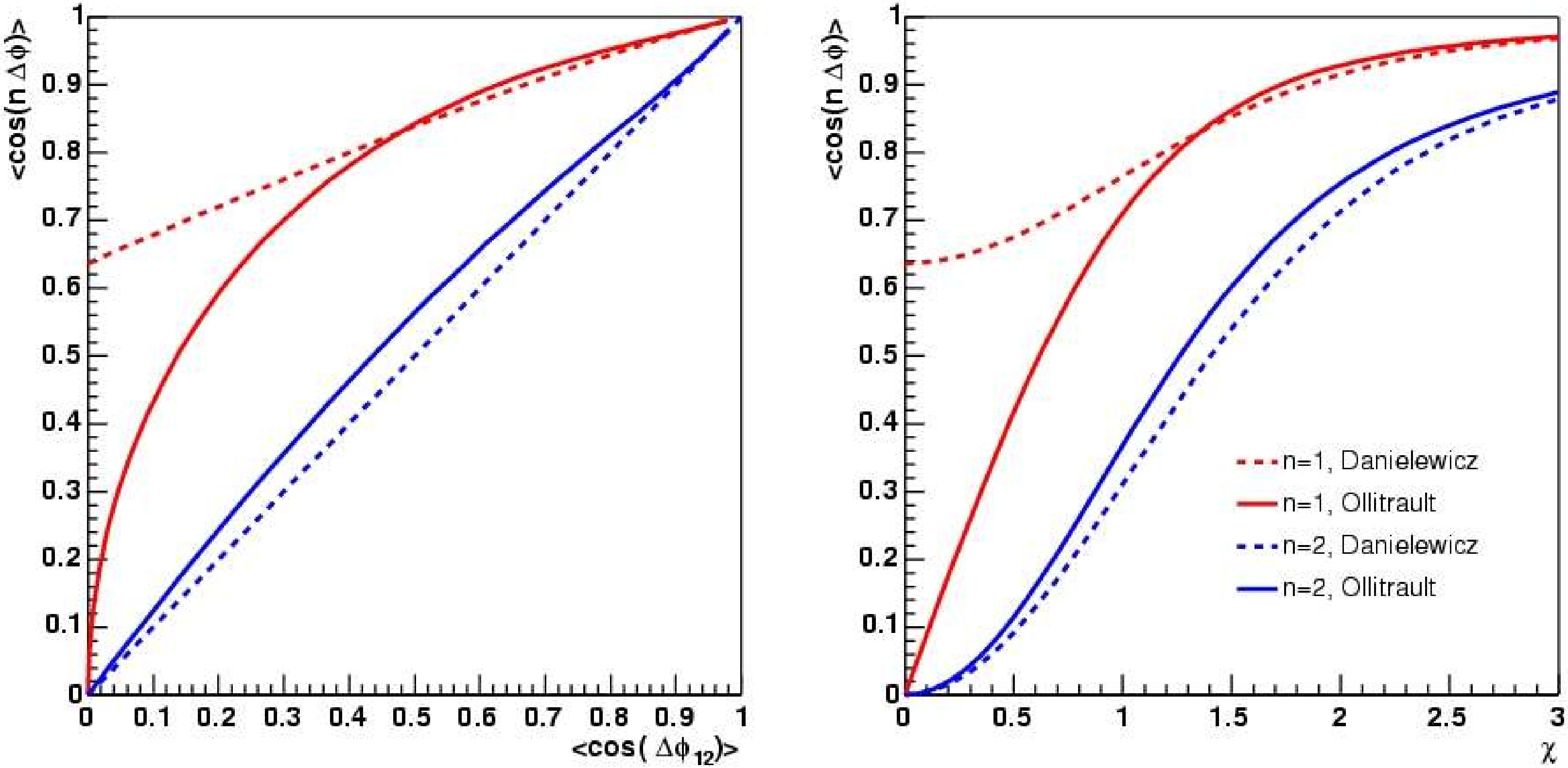
by fitting of:

$$\frac{dN}{d\Delta\phi_{12}} = \frac{e^{-\chi^2_{sub}}}{2} \left\{ \frac{2}{\pi}(1+\chi^2_{sub}) + z\left[I_0(z) + L_0(z)\right] + \chi^2_{sub}\left[I_1(z) + L_1(z)\right] \right\}$$

where: $z = \chi^2_{sub} \cos \Delta\phi_{12}$, $\chi = \sqrt{2}\, \chi_{sub}$
to the measured angular distribution of $\Delta\phi_{12}$ between the Q-vectors for **random** sub-events

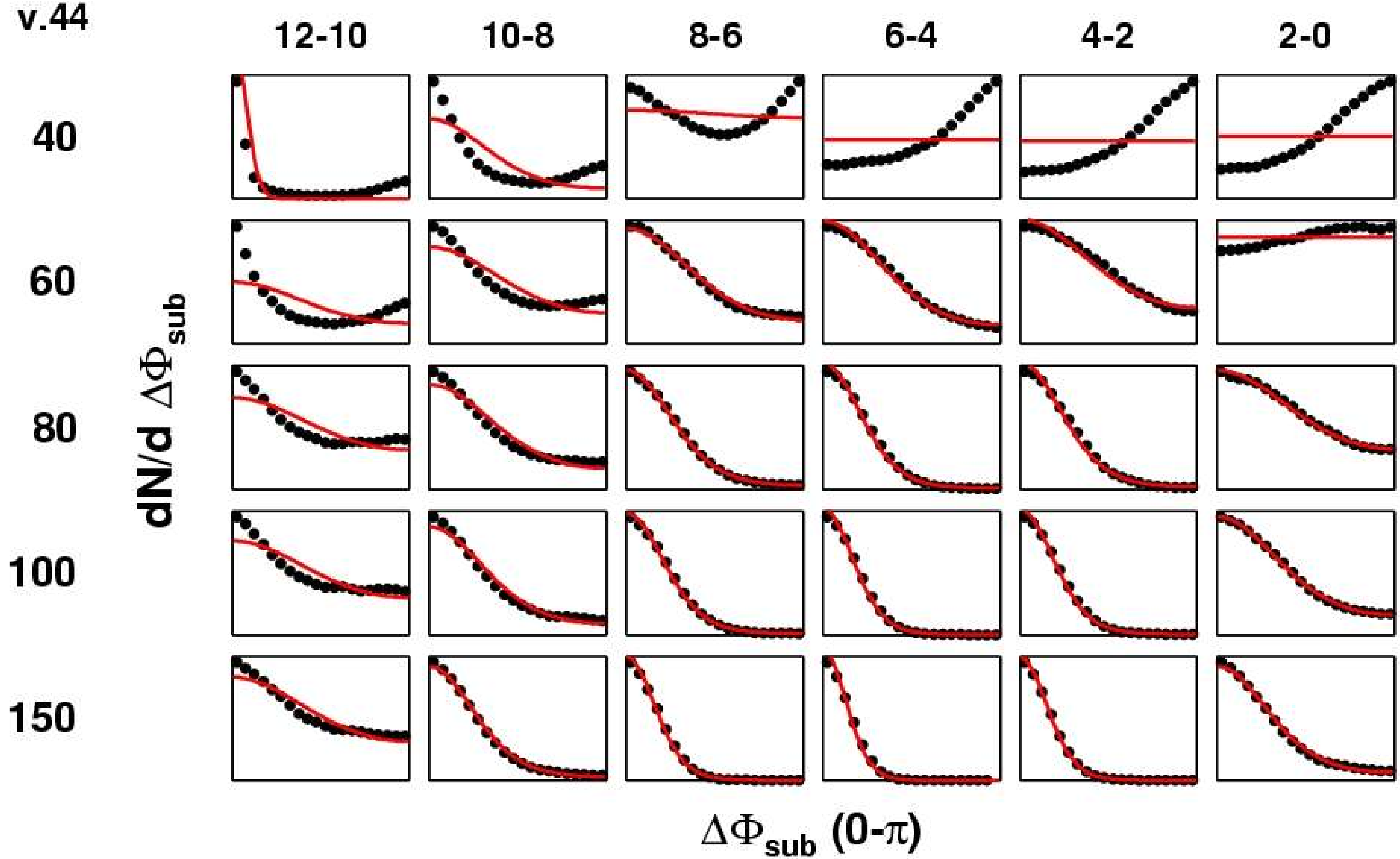
Danielewicz vs Ollitrault

Flow, reaction plane and corrections - standard methods



Ollitrault method (1-par fit, $\alpha = 1$, $\rho = 0$)

tests using CHIMERA-QMD



Further extension: accounting for correlations and non-isotropy

When **elliptic flow** and **correlations** between subevents are important:

$$\frac{d^4N}{d\vec{Q}_1 d\vec{Q}_2} = \frac{1}{\pi^2 \sigma^2_{xsub} \sigma^2_{ysub} (1-\rho^2)}$$

$$\exp\left(-\frac{(Q_{1x}-Q_{0sub})^2 + (Q_{2x}-Q_{0sub})^2 - 2\rho(Q_{1x}-Q_{0sub})(Q_{2x}-Q_{0sub})}{\sigma^2_{xsub}(1-\rho^2)}\right)$$

$$\exp\left(-\frac{Q_{1y}^2 + Q_{2y}^2 - 2\rho Q_{1y}Q_{2y}}{\sigma^2_{ysub}(1-\rho^2)}\right)$$

where the

correlation coefficient $\boxed{\rho = \frac{\langle \vec{Q}_1 \cdot \vec{Q}_2 \rangle - \langle \vec{Q}_1 \rangle \cdot \langle \vec{Q}_2 \rangle}{[(\langle \vec{Q}_1^2 \rangle - \langle \vec{Q}_1 \rangle^2)(\langle \vec{Q}_2^2 \rangle - \langle \vec{Q}_2 \rangle^2]^{1/2}}}$ $\in [-1, 1]$ and, besides the

resolution parameter $\boxed{\chi_{sub} \equiv \frac{Q_{0sub}}{\sigma_{xsub}}}$ one can introduce additional parameter

aspect ratio: $\boxed{\alpha \equiv \frac{\sigma_{xsub}}{\sigma_{ysub}}}$ since $\sigma_{xsub} \neq \sigma_{ysub}$ (elliptic flow)

$\chi = \chi_{sub}\sqrt{\frac{2}{1+\rho}}$

$$\frac{dN}{d\Delta\phi_{12}} = f(\chi_{sub}, \alpha, \rho)$$

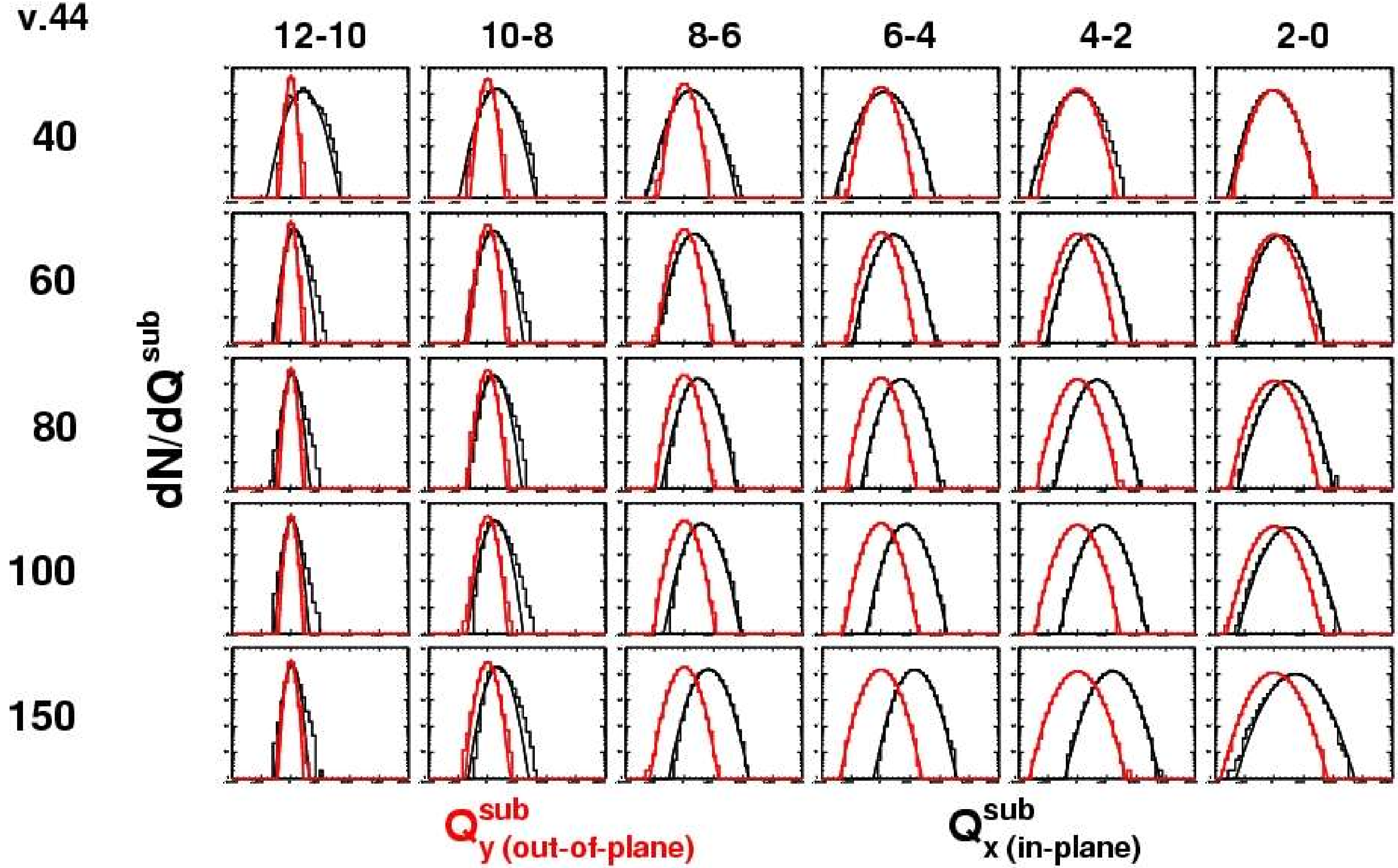
Testing the assumptions on the model

tests using CHIMERA-QMD

- ✔ Gaussian assumption for the distribution of sub-events
- ✔ Possible non-isotropy of sub-events
- ✔ Correlation between sub-events (and its nature)

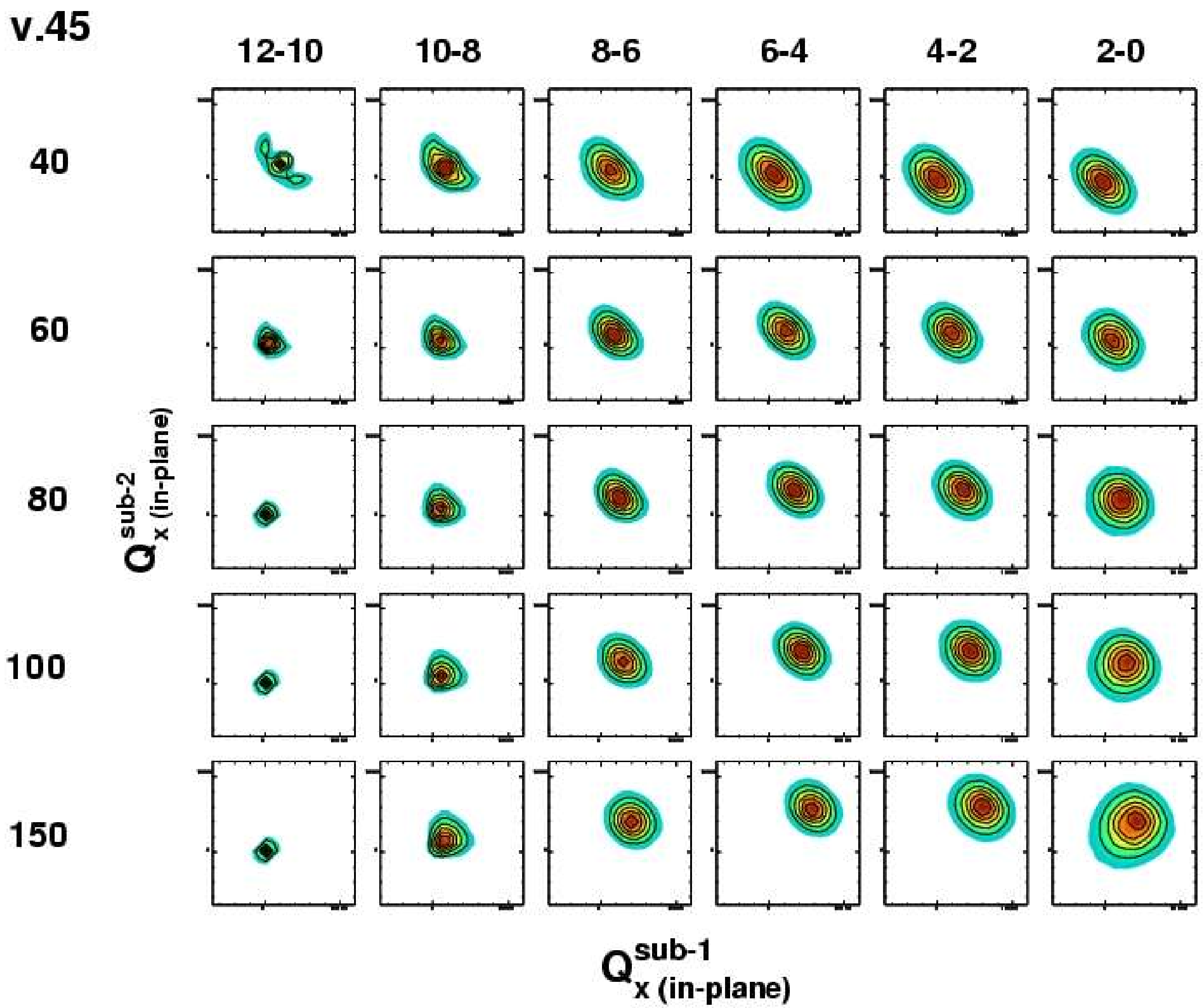
Projections of distributions of the sub-Q-vectors + Gaussian fits

tests using CHIMERA-QMD



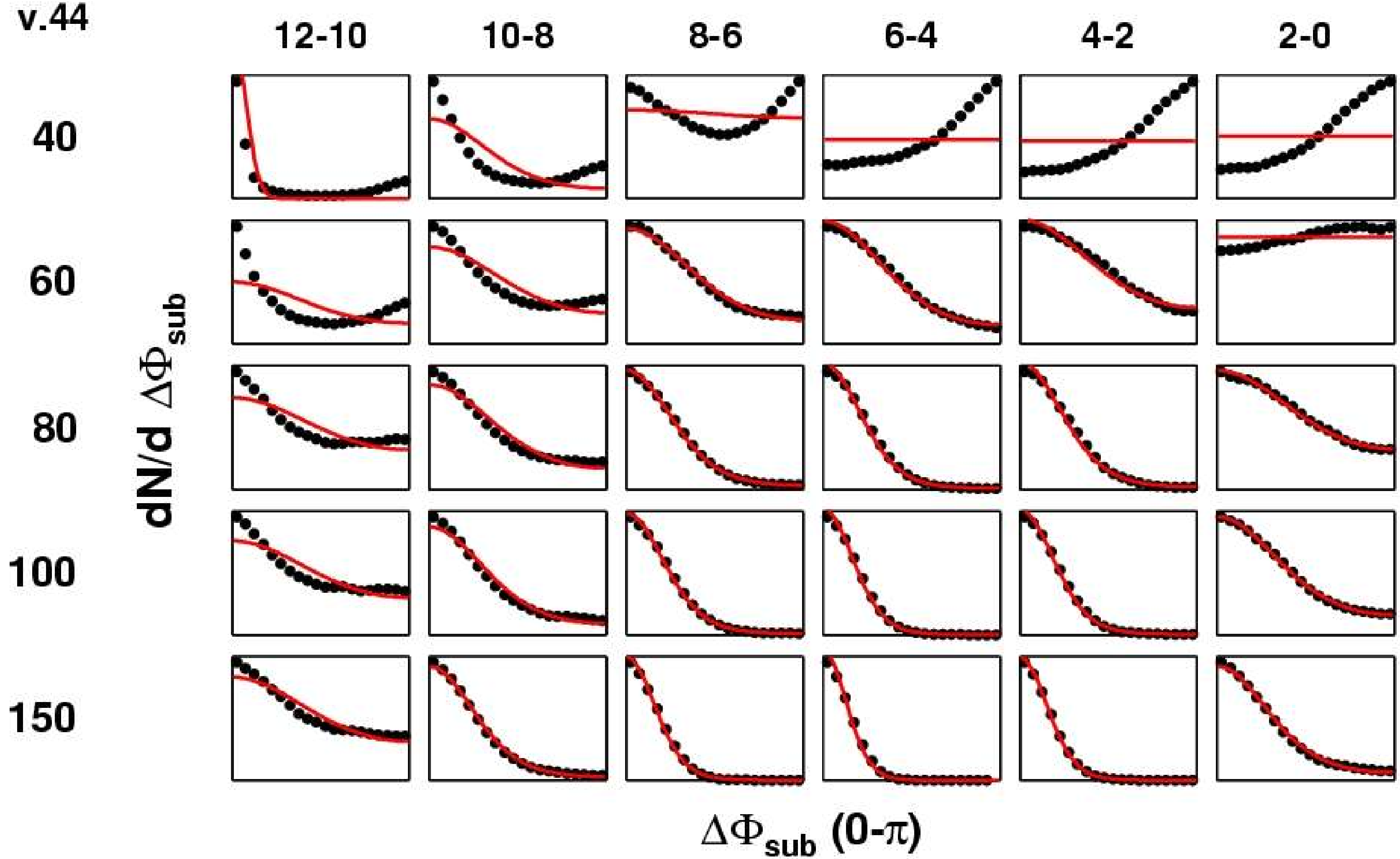
Correlation between the sub-Q-vectors (in-plane components)

tests using CHIMERA-QMD



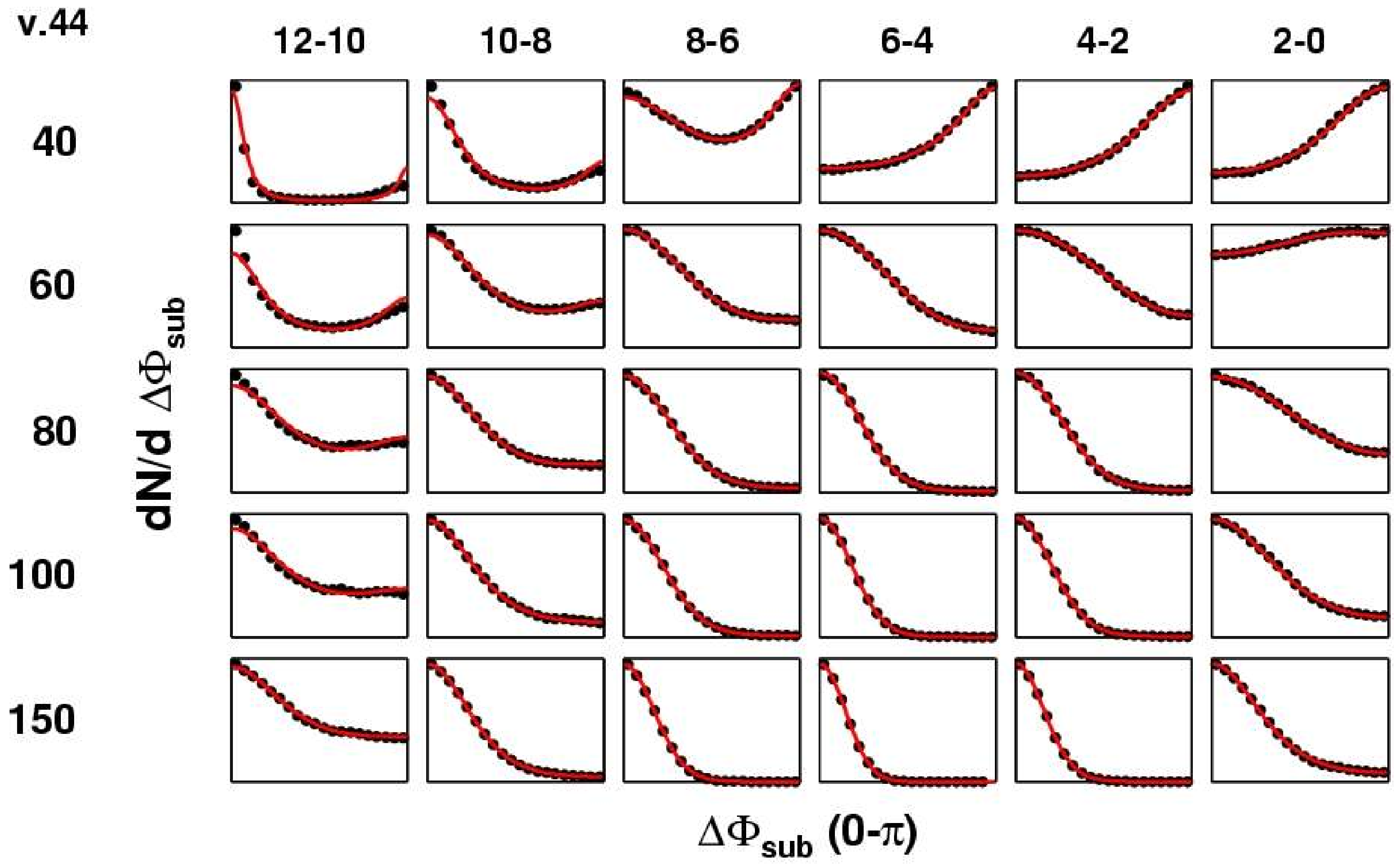
Ollitrault method (1-par fit, $\alpha = 1$, $\rho = 0$)

tests using CHIMERA-QMD



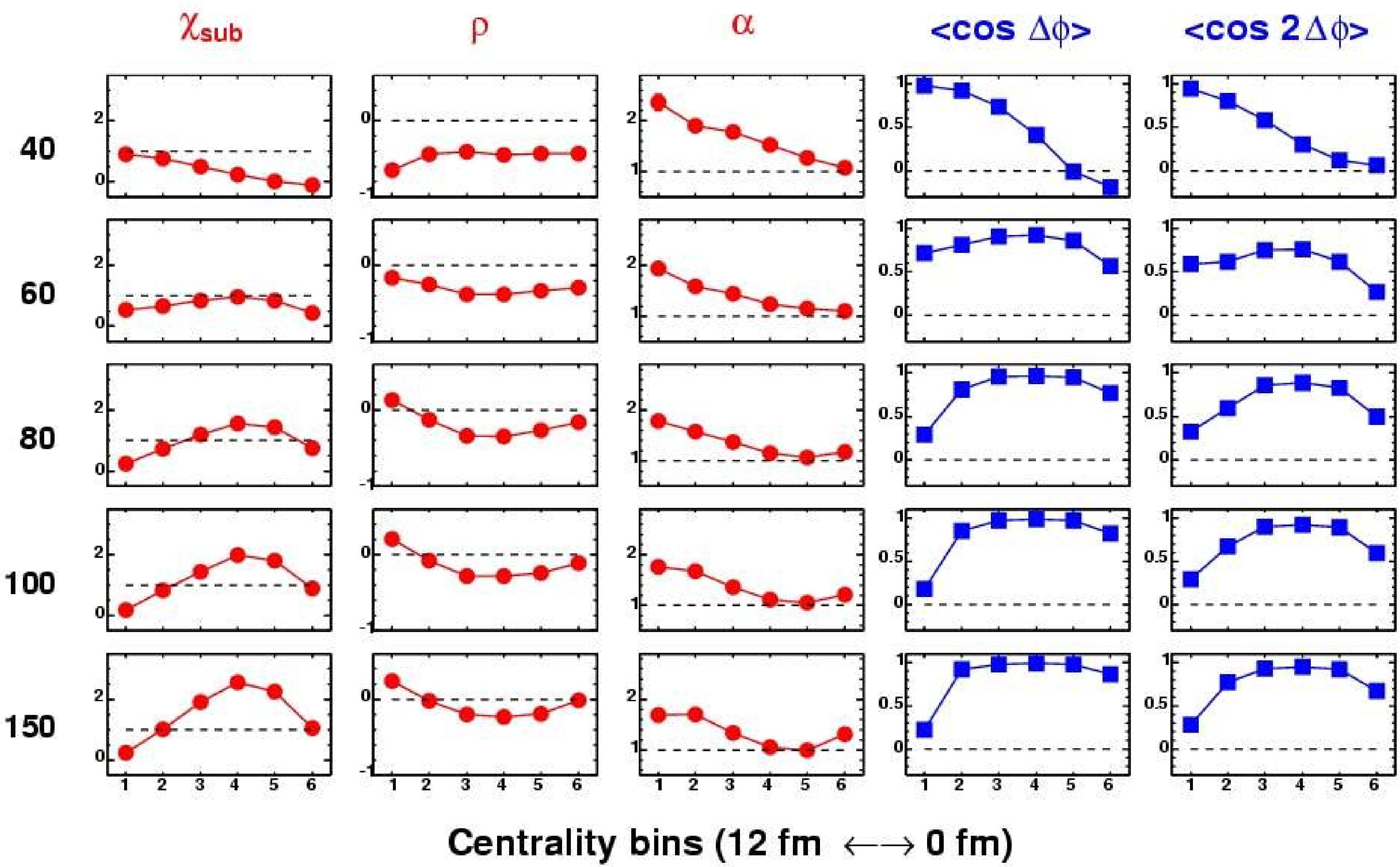
New method (3-par fit)

tests using CHIMERA-QMD



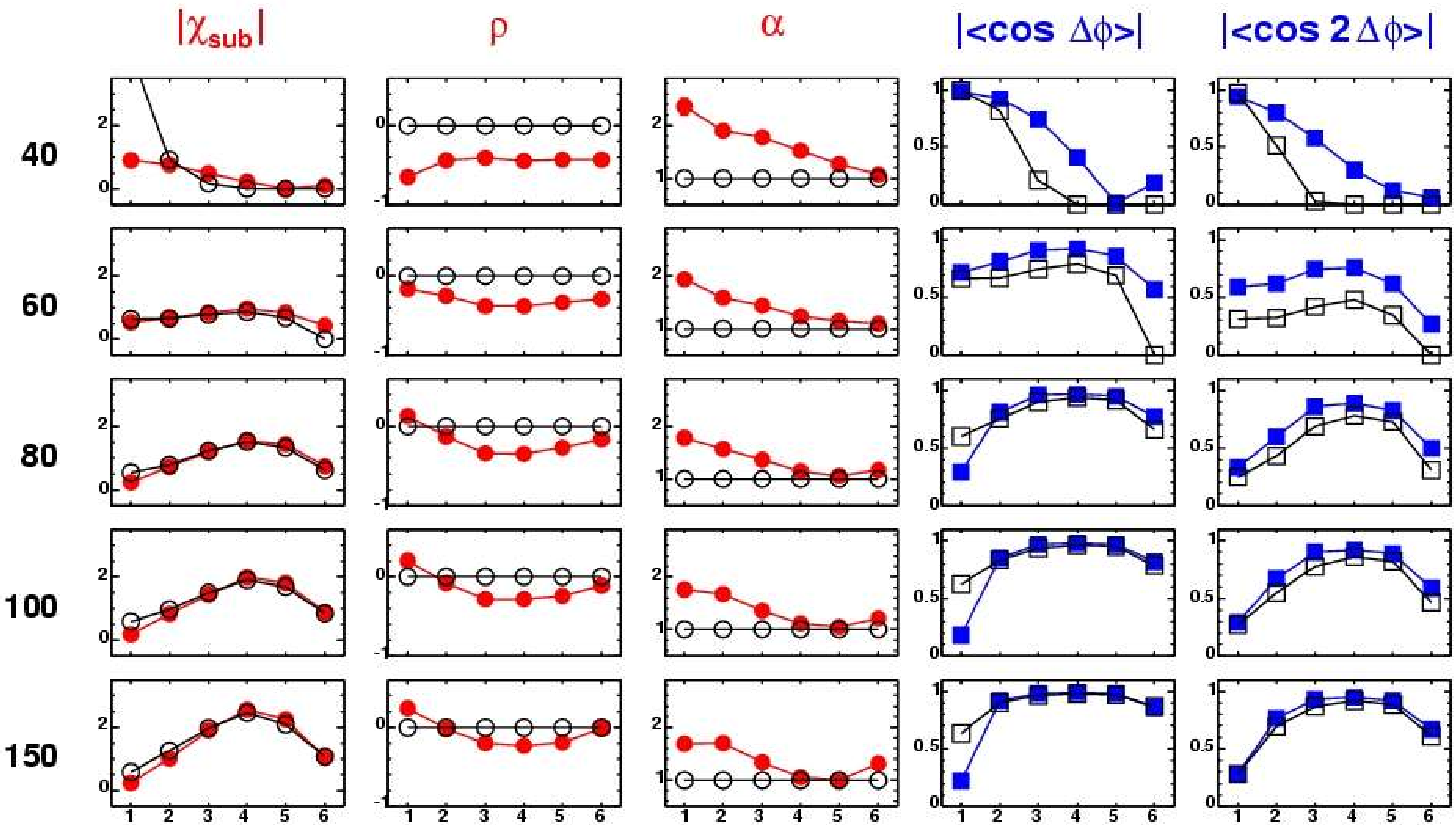
'True' parameters and corrections for v_1 and v_2

tests using CHIMERA-QMD



True parameters and corrections + fits using Ollitrault method (open symbols)

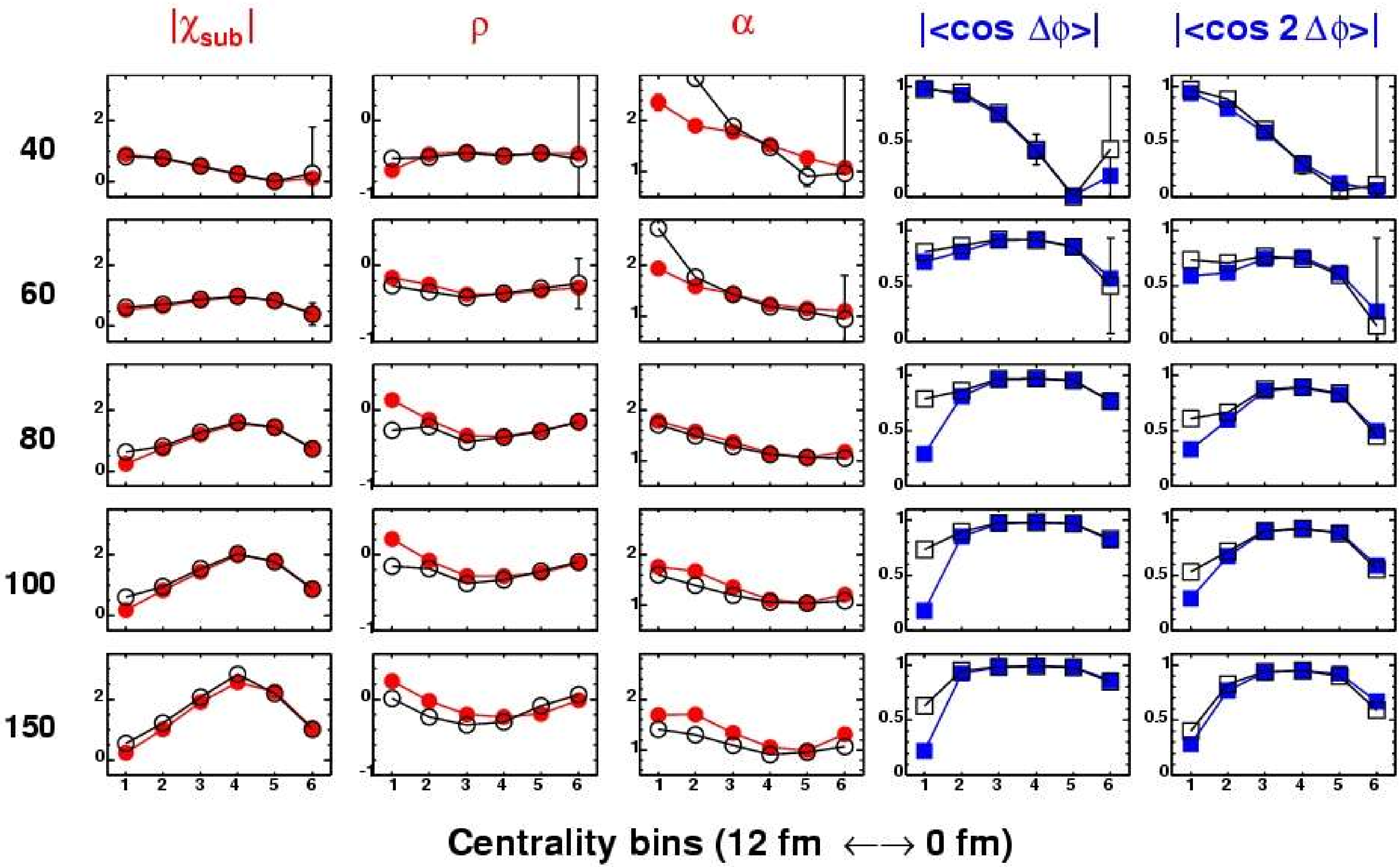
tests using CHIMERA-QMD



Centrality bins (12 fm ⟷ 0 fm)

True parameters and corrections + fits using new method ($\Delta\phi_{sub}$)

tests using CHIMERA-QMD



correction for loses due to multi-hits (v_2 case)

$$\frac{dM}{d(\phi-\phi_R)} = \frac{M_0}{2\pi}\left(1 + 2\langle v_1\rangle \cos(\phi-\phi_R) + 2\langle v_2\rangle \cos 2(\phi-\phi_R)\right)$$

$$M_{in} = \left(\int_0^{1/4\pi} + \int_{3/4\pi}^{5/4\pi} + \int_{7/4\pi}^{2\pi}\right) \frac{dM}{d(\phi-\phi_R)} d\phi = M_0 \frac{\pi + 4\langle v_2\rangle}{2\pi}$$

$$M_{out} = \left(\int_{1/4\pi}^{3/4\pi} + \int_{5/4\pi}^{7/4\pi}\right) \frac{dM}{d(\phi-\phi_R)} d\phi = M_0 \frac{\pi - 4\langle v_2\rangle}{2\pi}$$

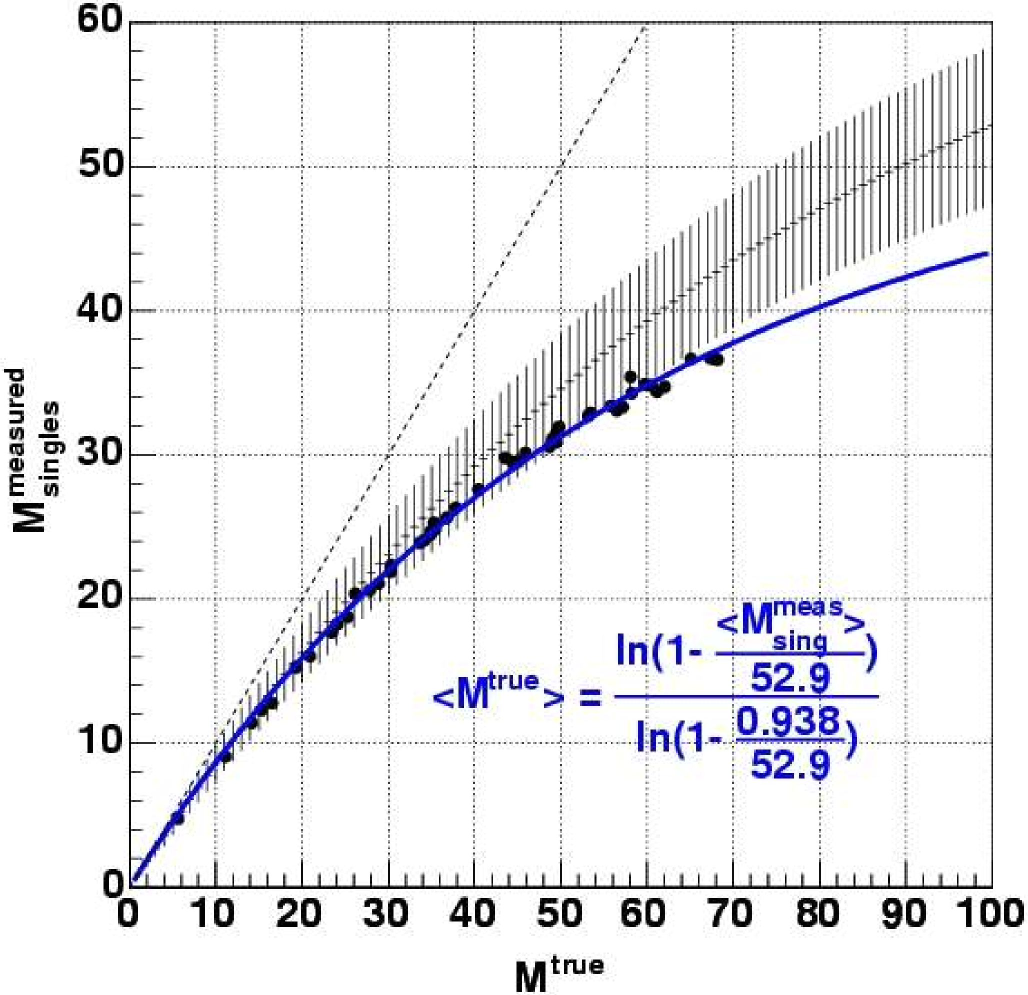
$$\langle v_2\rangle = \frac{\pi}{4}\frac{M_{in} - M_{out}}{M_{in} + M_{out}}$$

$$\Delta v_2 = \frac{\pi}{2}\frac{M_{out}^{meas} M_{in}^{true} - M_{in}^{meas} M_{out}^{true}}{(M_{in}^{meas} + M_{out}^{meas})(M_{in}^{true} + M_{out}^{true})}$$

$$\text{corr}_{fact} = \frac{\langle v_2^{true}\rangle}{\langle v_2^{meas}\rangle} = \frac{M_{in}^{true} - M_{out}^{true}}{M_{in}^{true} + M_{out}^{true}} \frac{M_{in}^{meas} + M_{out}^{meas}}{M_{in}^{meas} - M_{out}^{meas}}$$

how to get M_{in}^{true}, M_{out}^{true} ?

Measured MULT of single-hits vs "true" MULT in 1/2 of INDRA



how to exclude the POI?

$$\vec{Q} = \sum_{i=1}^{N} \omega_i \vec{p}_i, \qquad \vec{Q}_k = \sum_{i \neq k}^{N} \omega_i \vec{p}_i$$

$$\vec{Q}_k = \vec{Q} - \omega_k \vec{p}_k$$

assuming:

$$\frac{d^4 N}{d\vec{p}_k d\vec{Q}} \propto \frac{d^2 N}{d\vec{p}_k} \frac{d^2 N}{d\vec{Q}} \; e^{-\frac{1}{1-\rho_k^2}\left(\frac{(\vec{p}_k - \langle \vec{p}_k \rangle)^2}{\sigma_k^2} + \frac{(\vec{Q} - \langle \vec{Q} \rangle)^2}{\sigma_Q^2} - 2\rho_k \frac{(\vec{p}_k - \langle \vec{p}_k \rangle)(\vec{Q} - \langle \vec{Q} \rangle)}{\sigma_k \sigma_Q}\right)}$$

$$\boxed{\vec{Q}_k = \vec{Q} - \xi \cdot \vec{p}_k}$$

and require vanishing of the cross terms (shifting a'la Borghini)

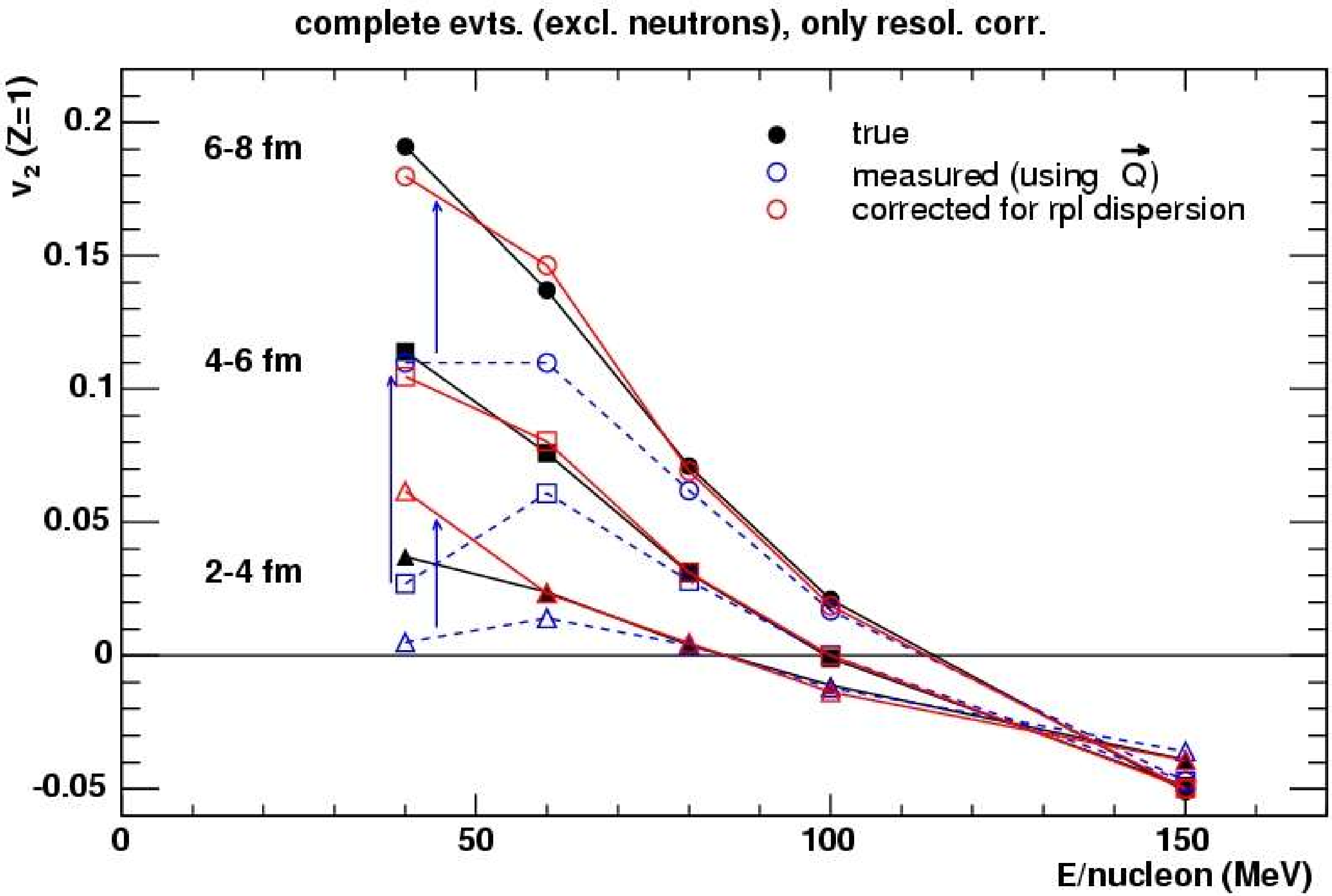
$$\xi(p_\perp, y) = \frac{\rho_k \, \sigma_Q}{\sigma_k} = \frac{\langle \vec{p}_k \cdot \vec{Q} \rangle - \langle \vec{p}_k \rangle \cdot \langle \vec{Q} \rangle}{\langle \vec{p}_k^{\,2} \rangle - \langle \vec{p}_k \rangle^2}$$

but depends on the reference frame... (gives good results in the true rpl frame)

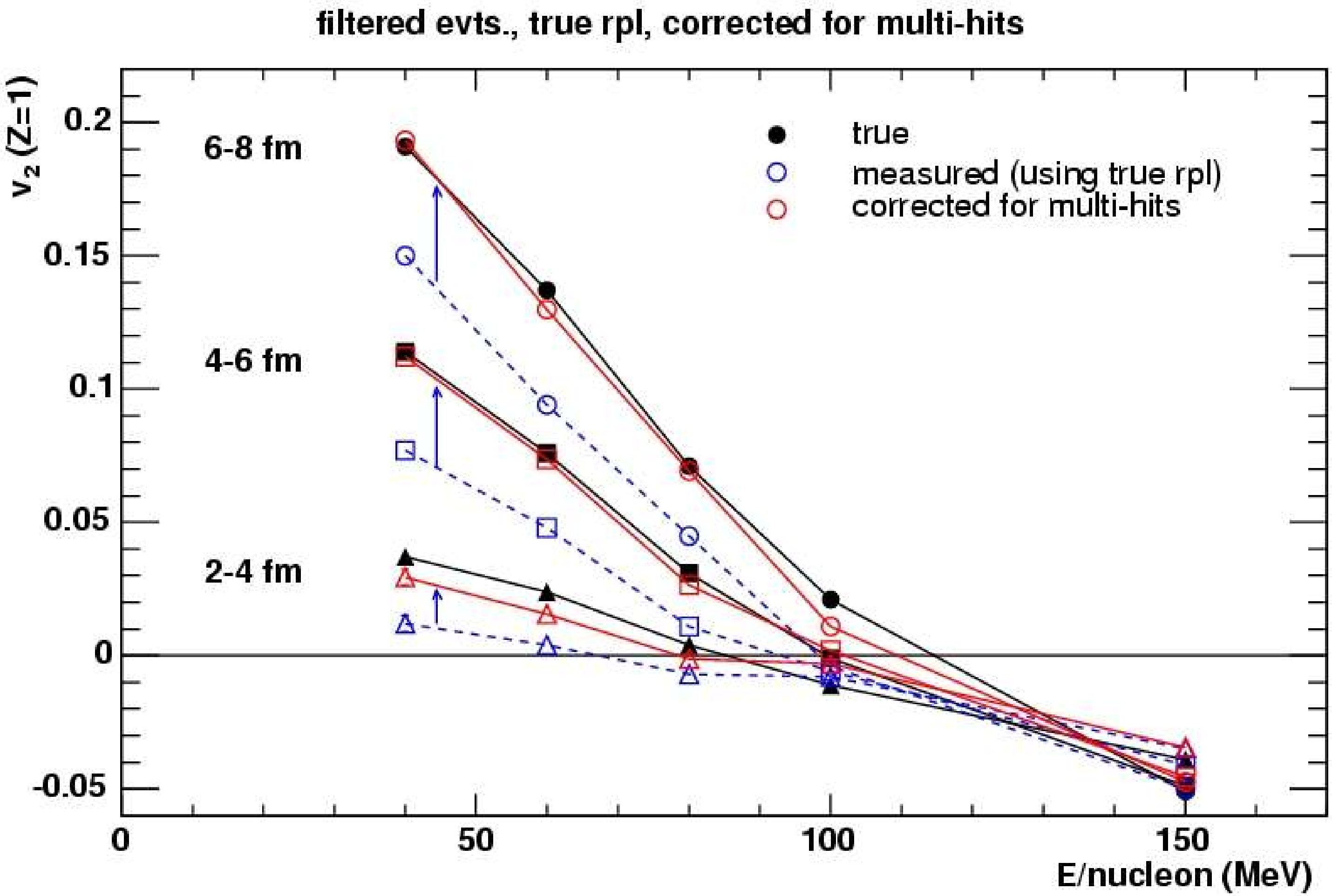
However, for high multiplicities:

$$\boxed{\xi \simeq \omega_k \frac{1+\rho}{1-\rho}}$$

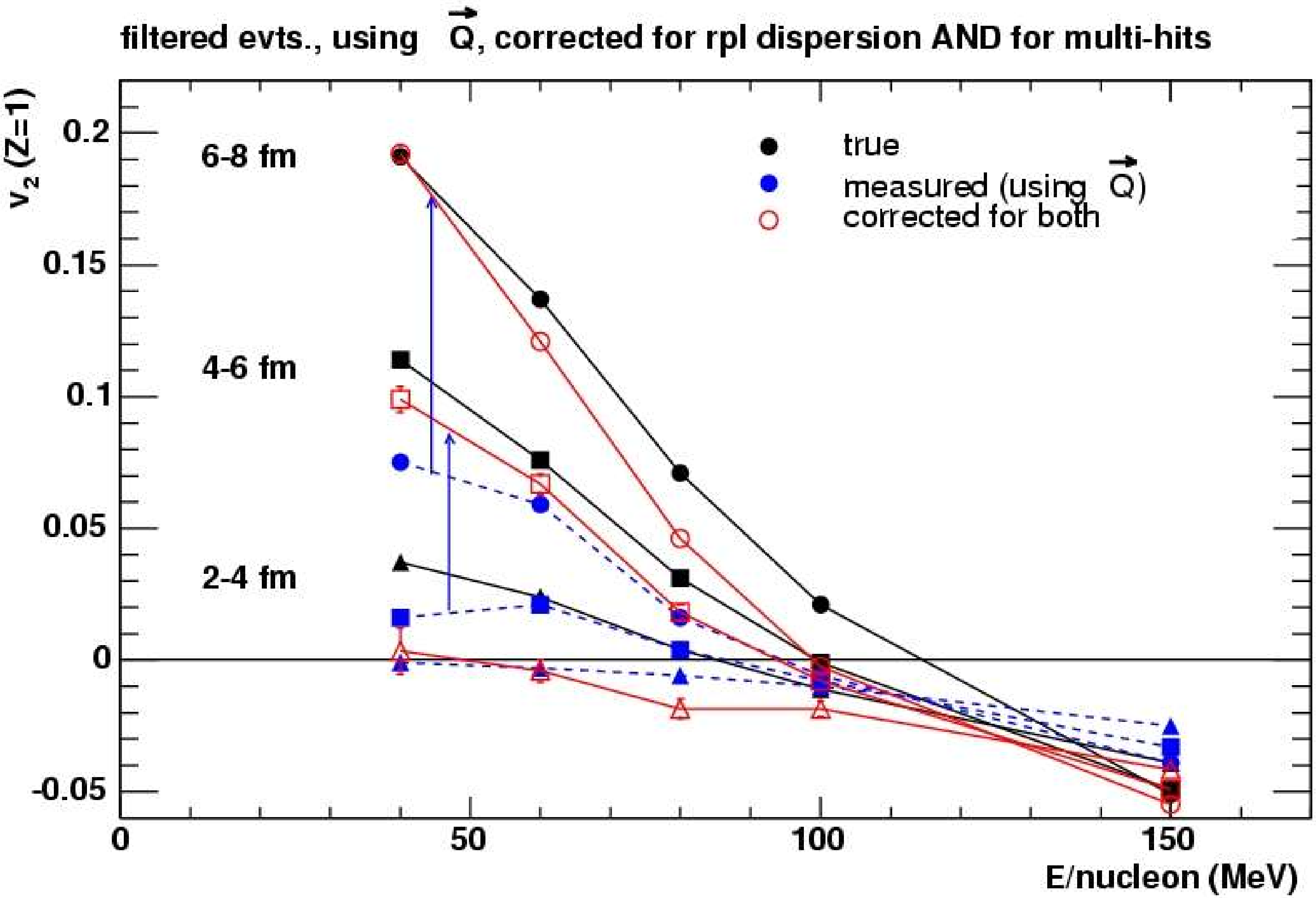
v_2 @ midrapidity, Z=1



v_2 @ midrapidity, Z=1



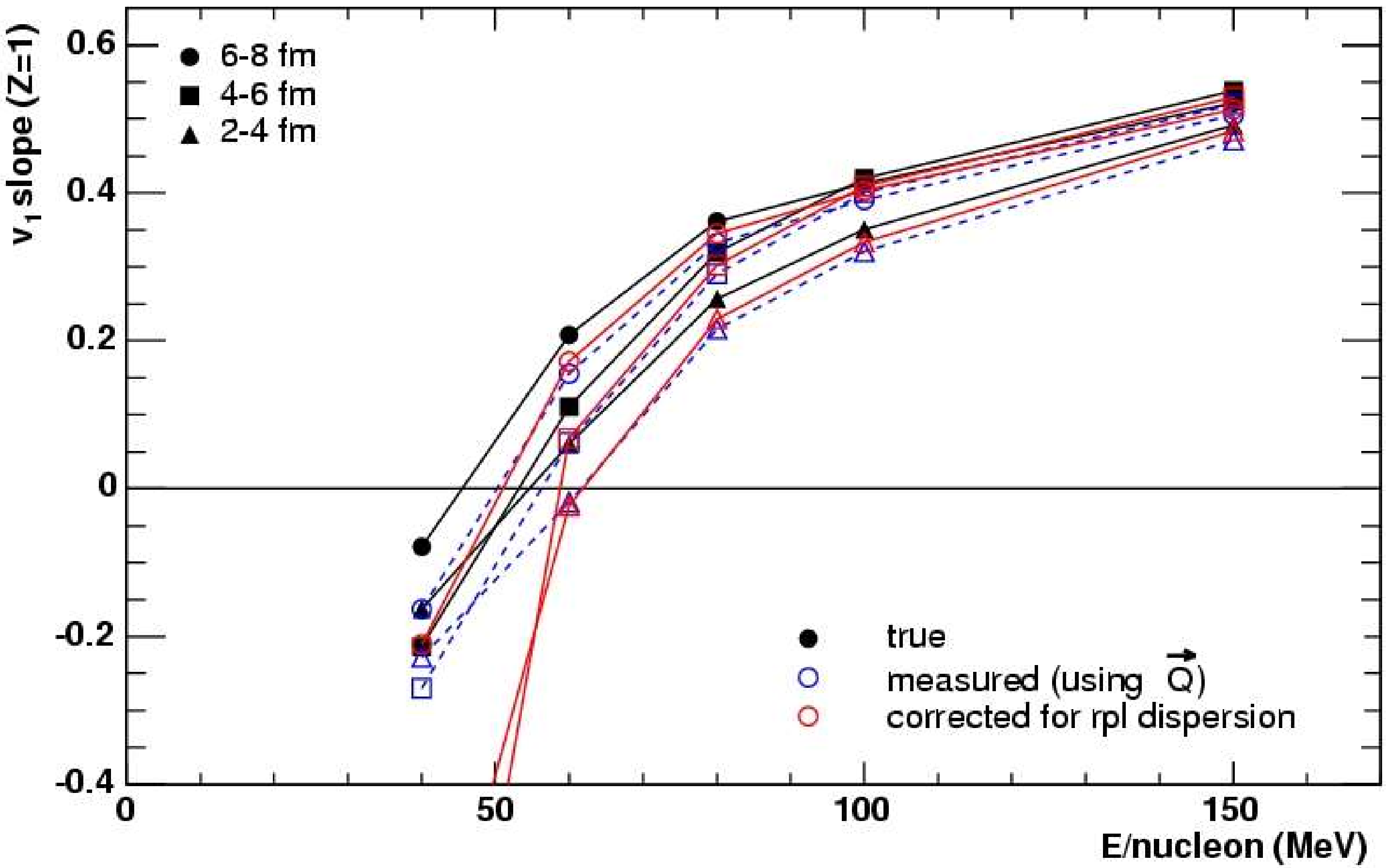
v_2 @ midrapidity, Z=1



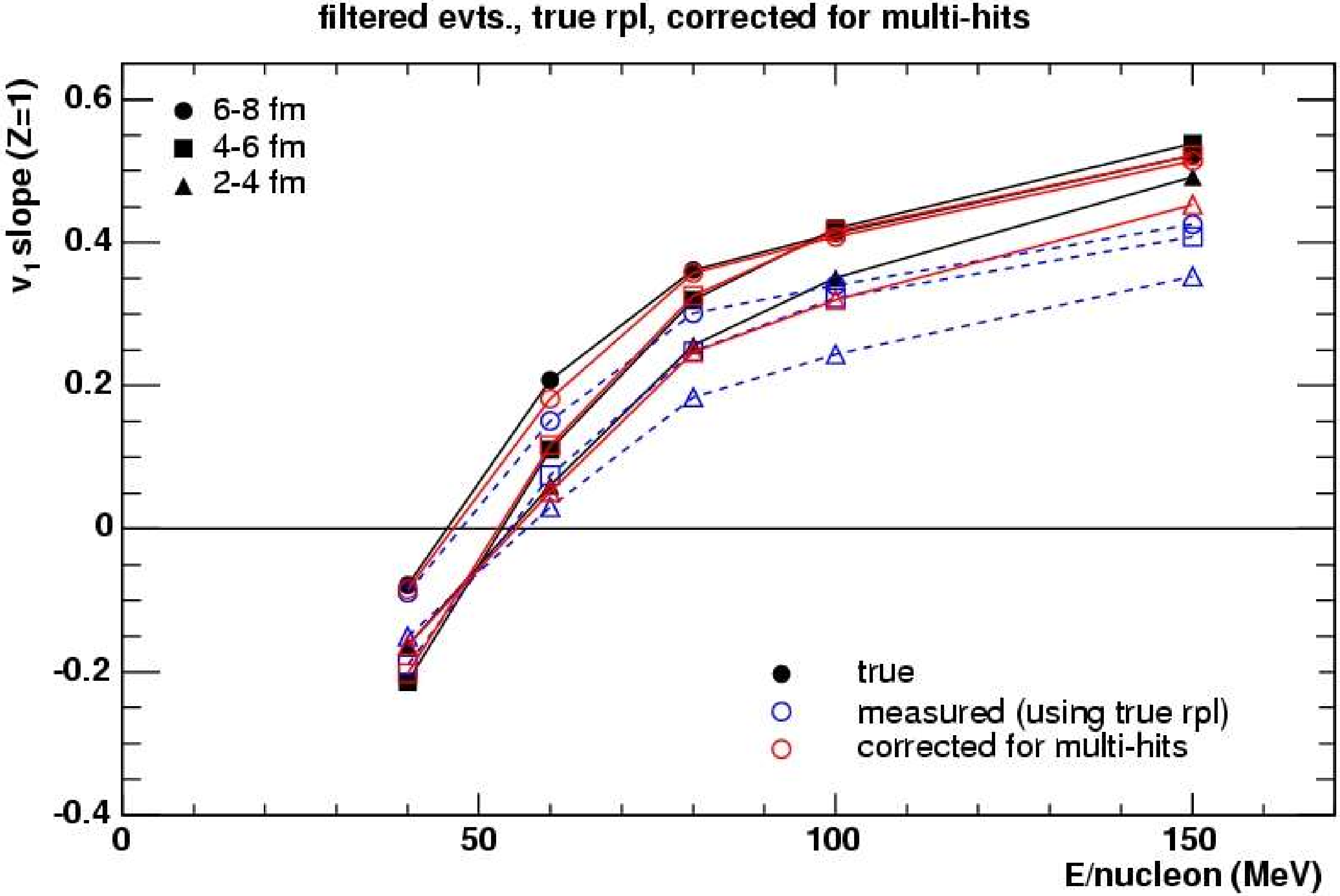
dv_1/dy @ midrapidity, Z=1

PROBLEM: correction for rpl dispersion is not enough → bad $v_2^{measured}$?

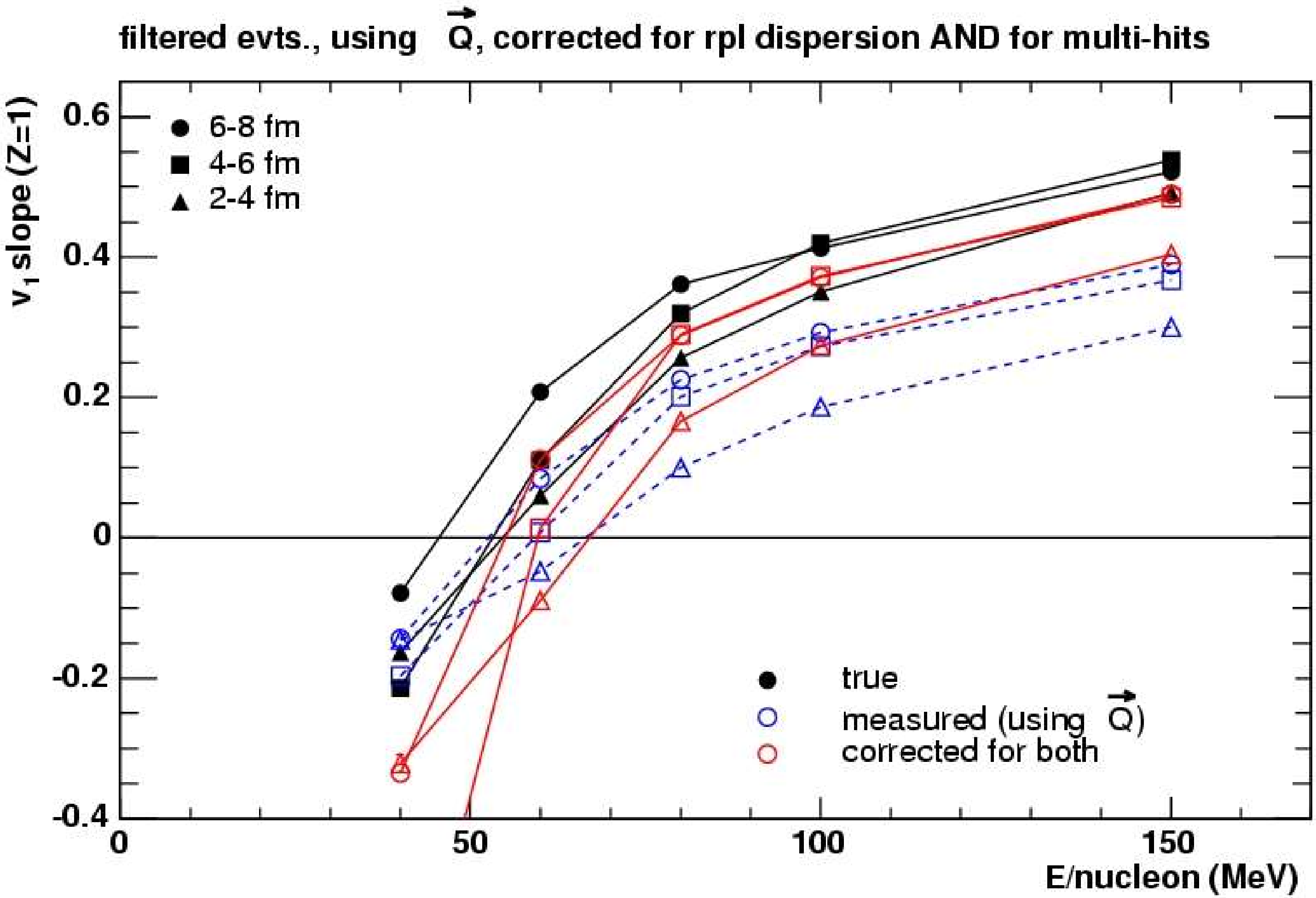
complete evts. (excl. neutrons), only resol. corr. (mom. cons. a'la Borghini (shifting))



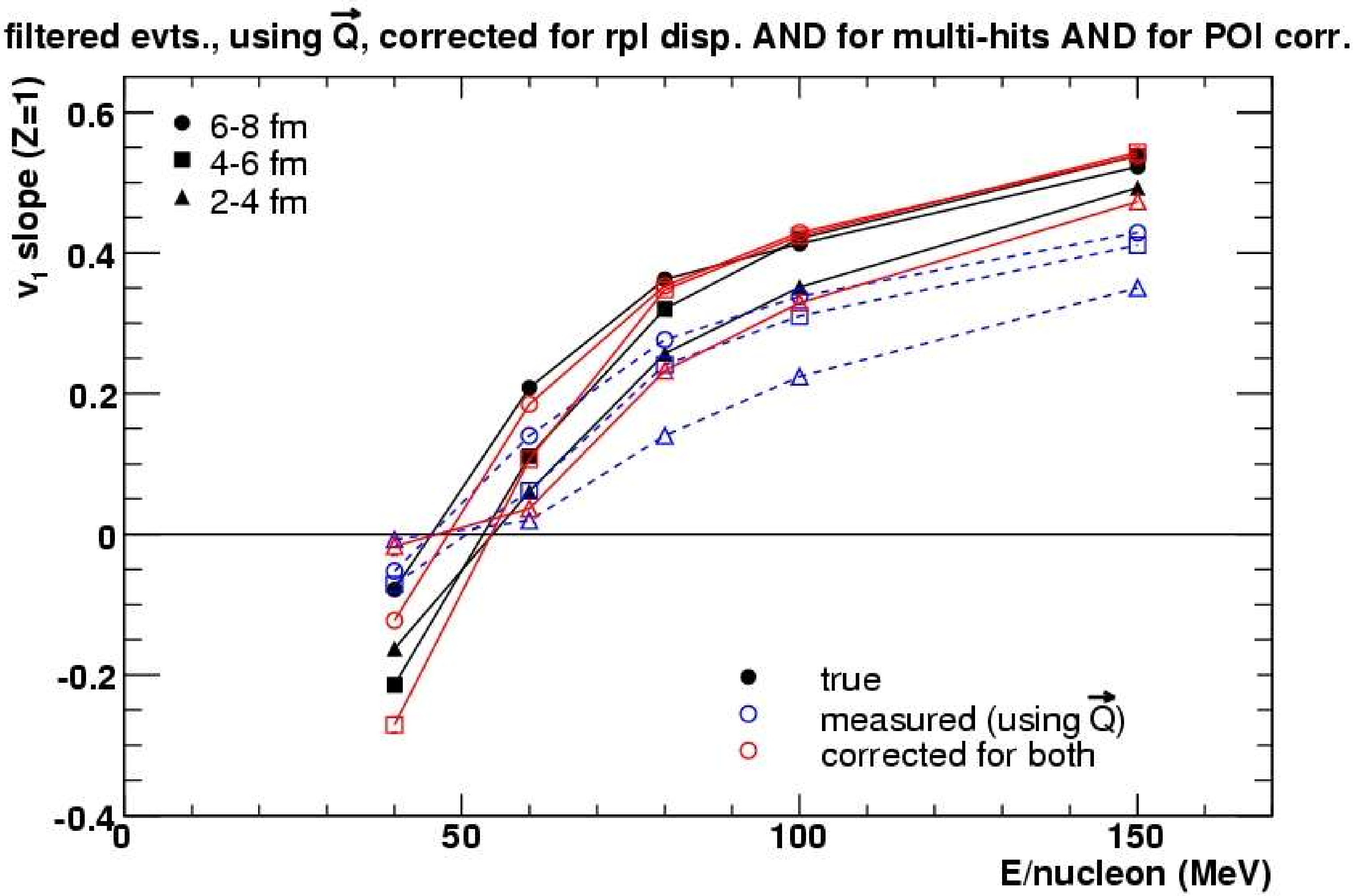
dv_1/dy @ midrapidity, Z=1
multi-hit correction



dv_1/dy @ midrapidity, Z=1
PROBLEM: bad $v_1^{measured}$ - exclusion of POI induces anti-correlations?



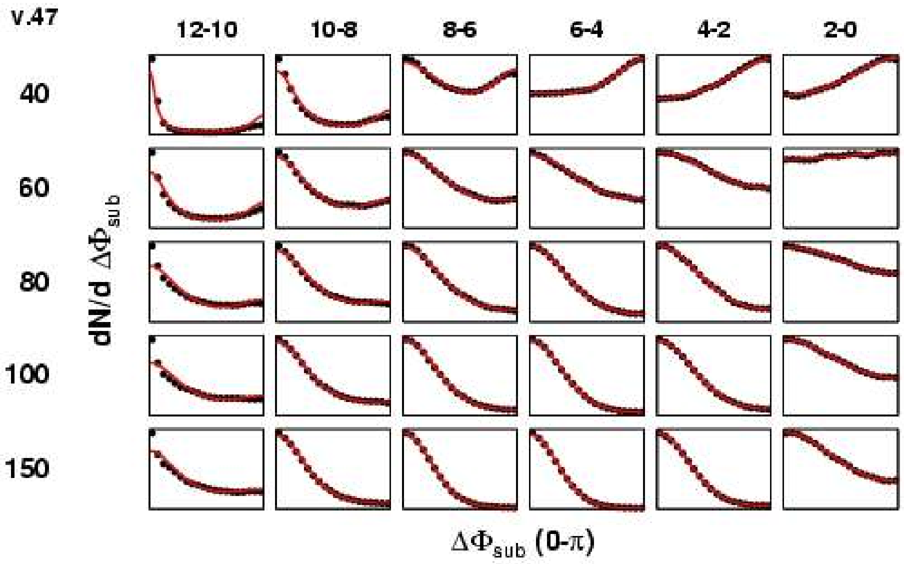
dv_1/dy @ midrapidity, Z=1
triple correction: rpl dispersion + multi-hit correction + POI correction

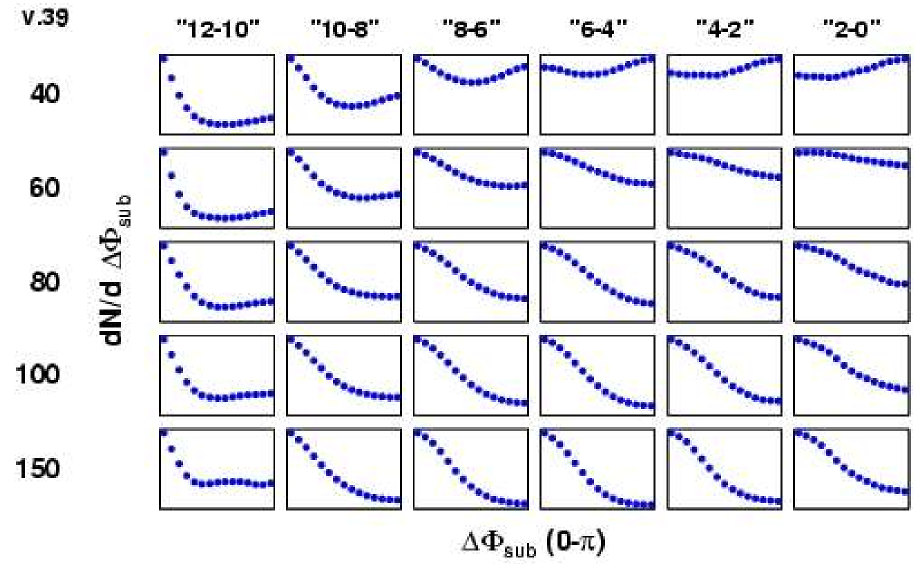


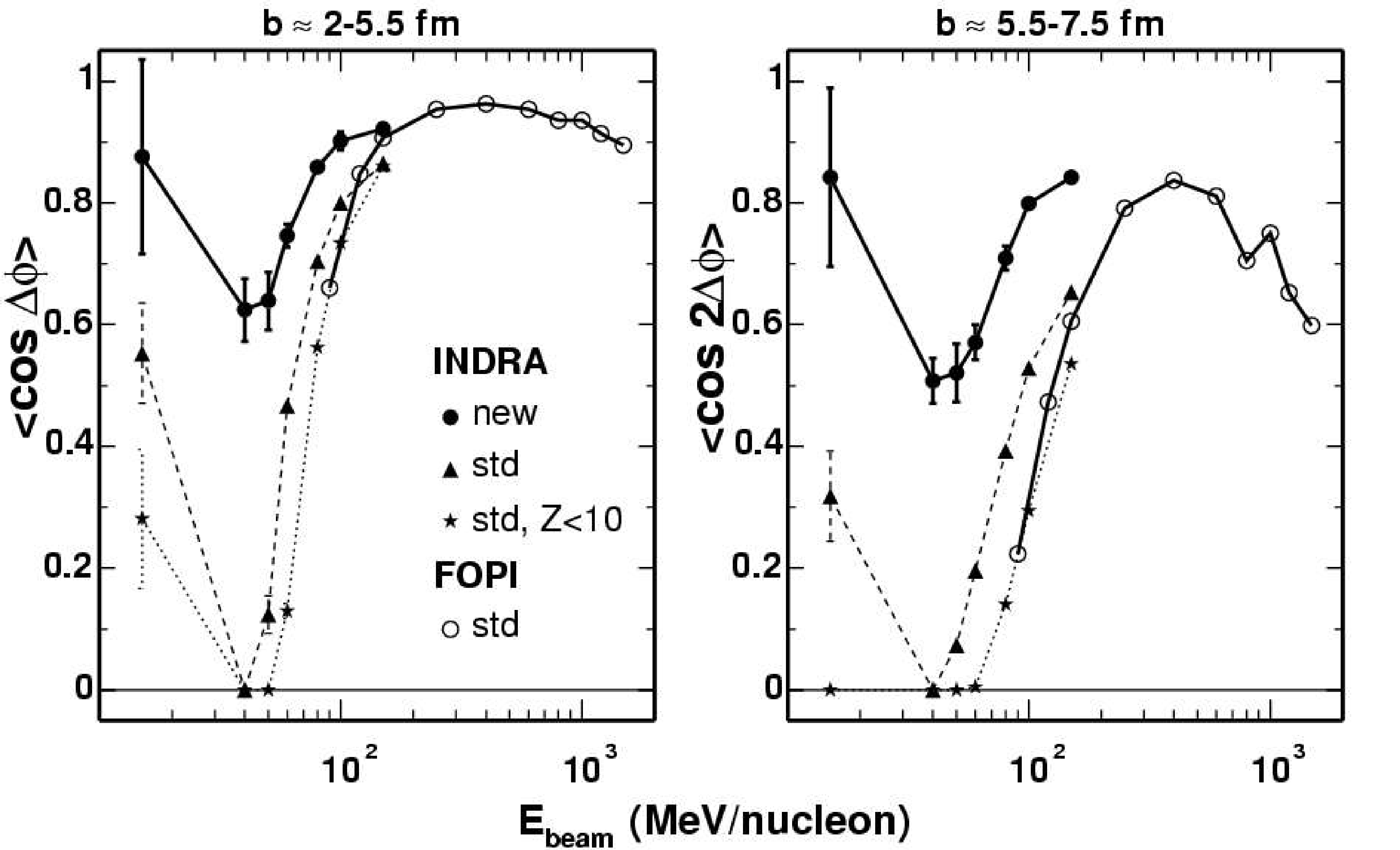
Reality...

CHIMERA-QMD vs INDRA

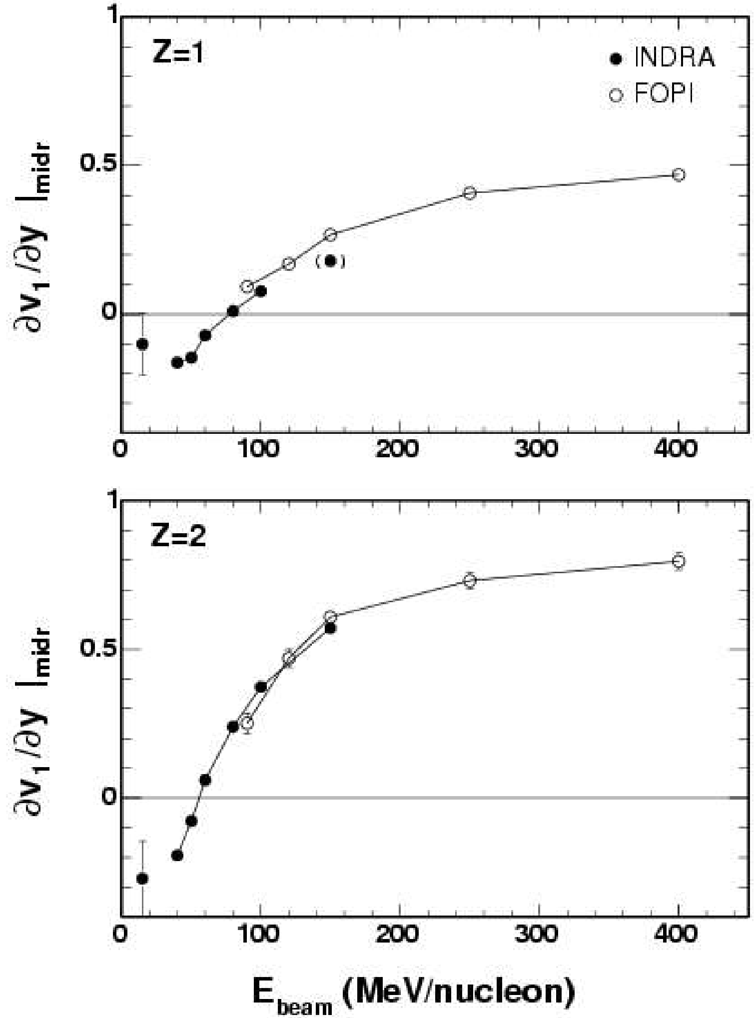
Correcting the experimental results (INDRA)





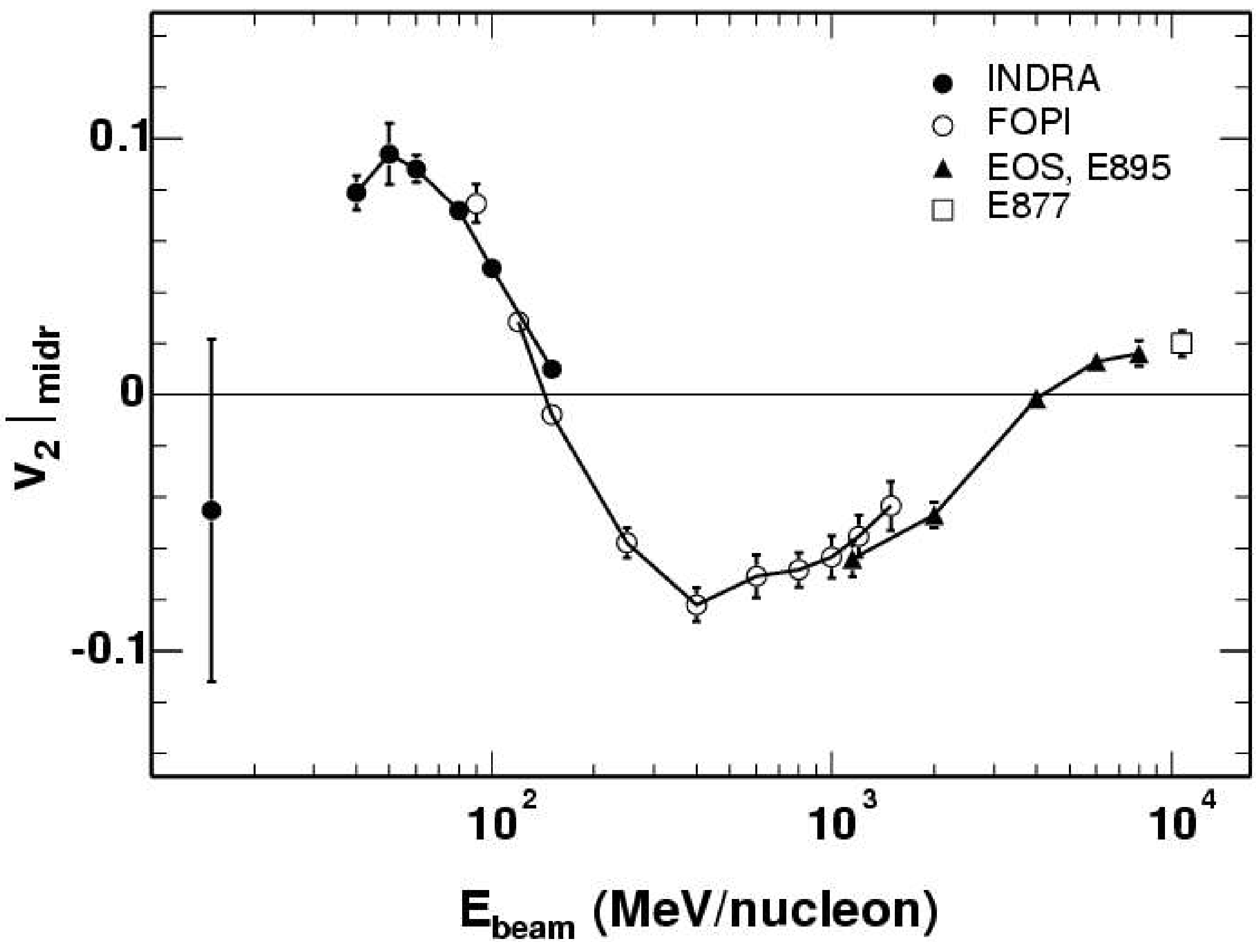

b ≈ 2-5.5 fm
b ≈ 5.5-7.5 fm
<cos Δφ>
<cos 2Δφ>
1
0.8
0.6
0.4
0.2
0
INDRA
new
std
std, Z<10
FOPI
std
10^2
10^3
E_beam (MeV/nucleon)

v_1 slopes at midrapidity, corrected (Au+Au)



FOPI data: A. Andronic *et al.*, Phys. Rev. C **64**, (2001) 041604.

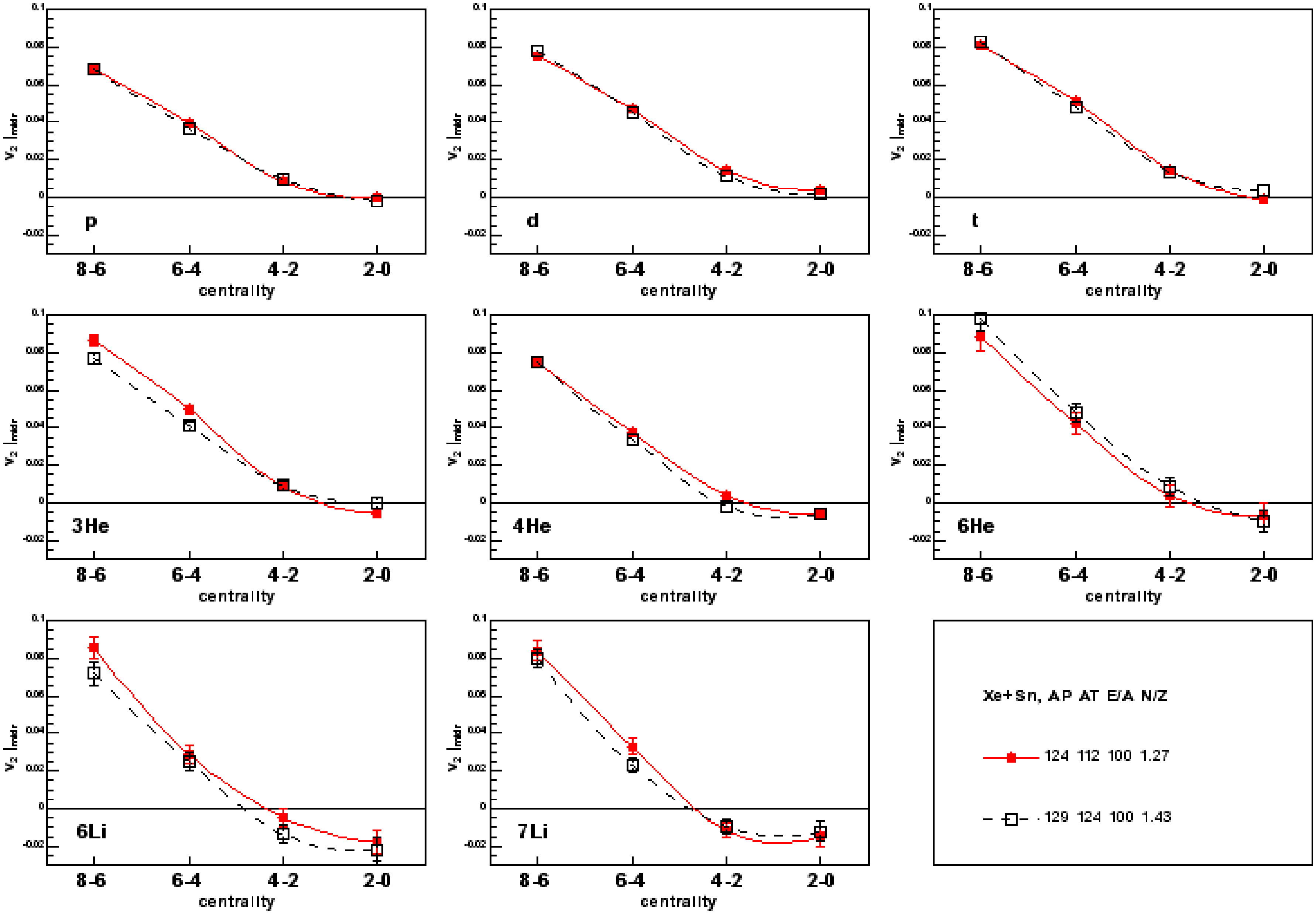
Elliptic flow parameter v_2 for Z=1, corrected (Au+Au)



FOPI data: A. Andronic *et al.*, Phys. Lett. B **612**, (2005) 173.
EOS, E895 data: C. Pinkenburg *et al.*, Phys. Rev. Lett. **83**, (1999) 1295.

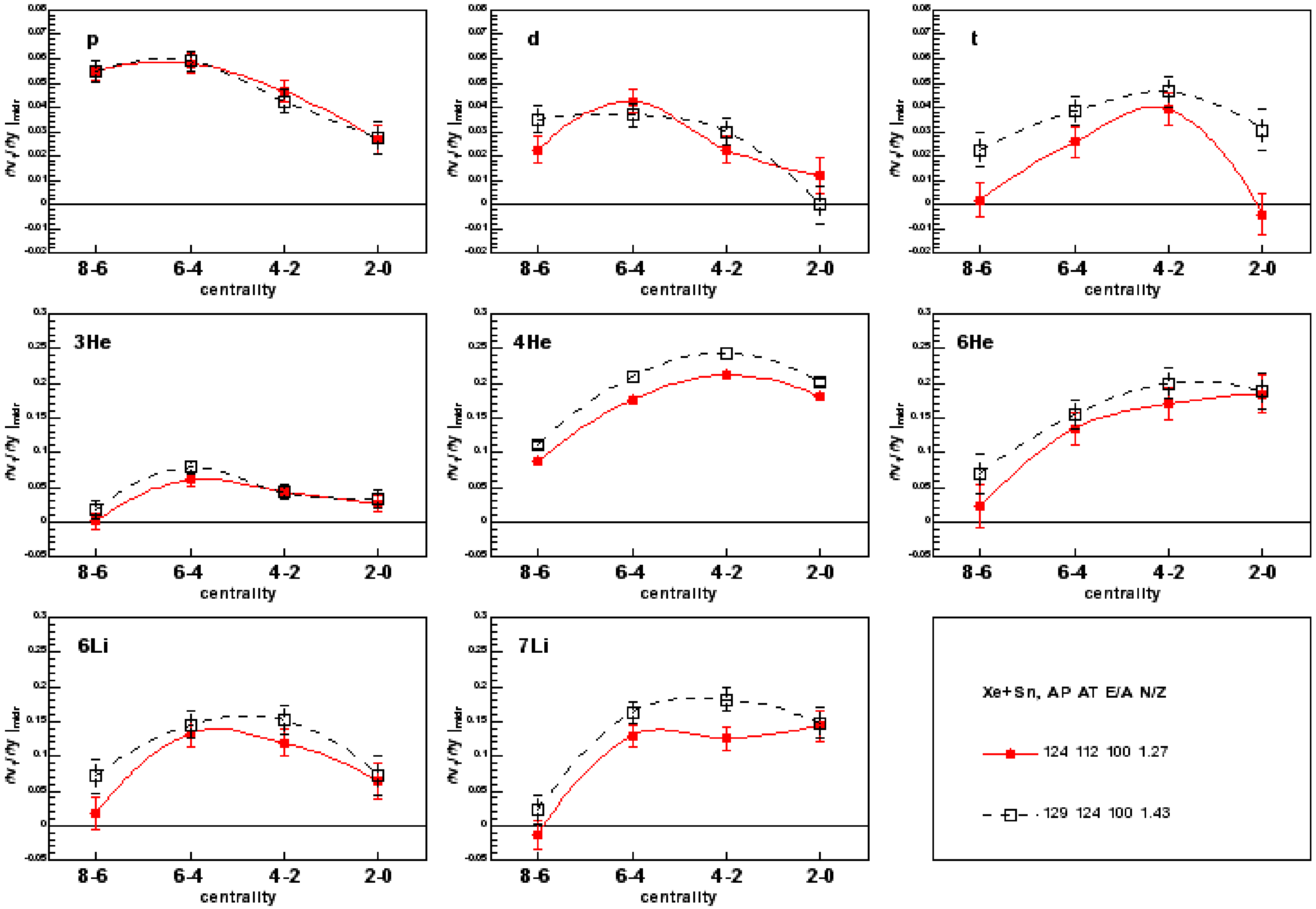
Xe+Sn @ 100 AMeV

v_2 @ midr, Xe+Sn



p
d
t
3He
4He
6He
6Li
7Li
centrality
8-6
6-4
4-2
2-0
$v_2|_{midr}$
Xe+Sn, AP AT E/A N/Z
124 112 100 1.27
129 124 100 1.43

Slope of v_1 @ midr, Xe+Sn



Summary

- Collective flow – a powerful (and demanding) observable
- New method to account for experimental limitations down to the balance energy
- INDRA+FOPI : good agreement
- Symmetry term: requires higher energies