Pulse Processing: Pulse Shaping

2.1 PULSE SHAPING

The shapes of signal pulses from detectors are usually changed or shaped by the signal conditioning or processing elements of the data acquisition system. It is very common, for example, to shape the output pulses of the preamplifier in the amplifier.

To assure complete charge collection from a detector, preamplifier circuits are normally adjusted to provide a long decay time for the pulse (typical decay times are on the order of 50

μs). Since the pulses occur at random times (radioactive decay is a random process) they will sometimes overlap (especially if the count rate is large). In such circumstances, a pulse train such as shown in Figure 2.1(a) may occur. The amplitudes of the pulses carry the basic information, the charge deposited in the detector (which often is proportional to the energy of the original radiation). Hence, the pile-up, saturation and subsequent non linear response shown in Figure 2.1(a) is very undesirable.

Shaping the pulses to produce a pulse train such as shown in Figure 2.1(b) can



alleviate the pile-up problem. With one exception, the pulses have been shaped in such a way that their total lengths have been reduced without affecting the pulse amplitude. Such shaping is normally carried out in a linear amplifier, usually using a variety of *RC* shaping networks. In this lecture, the operation of some of the commonly used pulse shaping networks will be described.

2.2 VOLTAGE AND CURRENT VERSUS TIME

It is useful to begin with a brief review of the behaviour of circuits, which contain resistive and capacitive elements and their response to an AC signal (e.g. a pulse from a detector).

2.2.1 Discharging

Consider the simple circuit shown in Figure 2.2, where a charged capacitor C is connected across a resistor R. The time evolution of the voltage is described by the differential equation,



Figure 2.2 Discharge of a capacitor across a resistor.

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = I = -\frac{V}{R} \tag{2.1}$$

To solve Equation (2.1): Rewrite it as
$$\frac{dV}{dt} + \frac{V}{\tau} = 0$$
, where $\tau = RC$.
Multiply by $e^{t/\tau}$: $e^{t/\tau} \left(\frac{dV}{dt} + \frac{V}{\tau} \right) = \frac{d}{dt} \left(V e^{t/\tau} \right) = 0$.
Integrate: $\left(V e^{t/\tau} \right) = K$ and, if $V = V_0$ at $t = 0$, $K = V_0$. Hence,

the solution to Equation (2.1) is:

$$V = V_0 \mathrm{e}^{-t/RC} \tag{2.2}$$

where V_0 is the initial voltage across the capacitor.

Therefore, a charged capacitor placed across a resistor will discharge, as shown in Figure 2.2, with a *time constant* $\tau = RC$.

For *R* in ohms and *C* in farads, the product *RC* is in seconds. For example, a 1 μ F capacitor placed across a 1k Ω resistor has a time constant of 1 ms.

2.2.2 Charging

Figure 2.3 shows a slightly different situation where a resistor and capacitor are connected across a battery (voltage source). At time t = 0, the switch is closed, connecting the battery. The equation for the current is

$$I = C\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V_{\mathrm{S}} - V}{R} \tag{2.3}$$

This equation can be solved in the same way as Equation (2.1) and is shown in the box below:

To solve Equation (2.3): Rewrite it as $\frac{dV}{dt} + \frac{V}{\tau} = \frac{V_s}{\tau}$, where $\tau = RC$. Multiply by $e^{t/\tau}$: $e^{t/\tau} \left(\frac{dV}{dt} + \frac{V}{\tau} \right) = \frac{d}{dt} \left(V e^{t/\tau} \right) = \frac{V_s}{\tau} e^{t/\tau}$. Integrate: $\left(V e^{t/\tau} \right) = V_s e^{t/\tau} + K$ and, if V = 0 at t = 0, $K = -V_s$. Hence,

the solution to Equation (2.3) is

$$V = V_{\rm S} (1 - e^{-t/RC})$$
(2.4)



Figure 2.3 Response of an RC circuit to a sudden voltage change

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As can be seen, after a long time ($t \gg 5RC$) the voltage approaches its final (equilibrium) value.

5RC rule: A capacitor charges or decays to within 1% of its final value after about five time constants.

2.3 CR DIFFERENTIATOR OR HIGH-PASS FILTER

A basic *CR* differentiator circuit is sketched in Figure 2.4(a). The input and output voltages are related by the equation

$$V_{\rm i} = \frac{Q}{C} + V_0 \tag{2.5}$$

where Q is the charge on the capacitor at time t. Differentiating gives

$$\frac{\mathrm{d}V_{\mathrm{i}}}{\mathrm{d}t} = \frac{I}{C} + \frac{\mathrm{d}V_{\mathrm{0}}}{\mathrm{d}t} \tag{2.6}$$

where the current I = dQ/dt.

Using $V_0 = IR$ and $\tau = RC$ we get

$$V_0 + \tau \frac{\mathrm{d}V_0}{\mathrm{d}t} = \tau \frac{\mathrm{d}V_i}{\mathrm{d}t}$$
(2.7)

This equation can be solved to give a general solution. However, useful insight is obtained by considering two limiting situations where the time constant τ is very much less than or very much greater than *T*, the duration of the pulse.

$\tau \ll T$:

If τ (= *RC*) is made sufficiently small, the second term on the left hand side can be neglected and the output voltage is proportional to the time derivative of the input, hence the name differentiator!

To meet this requirement, the time constant needs to be **short** compared to the **duration of the input pulse** (or to the Fourier frequency components of the pulse). However, care needs to be taken not to `load' the input by making *RC* too small (at the transition, the change in voltage across the capacitor is zero, so *R* is the load seen by the input).

$\tau \gg T$:

At the opposite extreme of a long time constant, the first term can be neglected giving

$$\tau \frac{\mathrm{d}V_0}{\mathrm{d}t} \approx \tau \frac{\mathrm{d}V_i}{\mathrm{d}t} \tag{2.8}$$

Integrating this and setting the constant of integration equal to zero gives

$$V_0 \cong V_i \tag{2.9}$$

In this limit, the network will pass the waveform without distortion.

The CR differentiator is a high-pass filter.

- High frequency components of pulses (edges) are passed without distortion.
- Low frequencies are attenuated away and any dc component is not passed.

The effect of a high-pass CR filter on different input waveforms is shown in Figure 2.4.



Figure 2.4 High-pass CR filter (differentiator): (a) basic circuit; (b) step input; (c) single (square) pulse (RC = T); (d) single pulse ($RC \gg T$); (e) single pulse ($RC \ll T$); (f) pulse train.

• For a step input, the output is

$$V_0 = V_1 e^{-t/\tau}, \qquad (2.10)$$

which approximately represents the shaping of a rapidly rising signal pulse by a single CR differentiator. Note that the fast leading edge is not differentiated because τ is not small compared to its rise time. Therefore, the leading edge is simply passed through while the shaping consists of differentiating away the long tail.

- For a single square pulse [curve (c)], the fast rise and fall are passed undistorted and the DC signal falls towards zero. Note that the areas above and below the baseline are equal, which is a result of the fact that any dc Fourier component of the input is not passed.
- Curve (d) shows that if the time constant is long compared with *T*, the pulse is passed with little distortion.
- By contrast, if RC « T [curve (e)], the pulse is strongly differentiated and the output is large only when dV_i/dt is large.
- A train of pulses is passed [curve (f)], shaped according to the value of *RC* relative to *T*, but **there is no baseline shift**, i.e. the dc component of the input signal is not passed.

The response of a high-pass filter to a rising input voltage $V_i = kt$, is shown in Figure 2.5.



Figure 2.5 Response of a high-pass CR filter to a rising ramp input.

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- At large t (» RC), the output reaches a saturation value equal to $\tau(dV_i/dt)$.
- Initially, (for *t* « *RC*), the output (solid curve) follows the input.
- Any real pulse rises from zero to its peak in a finite time T_r and there will be a difference between the output and input depending on the ratio of τ to T_r . This difference is called the **ballistic deficit** (shown as Δ in Figure 2.5). If $\tau > 5 \times T_r$, $\Delta < 1\%$ of the signal height.

Differentiators are useful for detecting leading and trailing edges in pulses. For example, in digital circuitry one sometimes sees networks like that shown in Figure 2.6. In this case, the differentiator generates spikes at the transitions of the input pulse and the output buffer converts these into short square pulses.

2.4 THE RC INTEGRATOR OR LOW-PASS FILTER

A passive RC network acts as an integrator when configured as shown in Figure 2.7. The circuit equation is now



Figure 2.6 Example of the use of a differentiator in digital circuitry.

$$V_{i} = IR + V_{0}. (2.11)$$

Since I = dV/dt represents the rate of charging of the capacitor. This can be rewritten as

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$$V_{\rm i} = \tau \frac{\mathrm{d}V_0}{\mathrm{d}t} + V_0$$

which can be rearranged as

$$\frac{\mathrm{d}V_0}{\mathrm{d}t} + \frac{V_0}{\tau} = \frac{V_\mathrm{i}}{\tau} \,. \tag{2.12}$$

Again, consider the extreme situations when *RC* is very large or very small (compared with the pulse duration, for example).

$\tau \gg T$:

When τ (= *RC*) is large, only the first term on the left-hand side matters and we see that the output voltage is proportional to the integral of the input voltage, hence the name `integrator'.

$$V_0 = \frac{1}{\tau} \int V_i dt \qquad (2.13)$$

$\tau \ll T$:

At the opposite extreme of very small time constants (low frequencies), only the second term is significant and once again

$$V_0 \cong V_i \tag{2.14}$$

so that the network will pass the signal without change.

The effect of a low-pass RC filter on different input waveforms is shown in Figure 2.7.



Figure 2.7 Low-pass RC filter (integrator): (a) basic circuit; (b) step input; (c) single (square) pulse (RC = T); (d) single pulse ($RC \gg T$); (e) single pulse ($RC \ll T$); (f) pulse train.

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For a step input [curve (b)], the fast rise is **not** passed and the output begins to increase only after the input has reached its maximum dc value. The capacitor C charges up at a rate dependent upon τ and, eventually, the output becomes equal to the input.

• For a single, square pulse [curve (c)], the output rises as C charges and then it falls after the end of the pulse.

- If $RC \gg T$, the output reaches a low value [curve (d)], because it varies inversely as $1/\tau$, according to Equation (2.13).
- By contrast, if $RC \ll T$ [curve (e)], the pulse is passed with little distortion, according to Equation (2.14).
- A train of pulses is passed [curve (f)], shaped according to the value of *RC* relative to *T*. In this case, **the baseline shift is passed**.

The response of a low-pass filter to a voltage that is rising for a time *T*, is shown in Figure 2.8.



Figure 2.8 Low-pass RC filter response to a linear rising ramp input.

- If *t* « *RC*, after a short delay, the output follows the input, according to Equation
 (2.14)
- If $t \gg RC$, the output is proportional to the integral of the input, according to Equation (2.13).

The integrator is used extensively in analog computation. It is a useful circuit that finds applications in control systems, analog/digital conversion, and in waveform generation.

2.5 CR-RC SHAPING

The combination of a CR differentiator and RC integrator is commonly used as a pulse shaper in linear amplifiers.

The output of a single CR differentiating circuit has several unwelcome features for pulse processing systems.

- The sharp top makes any subsequent pulse height analysis difficult (the maximum is only held for a short time).
- All high-frequency noise components are passed through by the circuit, implying that the signal-to-noise ratio is usually poor.

If an RC integrating stage follows the CR stage both of these features can be much improved. Such a combination, shown in Figure 2.9 is one of the most widely used methods for shaping preamplifier pulses.



Figure 2.9 A CR-RC shaping network.

The triangular symbol is an operational amplifier (op amp), which in this configuration has infinite input impedance and zero output impedance. The op amp serves to isolate the two individual networks (impedance isolation) so that neither influences the operation of the other.

The response of this CR-RC network to a step voltage of amplitude V_i is

$$V_0 = \frac{V_i \tau_1}{\tau_1 - \tau_2} (e^{-t/\tau_1} - e^{-t/\tau_2})$$
(2.15)

where τ_1 and τ_2 are the time constants of the differentiating and integrating circuits, respectively. For interested students, the derivation of this equation is given in the box below.

Figure 2.10 plots the response of Equation (2.12) for several different combinations of time

constants.

Derivation of Equation (2.15):

The first part of the circuit is a high-pass filter. The output voltage is given by Equation (2.10): $V_{\rm S} = V_{\rm i} e^{-t/\tau_{\rm I}}$, where $\tau_{\rm I} = R_{\rm I}C_{\rm I}$. The second part is a low-pass filter, whose behavior is given by Equation (2.12) with $V_{\rm i} = V_{\rm S}$, i.e. $\frac{dV_0}{dt} + \frac{V_0}{\tau_2} = \frac{V_{\rm S}}{\tau_2}$, where $\tau_2 = R_2 C_2$. Multiply by $\exp(t/\tau_2)$: $\exp(t/\tau_2) \frac{dV_0}{dt} + \frac{V_0}{\tau_2} \exp(t/\tau_2) = \frac{d}{dt} (V_0 \exp(t/\tau_2)) = \frac{V_{\rm S} \exp(t/\tau_2)}{\tau_2}$. Substitute for $V_{\rm S}$: $\frac{d}{dt} (V_0 \exp(t/\tau_2)) = \frac{V_{\rm i} \exp[t(1/\tau_2 - 1/\tau_1)]}{\tau_2}$. Integrate: $V_0 \exp(t/\tau_2) = \frac{V_{\rm i} \tau_1 \exp[t(1/\tau_2 - 1/\tau_1)]}{(\tau_1 - \tau_2)} + K$. Setting the condition: $V_0 = 0$ when t = 0, $K = -V_{\rm i}\tau_1/(\tau_1 - \tau_2)$ from which we get Equation (2.15).



Figure 2.10 Response of a CR-RC network to a step voltage. Curves are shown for several different combinations of time constants. Curves are labelled $\tau_1 + \tau_2$.

In some applications (a common situation in nuclear pulse processing), CR-RC shaping is carried out using equal time constants (τ say). For this particular case, the solution is

$$V_0 = V_i \frac{t}{\tau} e^{-t/\tau}.$$
 (2.16)

This response is also illustrated in Figure 2.10.

Derivation of Equation (2.16): If $\tau_1 = \tau_2 = \tau$, rewrite Equation (2.15) as $V_0 = \frac{V_i \tau_1}{\tau_1 - \tau_2} e^{-t/\tau_2} (e^{-t(1/\tau_1 - 1/\tau_2)} - 1)$ and expand the first term in brackets to first order in *t*: $V_0 = \frac{V_i \tau_1}{\tau_1 - \tau_2} e^{-t/\tau_2} \left[1 - t \left(\frac{1}{\tau_1} - \frac{1}{\tau_2} \right) - 1 \right] = \frac{V_i \tau e^{-t/\tau_2}}{\tau_1 - \tau_2} \left[\frac{t(\tau_1 - \tau_2)}{\tau^2} \right]$, giving Equation (2.16).

The choice of shaping time constants is determined primarily by the charge collection time in the detector. As always, there are competing factors that need to be considered:

• In order to minimize pile-up effects, the time constants should be kept **short** so that the pulse returns to baseline as quickly as possible.

• On the other hand, when the time constants become comparable to the rise time of the pulses from the preamplifier, the input to the network no longer looks like a step voltage and the result is that some of the amplitude of the signal is lost.

This is referred to as *ballistic deficit* (see Section 2.3) and can be avoided only by keeping the time constants **long** compared to the charge collection time of the detector.

Typical values for τ range from a few tenths of a μ s for small semiconductor detectors through to a few μ s for `large' Ge detectors to tens of μ s for some types of proportional counter.

2.6 GAUSSIAN SHAPING

When a single CR differentiator is followed by several stages of RC integration (four are sufficient), the output pulse shape approximates that of a mathematical Gaussian.

In theory, a Gaussian pulse shape has some potential advantages for improvement in signalto-noise ratio over simple CR-RC shaping. Gaussian or semi-Gaussian shaping is very commonly found in linear amplifiers used in nuclear pulse processing applications. A disadvantage, however, is that because of the greater pulse width, pile-up effects are more severe at high rates.

2.7 DOUBLE DIFFERENTIATION OR CR-RC-CR SHAPING

Double differentiation can be used to generate a *bipolar* pulse shape. Another differentiation stage is added with the result illustrated in Figure 2.11.

- The most common choice is to make all three time constants equal.
- The bipolar pulse shape makes **baseline shifts** (see later) much less severe.
- This type of signal is most useful at high count rates. However, at lower rates, its

signal-to-noise characteristics are usually not as good as single-stage RC shaping because the amplitude is lower.



Figure 2.11 Creation of a bipolar pulse using a CR-RC-CR shaping network.

2.8 SINGLE AND DOUBLE DELAY-LINE PULSE SHAPING

Some properties of coaxial cables can be used to shape pulses. Recall that a cable shorted at one end will produce a reflected pulse moving back toward the sending end of the cable with an equal amplitude but opposite polarity.

2.8.1 Single delay-line shaping

A single delay line can be used to generate a square pulse from a step voltage input as shown in Figure 2.12

The transmission line is assumed to be long enough so that the propagation time is long compare to the rise time of the step. The op amp provides impedance isolation and, since its output impedance is zero, the resistor Z_0 terminates the cable at its sending end. The width of the shaped pulse is just twice the propagation time through the length of the cable. For many applications, this is a microsecond or so. Special delay lines, with reduced pulse propagation velocities, are often used to avoid excessively long cables.



Figure 2.13 illustrates problems, which can arise if the delay time T is (a) too short or (b) too long.

At one extreme, T must not be shorter than the pulse rise time, otherwise **ballistic-deficit** effects will appear [see Figure 2.13(a)]. On the other hand, T must not be so long that it is comparable with the decay time of the input pulse, or there will be an **undershoot**, as illustrated in Figure 2.13(b). However, if it is not possible to achieve this condition, the undershoot can be alleviated by attenuating the reflected pulse, as illustrated in the lower two waveforms in Figure 2.13(b).



Figure 2.13 Single delay-line shaping where the delay time is (a) less than the pulse rise time and (b) is comparable with the input pulse decay time.

2.8.2 Double delay-line shaping

Bipolar pulses can be produced if two delay lines are used, as shown in Figure 2.14. If both delay lines have equal length, the resulting pulse will have equal positive and negative lobes and an average dc level of zero can be maintained. This will virtually eliminate any baseline shift in subsequent circuits. While DDL shaping has **excellent high counting rate** capabilities, it does not apply any high-frequency filtering to the signal and therefore, has **poorer signal-to-noise** characteristics than RC shaping networks. Therefore, it is seldom used in high resolution systems (i.e. Ge detectors).



2.9 DELAY-LINE PLUS RC SHAPING

Yet another option is to send the output of a delay-line network to an integrator or low-pass filter. This eliminates much of the high-frequency noise from the signal with a corresponding improvement in the response of the network.

- If the output of a DDL network is integrated, a triangular pulse can be produced as shown in Figure 2.15.
- As the time constants are increased, the approximation to a triangular shape gets better but the amplitude of the pulse is reduced.

• The signal-to-noise characteristics of triangular pulses are quite good and such pulses are sometimes used in high-resolution systems when the count rate is not excessively high.



Figure 2.15 (a) Production of triangular shaped pulses using a DDL network through an integrating RC network. (b) Similarly for an SDL network.

2.10 POLE-ZERO AND BASELINE RESTORATION.

2.10.1 Pole-zero

The simple CR-RC circuit described above produces a significant undershoot as the amplifier pulse attempts to return to zero. This is due to the long exponential decay of the preamplifier pulse. In high count rate situations it is possible (likely) that another pulse will arrive during this time and `ride' on the undershoot of the original pulse. Obviously, in this case, the apparent amplitude of the second pulse will be somewhat reduced, resulting in an undesirable broadening of the peaks in the energy spectrum.

The problem can be alleviated by use of a pole-zero cancellation network, such as shown in Figure 2.16. In Figure 2.16(a) the preamplifier signal is applied to the input of a normal CR differentiator circuit. The output shows the typical undershoot.



Figure 2.16 The benefit of pole-zero correction.

The following equation holds:

 $\frac{undershoot \ amplitude}{pulse \ amplitude} = \frac{differentiation \ time}{preamp \ pulse \ decay \ time}$

Therefore, for a longer preamp decay time, a longer shaping time in the amplifier leads to larger undershoots.

In Figure 2.16(b), the resistor R_{pz} placed in parallel with the capacitor can be adjusted to cancel the undershoot. The result is an output pulse with a simple exponential decay to zero. Virtually all spectroscopy amplifiers incorporate this pole-zero circuit (the term *pole-zero* comes from the mathematical representation of the circuit, the resistor `cancels' a pole in the expression).

2.10.2 Baseline restoration

To ensure good energy resolution and peak position stability, high-performance spectroscopy amplifiers are entirely dc coupled (except for the CR differentiator network located close to the input which is ac coupled almost by definition). Therefore, any DC offsets in the early stage of the system will be greatly amplified to cause a large unstable dc offset at the output. Since the amplitude of the pulses in nuclear applications carries much of the desired information such an offset is extremely undesirable.

The basic principle of the baseline restorer is illustrated in Figure 2.17. We consider two modes of operation:

Simple baseline restoration.

For a simple time-invariant baseline restorer, the switch **S1 is always closed** and the restorer behaves like a CR differentiator. The baseline between pulses is restored to zero by the resistor R_{blr} . The time constant $C_{blr}R_{blr}$ must be at least 50 times the shaping constant in order not to harm the signal-to-noise ratio.

Such a time invariant system does not adequately maintain the baseline at high counting rates. Since this simple circuit is essentially a CR differentiator, the average signal area above ground potential must equal that below (a capacitor cannot pass a dc level). For low count rates, the spacing between pulses is very long and so the baseline is restored to very close to ground potential. However, as the rate increases, the baseline must shift down, the amount of shift increasing with count rate.



Figure 2.17 Operation of a base-line restorer in simple mode and in gated mode.

Gated baseline restoration:

The gated baseline restorer virtually eliminates the problem caused by changing count rates. In this case, the switch S1 in Figure 2.17 is opened (O) only during the duration of each amplifier pulse and is closed (C) otherwise. Therefore, the CR differentiator is active only on the baseline in between pulses and the effect of signal pulse is virtually eliminated. The gated baseline restorer `thinks' that it is operating at zero counting rate!

2.11 GATED INTEGRATOR

Performance at high count rate can be improved using a gated integrator at the output stage of the amplifier shaping network. The principle is illustrated in Figure 2.18.

The unipolar output of a shaping pre-filter is integrated on a capacitance C, which is part of an active circuit, for a time that encompasses the duration of the pulse. At the end of this interval, the capacitance is discharged by closing a switch. The amplitude of the pulse from the gated integrator is now proportional to the **area** of the unipolar pulse from the shaping circuit rather than its peak value. This is much less sensitive to the ballistic deficit caused by variable charge

collection times in the detector.

As an illustrative example, consider the two delay-line shaped pulses shown in Figure 2.18. The top waveform shows a pulse with a fast rise time. The area of the pulse is VT, where T is the delay time and V is the pulse amplitude. The central waveform shows a pulse with a ballistic deficit, because it has a rise time T_r that exceeds T. The pulse reaches an amplitude mT, where m is the voltage gradient of the leading edge, and there is a ballistic deficit because $T_r > T$. The pulse falls to zero at time (V/m)+T after its initial rise. The area of the pulse is the area of the trapezoid ABCD, which is VT, i.e. it is independent of T_r . The output of the gated integrator is shown in the bottom waveform. Before the start of the pulse, switch S1 is open and S2 is closed. During integration, from t_1 to t_2 , S1 is closed and S2 is open and at the end of the pulse, S1 is open and S2 is closed, awaiting the arrival of the next pulse. Provided that $(t_2 - t_1)$ exceeds the total pulse duration, the output of the gated integrator is independent of ballistic-deficit effects of using a shaping time comparable with T_r .



Figure 2.18 Principles of operation of a gated integrator.

Figure 2.19 shows the timing and output pulse from a gated integrator acting on a unipolar pulse from the output of a Gaussian shaping network The integration continues for a time that is about 8 to 10 times the shaping time used in the Gaussian network (or 2 to 3 times the peaking time). With a gated integrator, much shorter shaping times can be used than if a gated integrator is not used, which will minimize the effect of pile up on the resolution. The waveform shown in

Figure 2.19 uses a Gaussian shaping time of 0.25 μ s resulting in a total time of 5 μ s above the baseline for the output of the gated integrator. Using a conventional Gaussian shaping amplifier, would need a much longer shaping time of 3 μ s (20 μ s above the baseline) to achieve the same resolution. Thus, use of a gated integrator enables higher-rate operation before the effects of pile up become a limiting factor. Systems using gated integrators can operate with germanium detectors up to a count rate of 10⁶ pulses/sec with good energy resolution.



T_P is, typically, 2×amplifier shaping time.