WEIGHTING POTENTIALS

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Principle of Signal induction

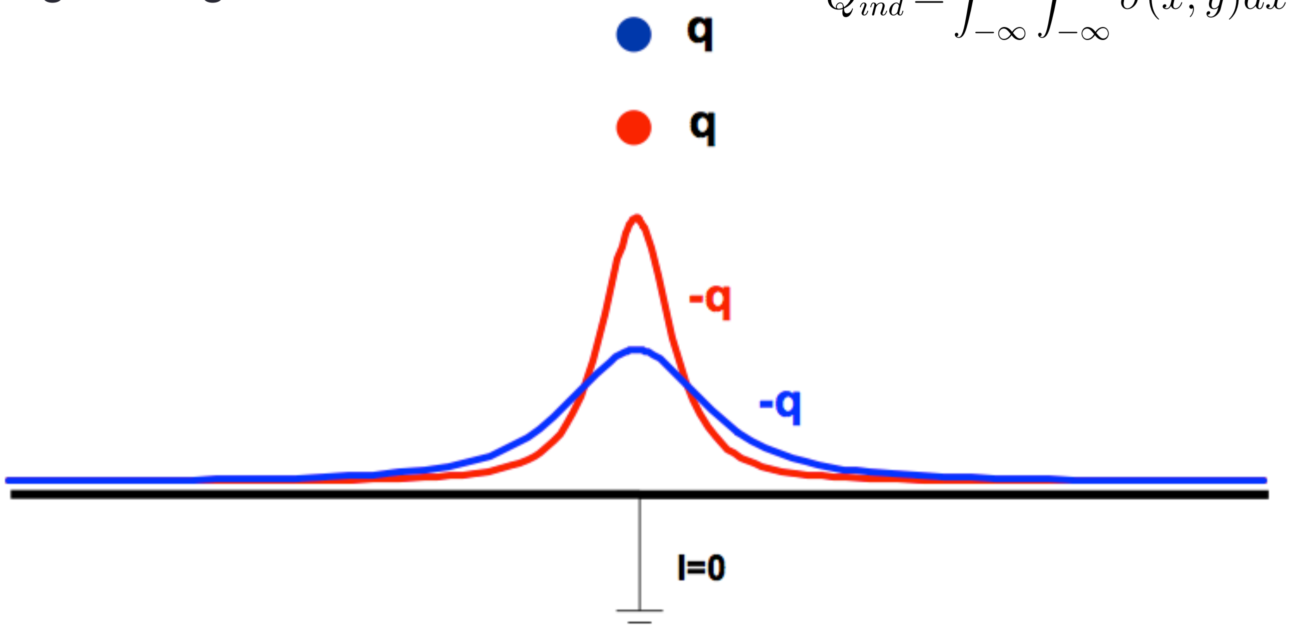
- A point **charge q at distance z_0** above a **grounded metal plate** induces a surface charge
- Different positions of q yield different charge distributions
- Here image charges can be used

$$E_z(x,y) = -\frac{q z_0}{2\pi\varepsilon_0 (x^2+y^2+z_0^2)^{3/2}}$$

$$E_x, E_y = 0$$

$$\sigma(x,y) = \varepsilon_0 E_z(x,y)$$

$$Q_{ind} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sigma(x,y)\,dx\,dy = -q$$



Principle of Signal induction

If we segment the metal plate
and keep individual strips grounded:

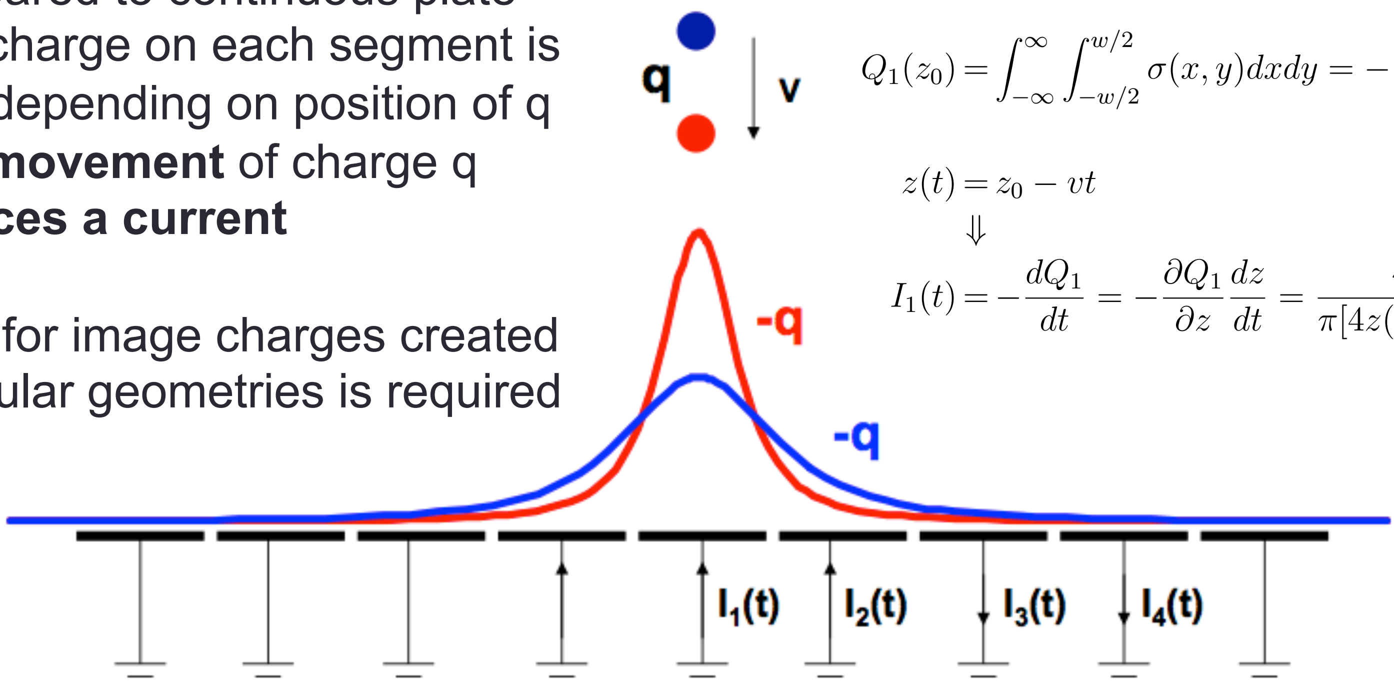
- Surface charge does not change compared to continuous plate
- The charge on each segment is now depending on position of q
- The **movement** of charge q **induces a current**

$$E_z(x,y) = -\frac{q z_0}{2\pi\varepsilon_0 (x^2+y^2+z_0^2)^{3/2}}$$

$$Q_1(z_0) = \int_{-\infty}^{\infty}\int_{-w/2}^{w/2} \sigma(x,y)\,dx\,dy = -\frac{2q}{\pi}\arctan\frac{w}{2z_0}$$

$$z(t) = z_0 - vt$$
$$\Downarrow$$
$$I_1(t) = -\frac{dQ_1}{dt} = -\frac{\partial Q_1}{\partial z}\frac{dz}{dt} = \frac{4qw}{\pi[4z(t)^2+w^2]}\cdot v$$

Method for image charges created for irregular geometries is required



Assumption

- Maxwell -> Quasi steady state approximation: $\frac{\partial \mathbf{B},\mathbf{D}}{\partial t} \simeq 0$

Pauli lectures on physics, Volume 1, chapter 3:

> The fields to be considered in this chapter are assumed to change but little during the time required for light to traverse a distance equal to the maximum dimension of the body under consideration. Thus, the finite velocity with which the fields propagate need not be considered.

$$n_{Ge} = 4.0$$
$$c = 75\, m/\mu s$$
$$v_{drift} \simeq 10\, cm/\mu s$$
$$d \simeq 10\, cm$$

➔ Steady state ✓

- Allows separation of problem:
  - First calculate path z(t) of free charges
  - Secondly: For each position of the trajectory, calculate the induced charge $Q_{ind}(t)$ in the electrode of interest.

The Shockley-Ramo theorem

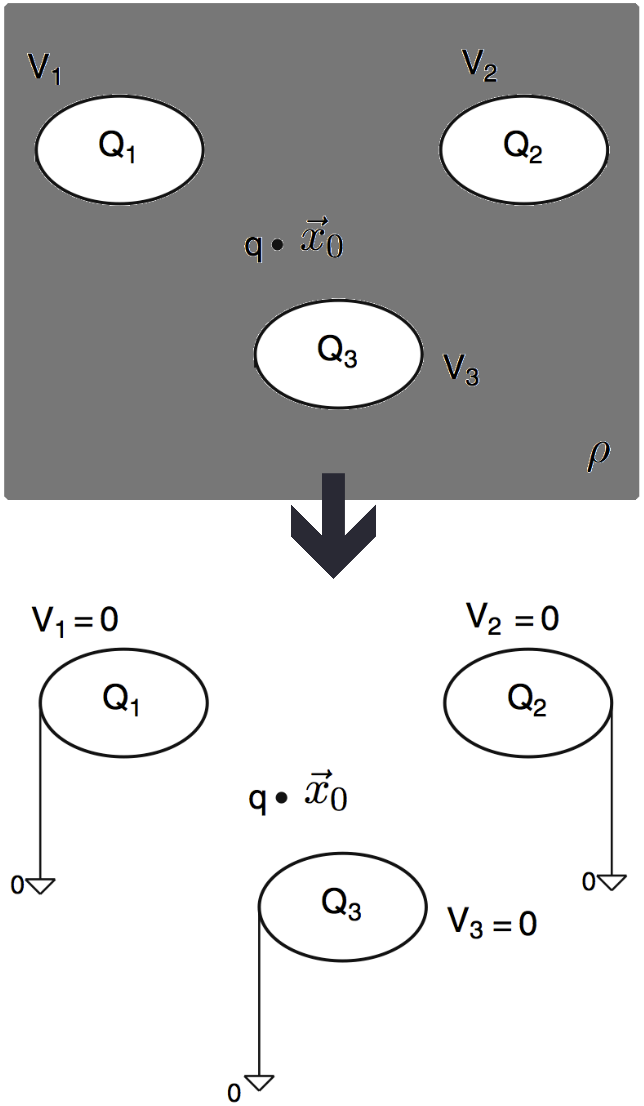
- The problem:

$$\nabla^2\phi(\vec{x}) = -[\rho(\vec{x}) + q\delta(\vec{x}-\vec{x}_0)]/\epsilon$$

- The superposition principle:
  - Charges / current that flow during power up are not of interest

$$\begin{aligned}
\phi(\vec{x}) &= \quad \phi_0(\vec{x}) + \phi_q(\vec{x}) && \text{with} \\
\nabla^2\phi_0(\vec{x}) &= \quad -\rho(\vec{x})/\epsilon && \phi|_{S_j} = V_j \\
\nabla^2\phi_q(\vec{x}) &= \quad -q\delta(\vec{x}-\vec{x}_0)/\epsilon && \phi|_{S_j} = 0
\end{aligned}$$

- Signal shapes are independent of the space charge in the detector



The Shockley-Ramo theorem

- Consider V the volume excluding all electrodes
- Consider the potentials Φ, Ψ corresponding to:

potential X	ΔX	$X\|_{S_j}$	$\frac{\partial X}{\partial n}\|_{S_j}$
Φ	$-q\delta(\vec{x}-\vec{x}_0)/\varepsilon$	0	$-\sigma_{q,j}/\varepsilon$
Ψ	0	$\delta_{i,j}$	$-\tau_j/\varepsilon$

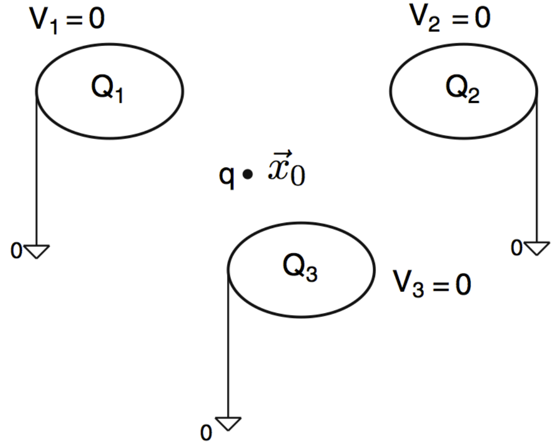
- Relate both potentials using **Greens 2nd identity**:

$$\int_V \underset{=0}{\Phi\Delta\Psi} - \Psi\Delta\Phi \; dV = \oint_S \underset{=0}{\Phi\frac{\partial\Psi}{\partial n}} - \Psi\frac{\partial\Phi}{\partial n} \; dS$$

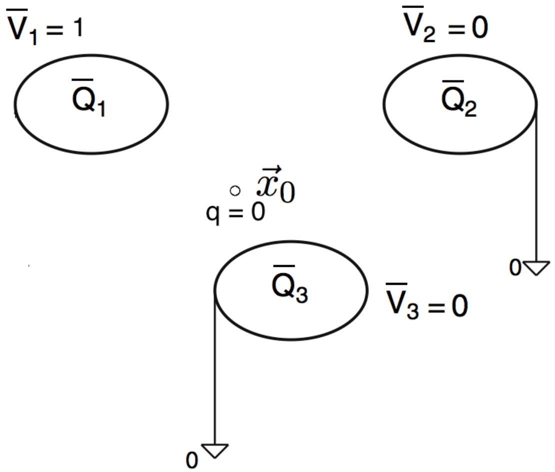
$$\int_V \Psi\cdot q\delta(\vec{x}-\vec{x}_0)\; dV = -\sum_j \oint_{S_j} \delta_{i,j}\cdot\sigma_{q,j}\; dS$$

$$\boxed{q\Psi(\vec{x}_0) = -\oint_{S_i} \sigma_{q,i}\; dS = -Q_{ind,i}}$$

Potential Φ



Potential Ψ



The Shockley-Ramo theorem

- The induced charge Q_{qi} on electrode i
  by a point charge q located at position x_0 is

$$Q_{qi} = -q \cdot \psi_i(\vec{x}_0)$$

- With **weighting potential** ψ_i defined by

$$\nabla^2 \psi_i(\vec{x}) = 0 \qquad \phi|_{S_j} = \delta_{i,j}$$

- The current $I_{qi}(t)$ to electrode i is then given by

$$I_{qi} = \frac{dQ_{qi}}{dt} = -q \cdot \left(\frac{\partial \Psi_i}{\partial x_0} \frac{dx_0}{dt} + \frac{\partial \Psi_i}{\partial y_0} \frac{dy_0}{dt} + \frac{\partial \Psi_i}{\partial z_0} \frac{dz_0}{dt} \right)$$
$$= q \; \vec{E}_{\Psi i}(\vec{x}_0) \cdot \vec{v}_{drift}$$

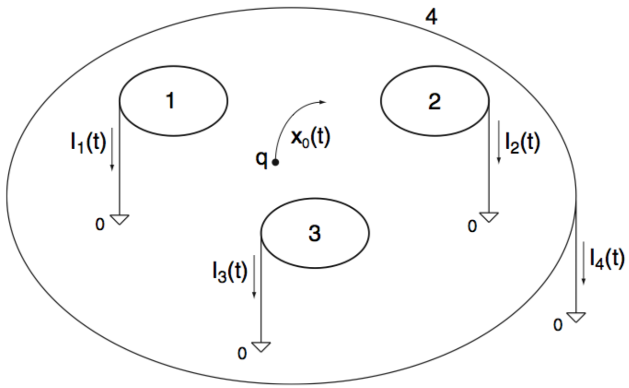
- The function $\vec{E}_{\Psi i} = -\nabla \Psi_i$ is called the **weighting field**

Weighting field properties

For a set of electrodes completely enclosing the detector volume V:

- The sum of weighting potentials is 1 everywhere on V

$$\Psi(\vec{x}) = \sum_i \Psi_i(\vec{x}) \equiv 1$$



- The total current is 0 at any time

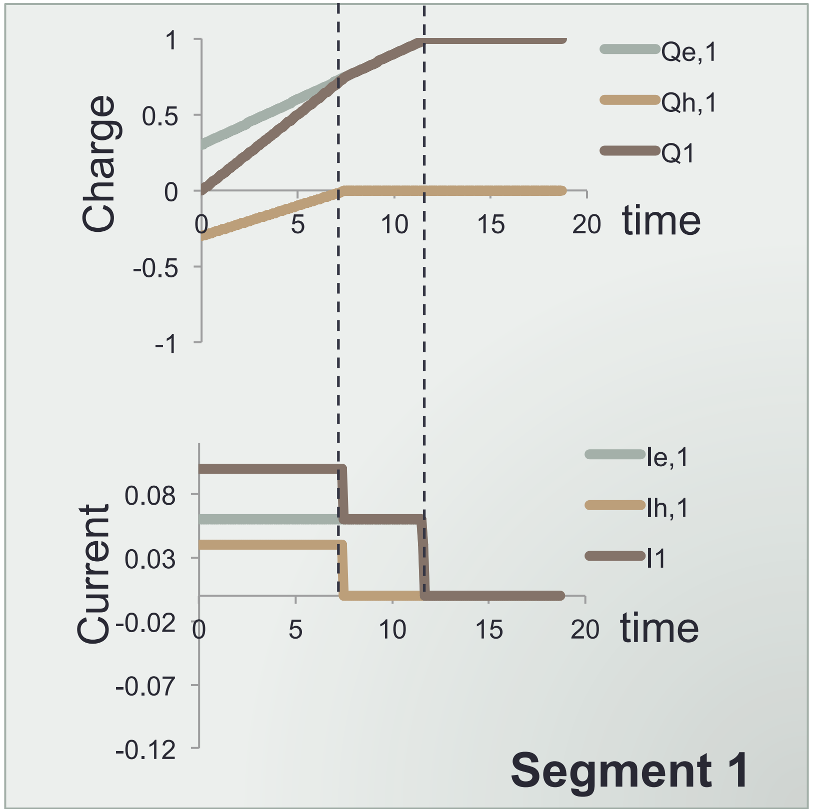
$$I_{tot}(t) = \sum_i I_{q,i} \propto \nabla\Psi \equiv 0$$

- The total induced charge is 0 at any time

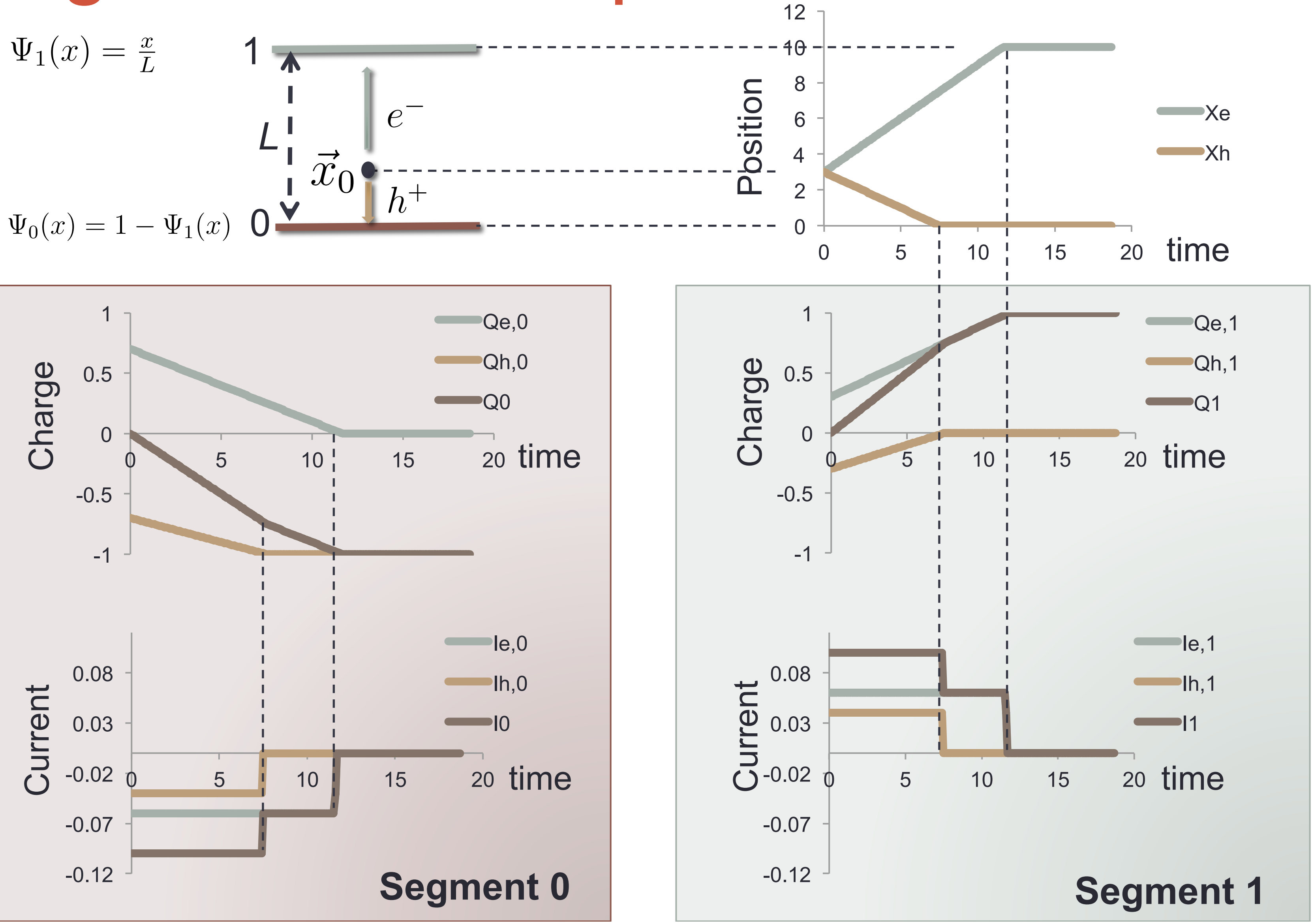
$$Q_{tot}(t) = \sum_i Q_{q,i} \equiv 0$$

Signal formation: planar detector

- (Assumed constant drift velocities)
- Electrons and holes are created in equal amounts, at equal positions: Charge signals always start from 0.
- When all charges are collected, the charge signal has the amplitude equal to the collected charge, but with opposite sign of the collected charge (but it is not a collection process)
- Steepest slope method: The change in slope can be used to calculate the collection time and thus the initial starting position

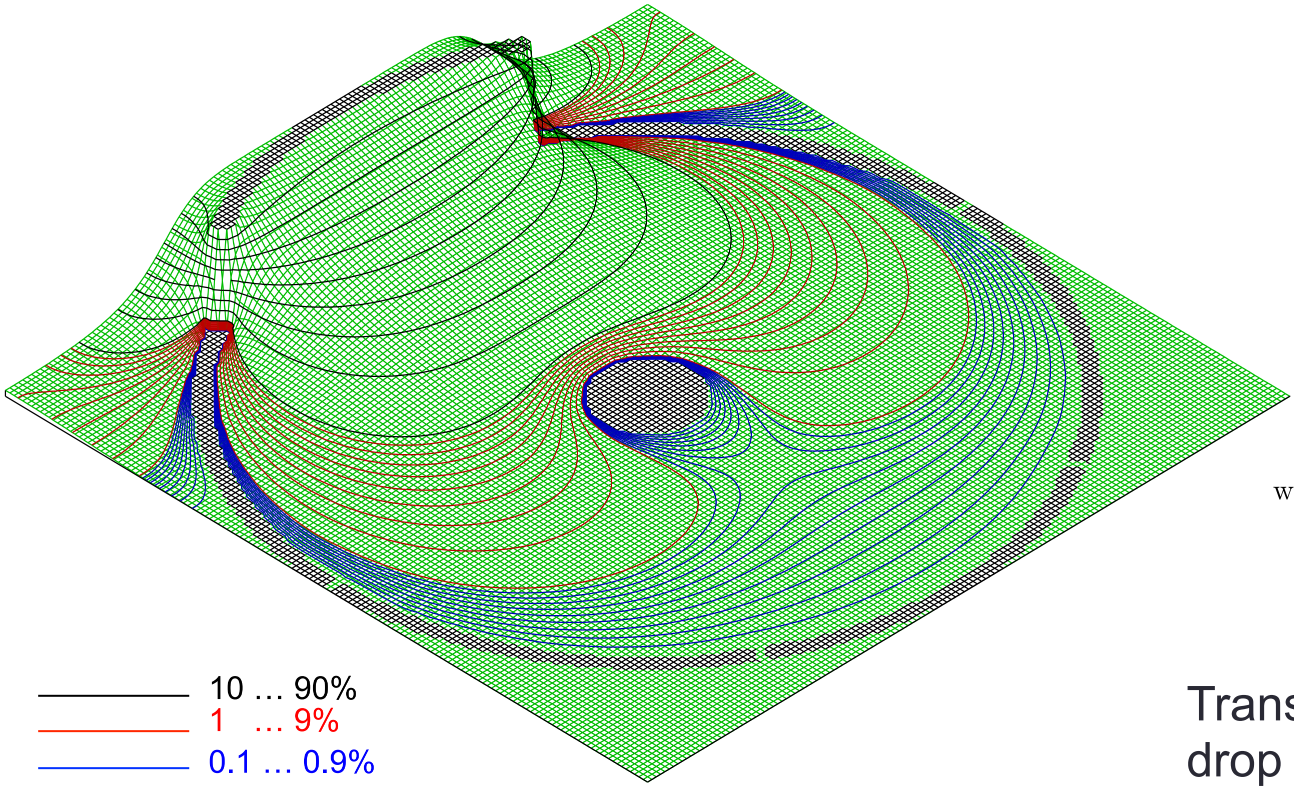


Signal formation: planar detector



Weighting potentials: examples

- Detector Simulation Software ADL:
  http://www.ikp.uni-koeln.de/research/agata/ ➔ Downloads



Coaxial detector
(6x segmented)

Core weighting potential

$$\Psi_0(r) = 1 - \ln \frac{r}{r_{min}} / \ln \frac{r_{max}}{r_{min}}$$

Segment weighting pot:

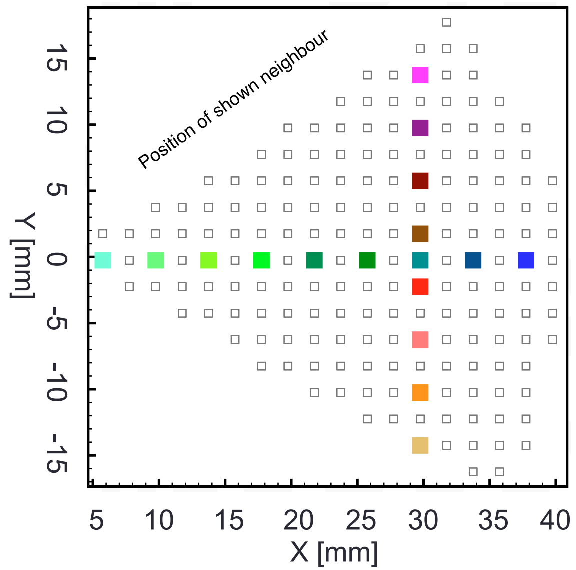
$$\psi_1(r,\theta) = \frac{\ln(r/r_{min})}{6\ln(r_{max}/r_{min})} + \sum_{n=1}^{\infty} B_n \left[\left(\frac{r_{min}}{r}\right)^n - \left(\frac{r}{r_{min}}\right)^n \right] \cos(n\theta)$$

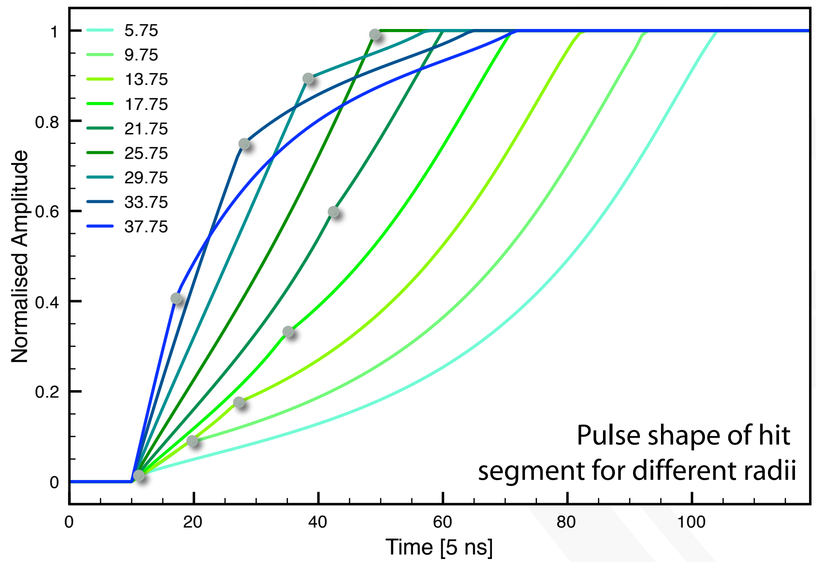
with $$B_n = \frac{2\sin(n\pi/6)}{n\pi \left[\left(\frac{r_{min}}{r_{max}}\right)^n - \left(\frac{r_{max}}{r_{min}}\right)^n \right]}$$

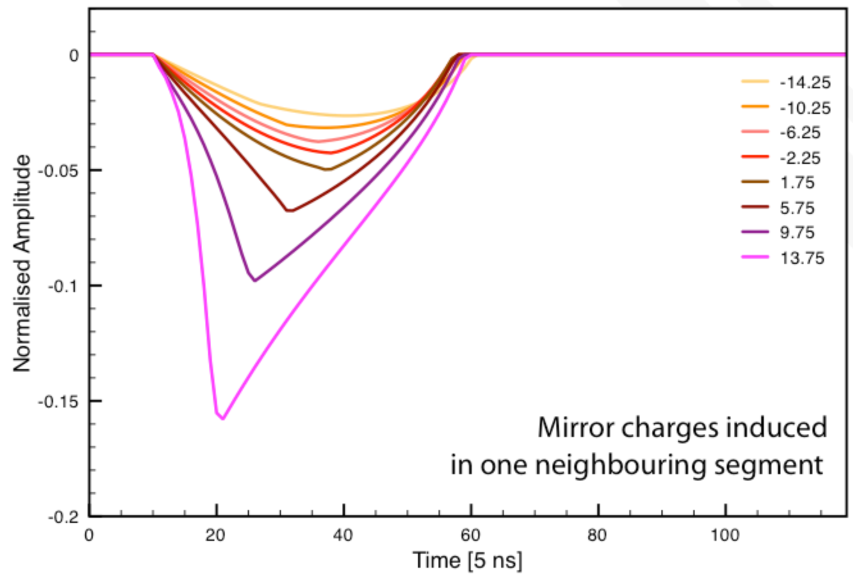
Transient Signal amplitudes drop 1 order for each segment one moves away from the hit segment

Examples: AGATA

- Signal shapes from AGATA detector as function of position
- Simulation using ADL
- (Steepest slope not always clear)
- (remark: segment preamps are inverting)

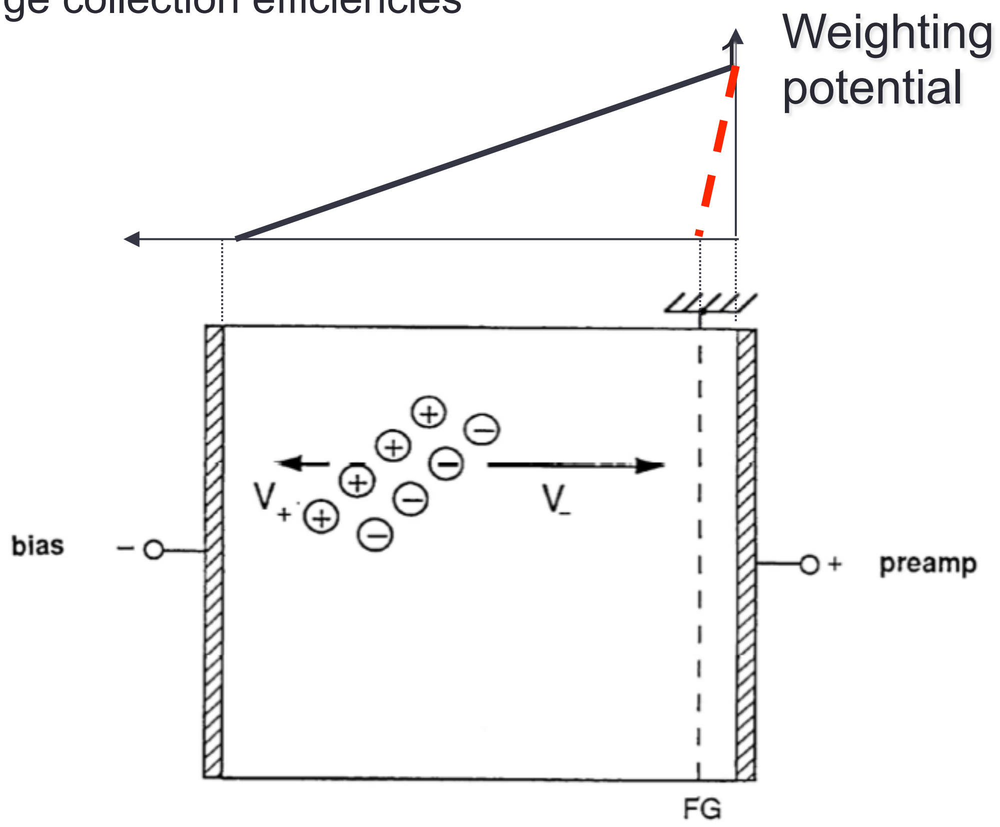






Weighting potentials: examples

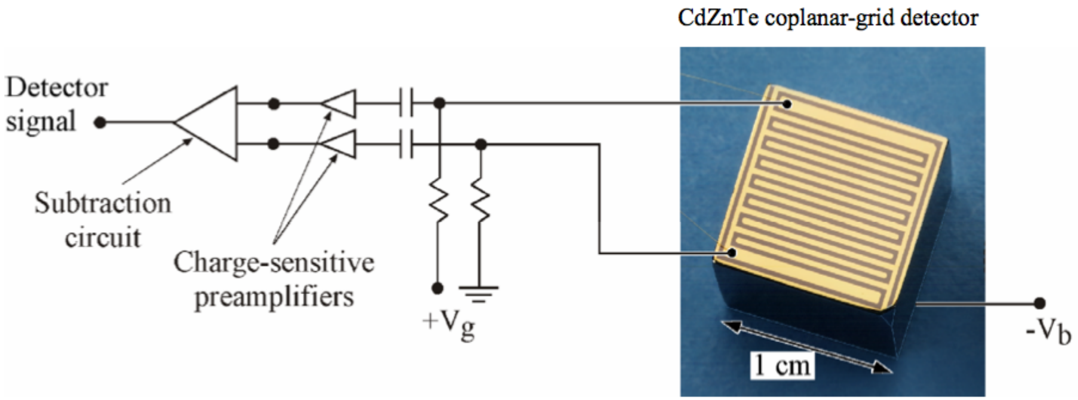
- In gas detector: ions ~1000x slower than electrons
- A **Frish grid** makes the signal only depending on the fast electrons:
  - Better timing resolution
  - Higher charge collection efficiencies

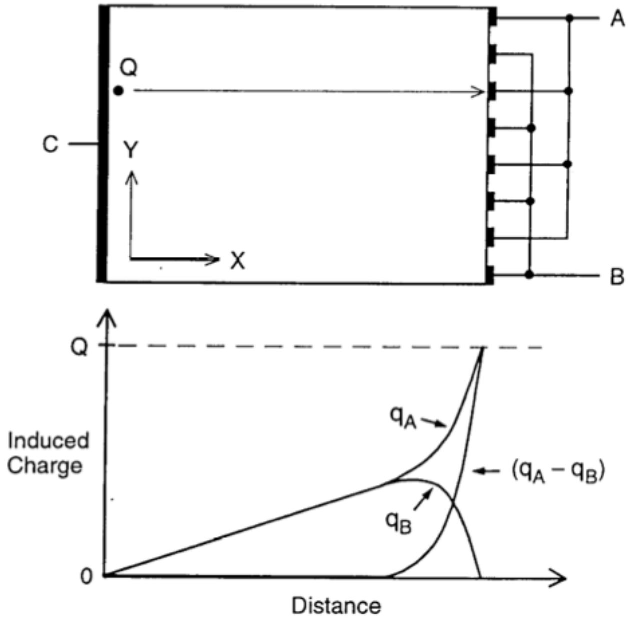


Examples: CdZnTe

- Frish grids also of interest in CdZnTe semiconductor detectors:
  - CdZnTe Mobility holes = 120 cm^2/Vs
  - CdZnTe Mobility electrons = 1350 cm^2/Vs

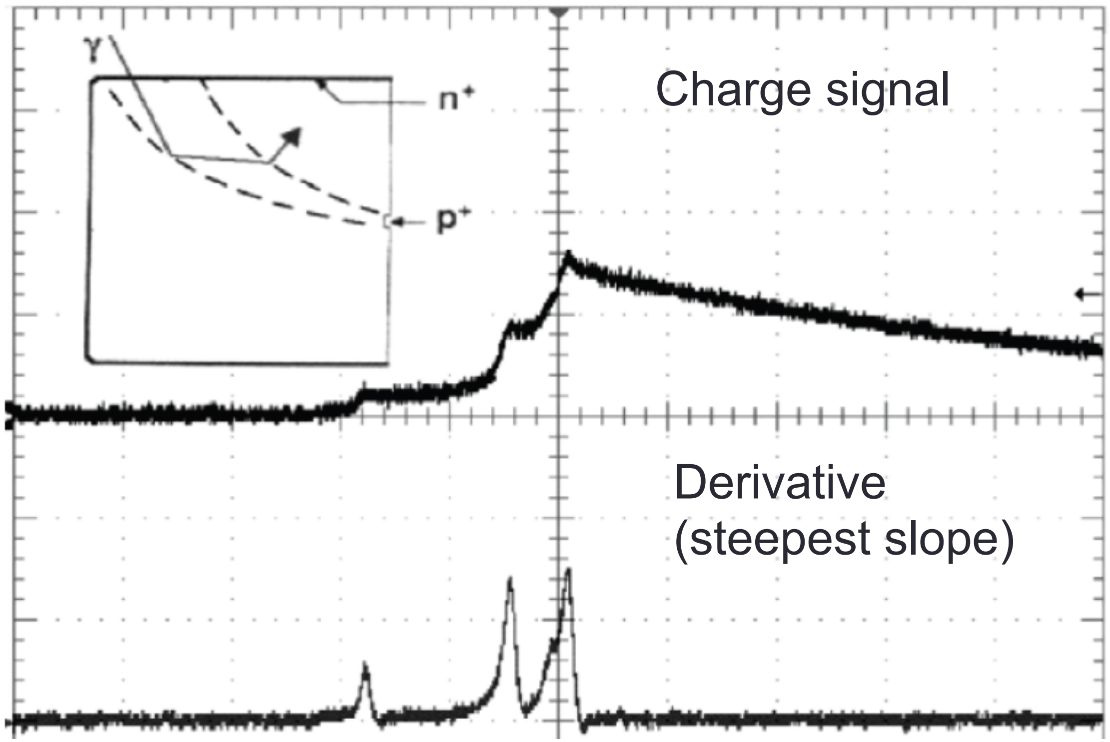
- Reduced influence of trapping





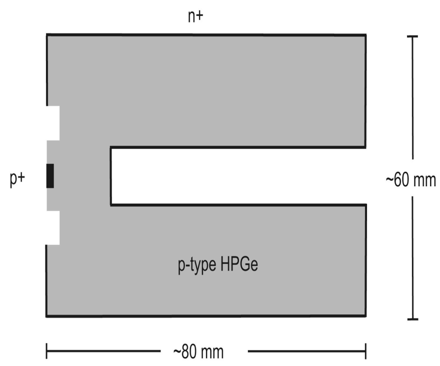
Examples: GERDA

- Point contact detectors: also very local weighting potential
- Identification of multiple hits via steepest slope method
- Very small capacity ➔low serial noise

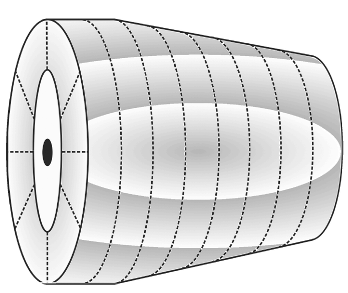


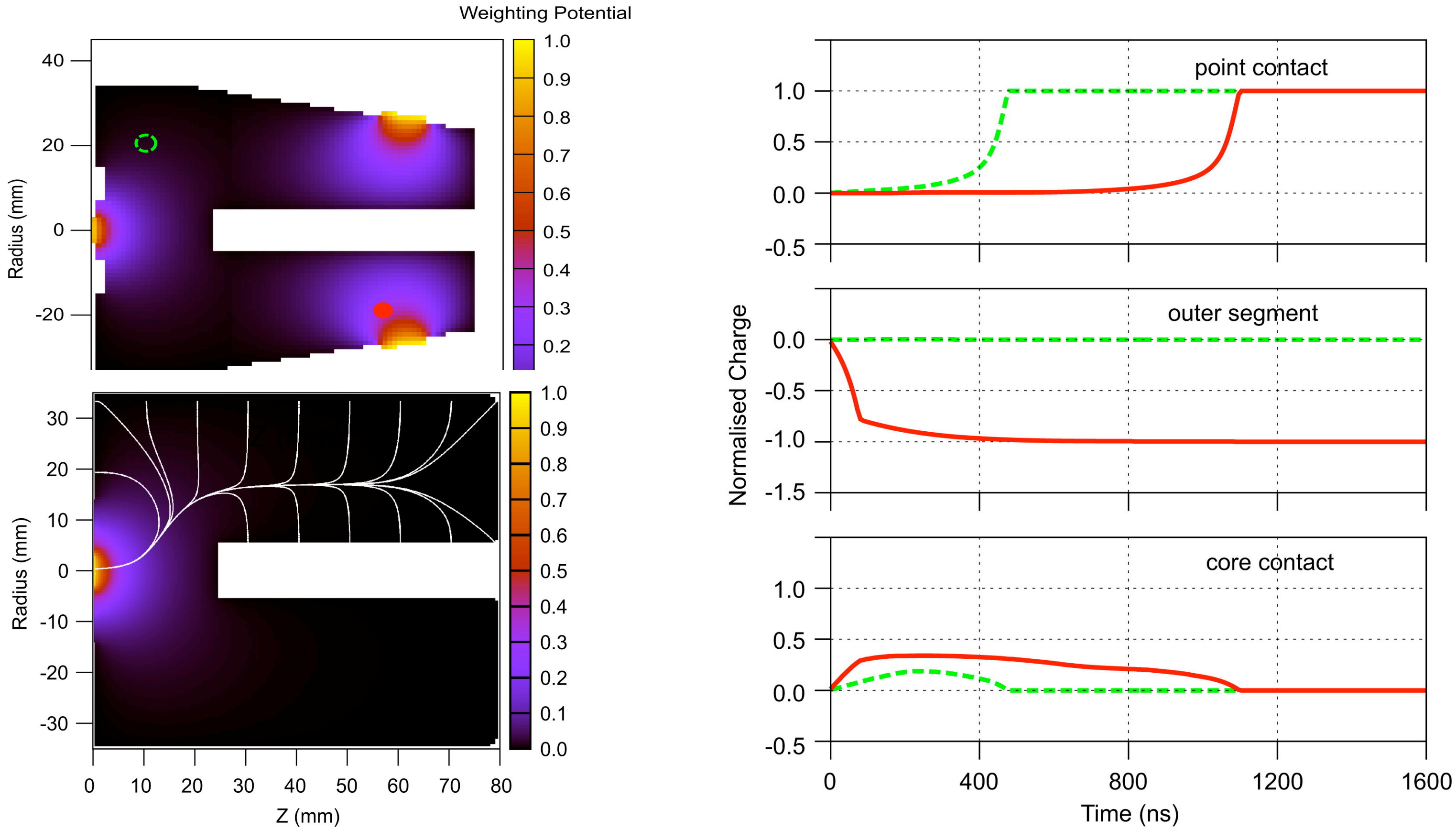
Examples: Gerda++

- Point contact detectors: new generation NIM A doi:10.1016/j.nima.2011.10.008
- Sub mm position resolution



+segmentation (20x)





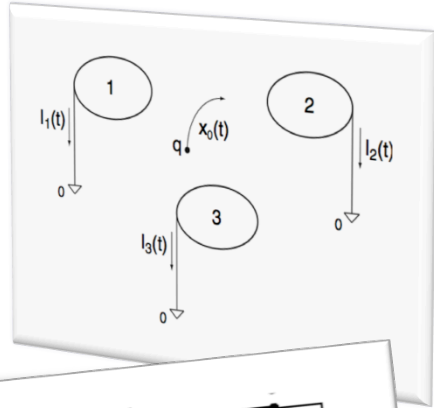
Extended Ramo theorem

- Describes detectors in a realistic electronic network.

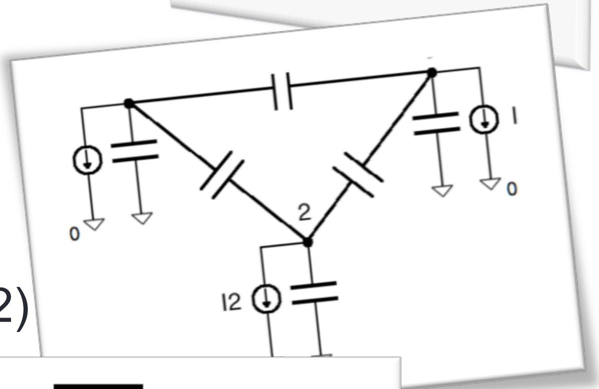
- In 3 steps:
  - 1) Apply the Ramo theorem:
    Calculate the induced currents in each electrode
  - 2) Equivalent electronics scheme:
    Proof: see Gatti and Padivini, NIM 193 (1982) 651-653
    -Determine the capacitances of your detector,
    -Add the current sources found from 1)
  - 3) Realistic electronics scheme:
    Change the above simplified scheme
    into a realistic model

- Result = realistic signals

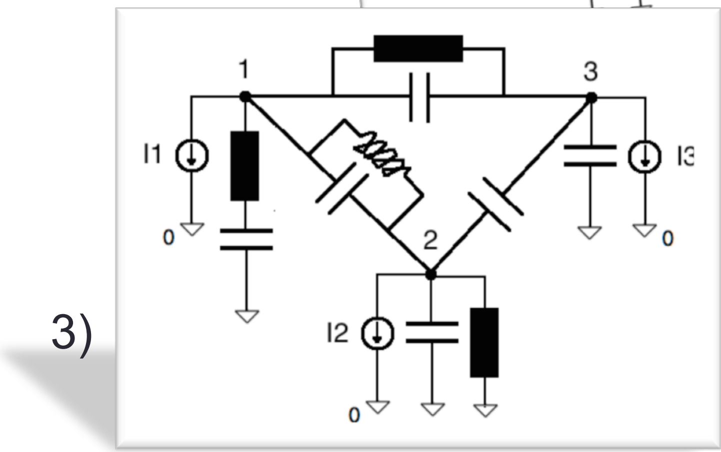
1)



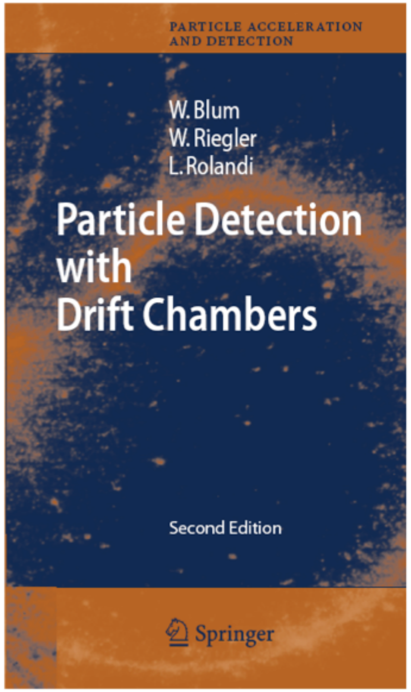
2)

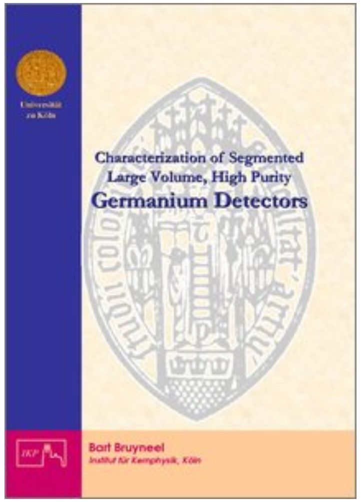


3)



Recommended Literature





http://kups.ub.uni-koeln.de/1858/

www.ikp.uni-koeln.de/research/agata/
➔publications

Makes-Cows-Weighting fields: