ELECTRONICS

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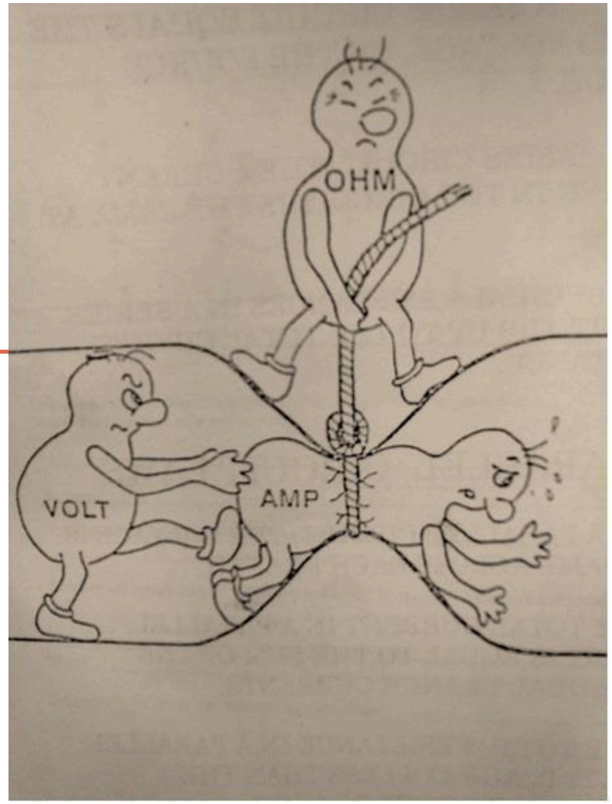


Recapitulation: Extended Ramo theorem

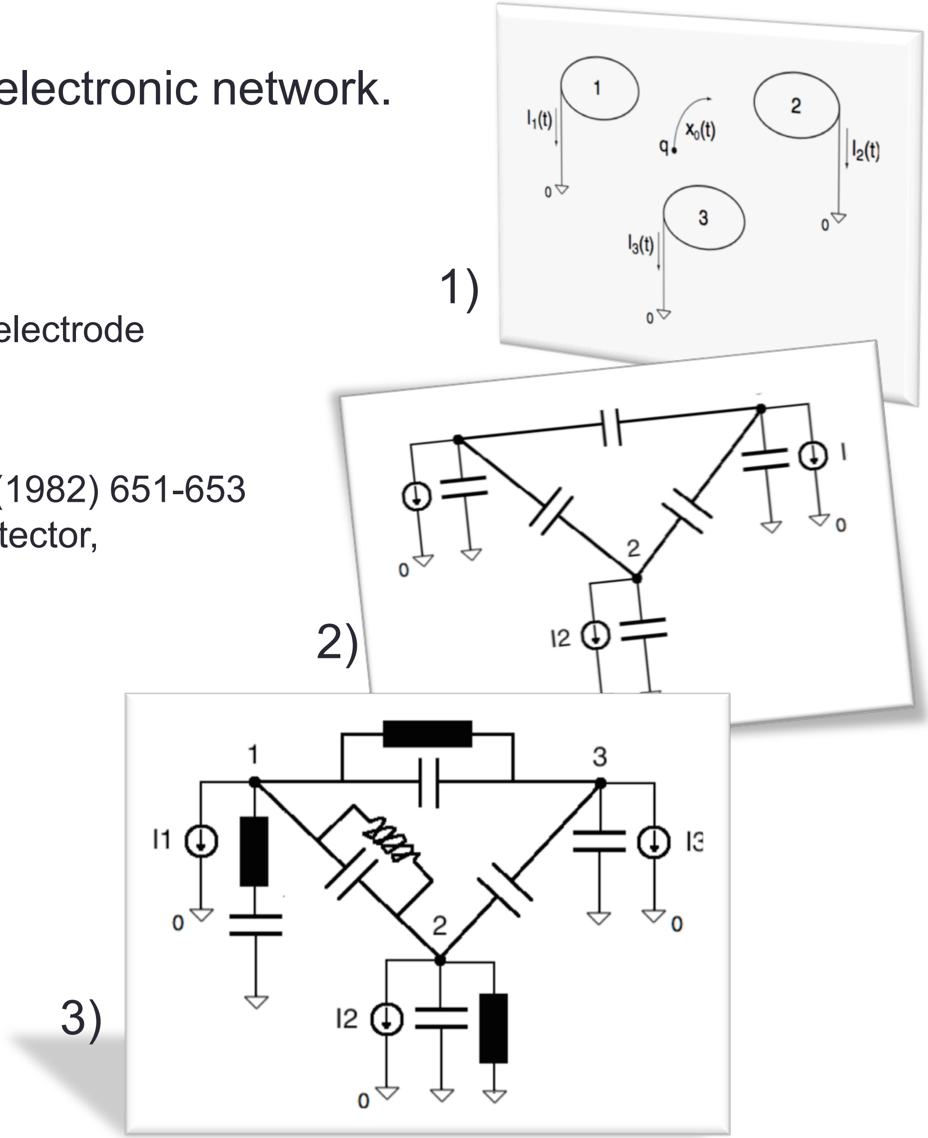
- Describes detectors in a realistic electronic network.

- In 3 steps:
  - 1) Apply the Ramo theorem:
    Calculate the induced currents in each electrode

  - 2) Equivalent electronics scheme:
    Proof: see Gatti and Padivini, NIM 193 (1982) 651-653
    -Determine the capacitances of your detector,
    -Add the current sources found from 1)

  - 3) Realistic electronics scheme:
    Change the above simplified scheme
    into a realistic model

- Result = realistic signals



The Laplace Transform

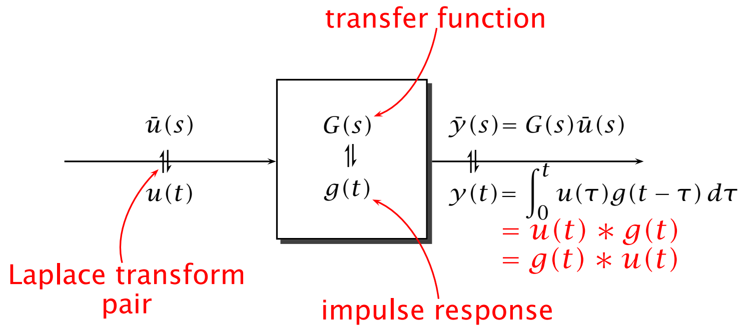
- Laplace Transform of f(t): $F(s) = \int_0^\infty f(t) e^{-st}\, dt$ with complex frequency $s \in \mathbf{C}$

- Transforms integral and differential equations into algebraic equations:
  L, R, C circuits easily solved with **impedances**

$$i(t) = C\frac{du(t)}{dt} \rightarrow I(s) = C \cdot s \cdot U(s) \qquad \text{or} \qquad \frac{U(s)}{I(s)} = \frac{1}{sC}$$

X	$Z_X(s)$
R	R
C	1/sC
L	sL

- Convolution becomes multiplication



- Fourier transform is for periodic functions, while
  Laplace transform for signals “switching on” at time t=0 : f(t<0) = 0

The Laplace Transform rules

$\mathcal{L}(f(t)+g(t)) = \mathcal{L}(f(t)) + \mathcal{L}(g(t))$	Linearity. The Laplace of a sum is the sum of the Laplaces.
$\mathcal{L}(cf(t)) = c\mathcal{L}(f(t))$	Linearity. Constants move through the $\mathcal{L}$-symbol.
$\mathcal{L}(y'(t)) = s\mathcal{L}(y(t)) - y(0)$	The t-derivative rule. Derivatives $\mathcal{L}(y')$ are replaced in transformed equations.
$\mathcal{L}\left(\int_0^t g(x)dx\right) = \frac{1}{s}\mathcal{L}(g(t))$	The t-integral rule.
$\mathcal{L}(tf(t)) = -\frac{d}{ds}\mathcal{L}(f(t))$	The s-differentiation rule. Multiplying f by t applies $-d/ds$ to the transform of f.
$\mathcal{L}(e^{at}f(t)) = \mathcal{L}(f(t))\vert_{s\to(s-a)}$	First shifting rule. Multiplying f by e^{at} replaces s by $s-a$.
$\mathcal{L}(f(t-a)H(t-a)) = e^{-as}\mathcal{L}(f(t))$, $\mathcal{L}(g(t)H(t-a)) = e^{-as}\mathcal{L}(g(t+a))$	Second shifting rule. First and second forms.
$\mathcal{L}(f(t)) = \frac{\int_0^P f(t)e^{-st}dt}{1-e^{-Ps}}$	Rule for P-periodic functions. Assumed here is $f(t+P) = f(t)$.
$\mathcal{L}(f(t))\mathcal{L}(g(t)) = \mathcal{L}((f*g)(t))$	Convolution rule. Define $(f*g)(t) = \int_0^t f(x)g(t-x)dx$.

Laplace Transform lookup table

$F(s)$	$f(t) \quad 0 \le t$
1. 1	$\delta(t)$ unit impulse at $t = 0$
2. $\frac{1}{s}$	1 or $u(t)$ unit step starting at $t = 0$
3. $\frac{1}{s^2}$	$t \cdot u(t)$ or t ramp function
4. $\frac{1}{s^n}$	$\frac{1}{(n-1)!}t^{n-1}$ n = positive integer
5. $\frac{1}{s}e^{-as}$	$u(t-a)$ unit step starting at $t = a$
6. $\frac{1}{s}(1-e^{-as})$	$u(t)-u(t-a)$ rectangular pulse
7. $\frac{1}{s+a}$	e^{-at} exponential decay
8. $\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ n = positive integer
9. $\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$
10. $\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}(1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt})$
11. $\frac{s+\alpha}{s(s+a)(s+b)}$	$\frac{1}{ab}[\alpha-\frac{b(\alpha-a)}{b-a}e^{-at}+\frac{a(\alpha-b)}{b-a}e^{-bt}]$
12. $\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}(e^{-at}-e^{-bt})$
13. $\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b}(ae^{-at}-be^{-bt})$

…or use mathematica or similar

Steve Jobs, 1955 – 2011

AC small signal equivalent scheme…

- Analog electronics schemes can be separated in a **DC scheme + AC small signal** equivalent scheme

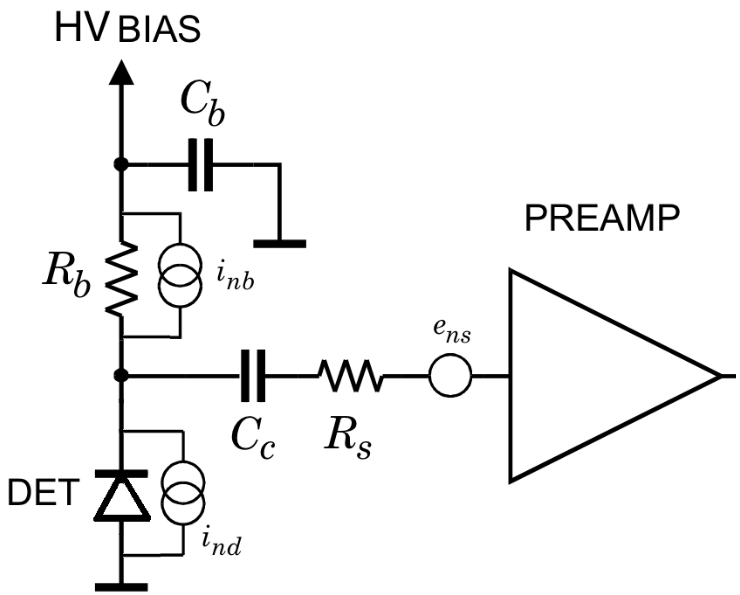
- The **DC scheme** serves to properly bias all the components
  - only DC currents and potentials are regarded
  - AC current / potential sources are taken out

- The **AC scheme equivalent** for small signal perturbations
  - DC bias values are used to define a linearization of non-linear elements
  - DC current / potential sources are taken out

- (Superposition principle) the solution for the whole circuit = **The small signal solution superposed on the DC bias values**.

AC equivalent scheme… example

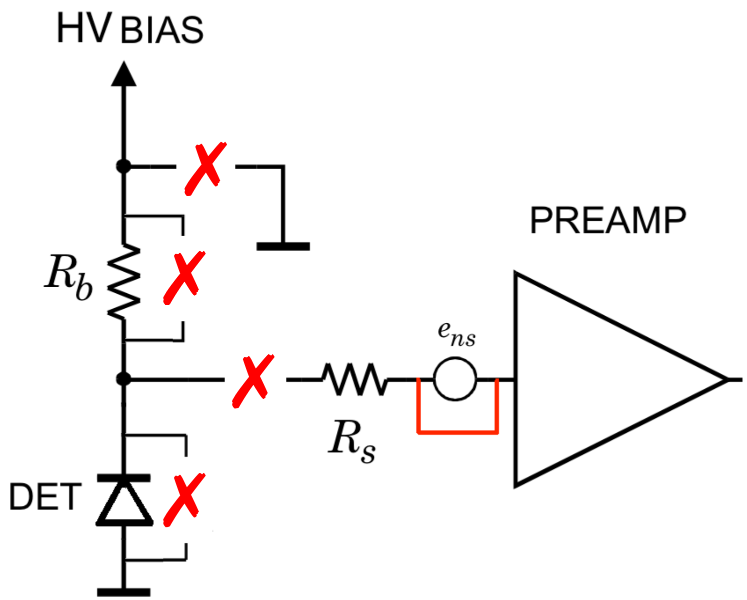
- EXAMPLE: Noise analysis of a detector.



- (Convention: use capital letters for DC quantities,small letters for AC quantities)
- C_b = HV filter
- R_b = HV load resistor (1G ohm)
- C_c = HV coupling capacitor (1000 pF)
- R_s = any resistivity betw. det and preamp (e.g. resistivity wires, det. Electrodes,..)
- i,e = AC current / voltage noise sources (leave out if you don't want to study noise)

1) DC scheme

- Reduction to DC equivalent scheme

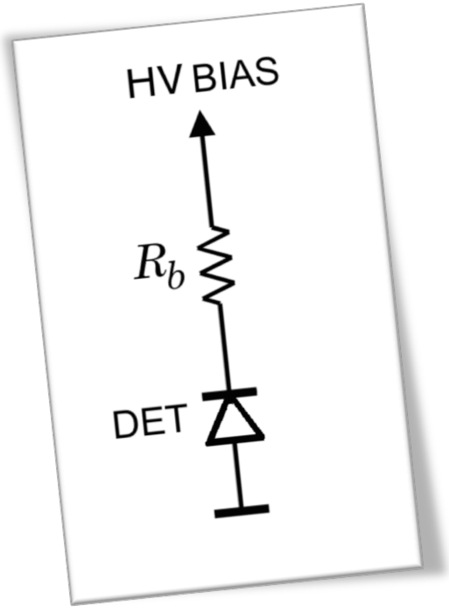


Consider:

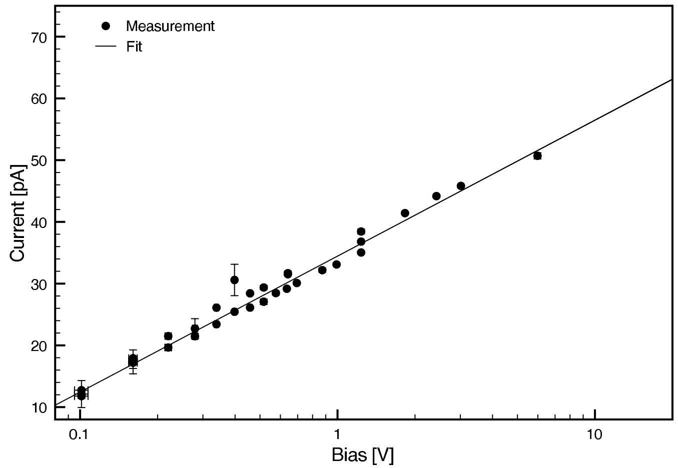
- Capacities as open circuit, inductances as short circuit.
- AC current sources as open circuit, AC voltage sources as short circuit
- Remark: a DC coupled preamp would remain in the scheme. It can be used to monitor the leakage current

1) DC scheme

- Solution:



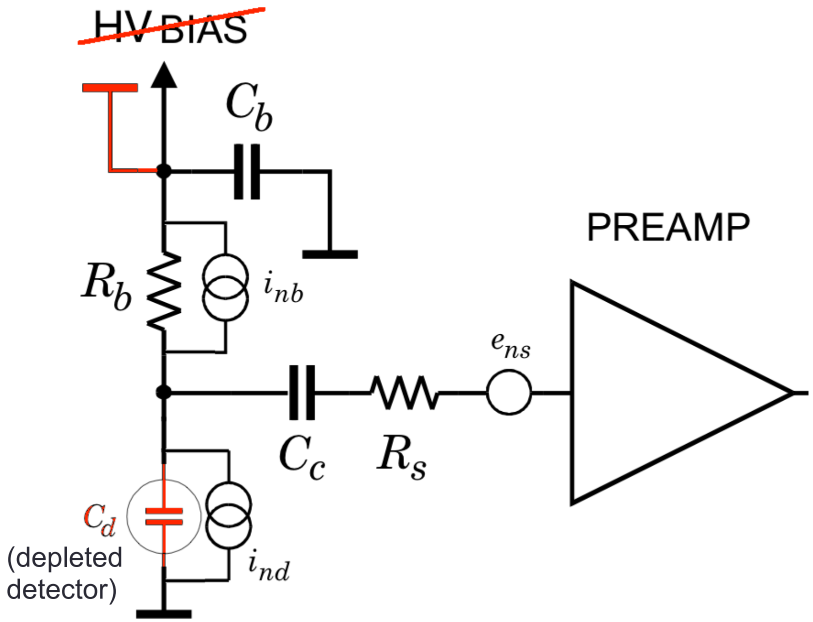
- DC leakage current I_d found by analysis of "load line"
- In reality: take a HV module with current readout
- For AGATA detector, I_d = 100 pA (not measureable with HV module)
- I_d known, Potential drop over R_b is:
  $V_{drop} = 10^{-10}\text{A} \times 10^{9}\Omega = 0.1\text{V}$
  (here negligible, but not always…)



Leakage current of an AGATA detector
Birkenbach, et al. NIM A Vol.40, Iss. 1 (2011), p.176

2) AC equivalent scheme

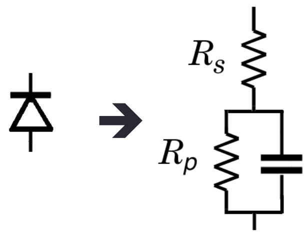
- Reduction to AC equivalent scheme.



- Consider:
  DC current sources as open circuit,
  DC potential sources as short connected

- Replace nonlinear elements by their small signal equivalent (parameters depend on DC bias values)

- Small signal equivalent of a detector:

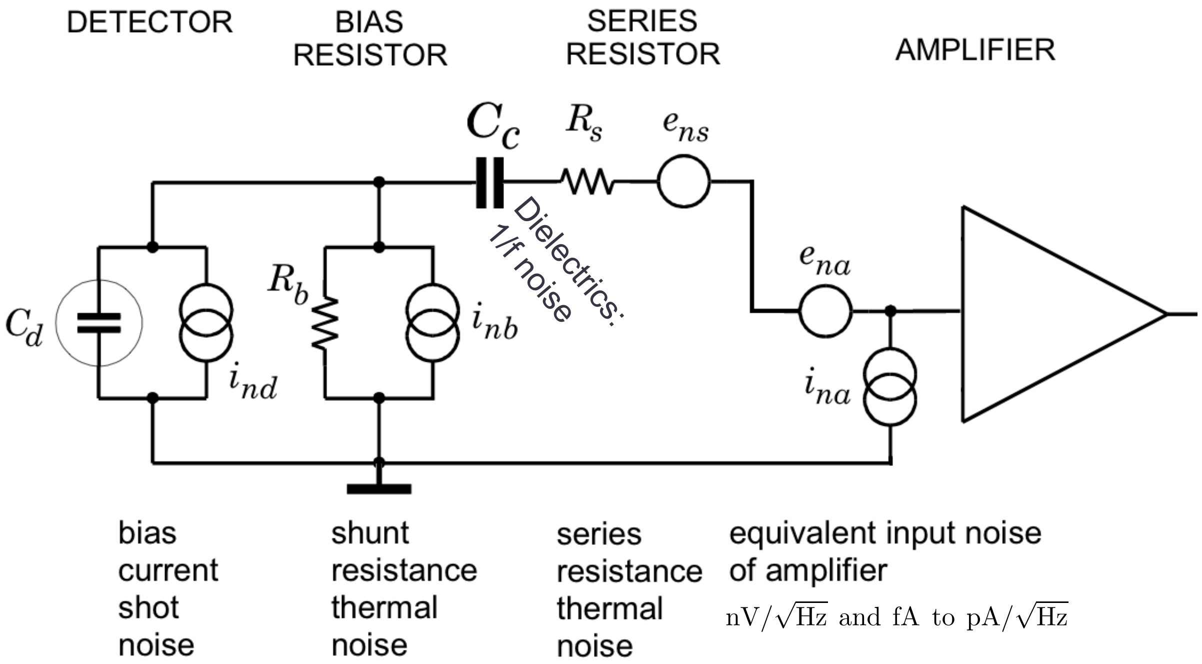


- Series resistance
  (if detector not fully depleted)
- Detector Capacitance
- Parallel resistance
  (allows leakage current)
  (AGATA: > 1TΩ : leave out)

- Series resistance and Capacitance measured by manufacturer as function of V_{bias}
  AGATA: $R_s = 0$, C~46pF at full depletion

2) AC equivalent scheme

- Solution:



- "White" noise sources $i_{nd}^2 = 2eI_d$ $\quad i_{nb}^2 = \frac{4kT}{R_b}$ $\quad e_{ns}^2 = 4kTR_s$
- Remark: in AC scheme, R_b becomes **in parallel** with the detector !!
- Presence of C_d makes the white noise frequency dependent at the output
- Traditional calculation ends here:
  noise sources are statistical and amplitudes add **quadratically** !!!

Noise analysis

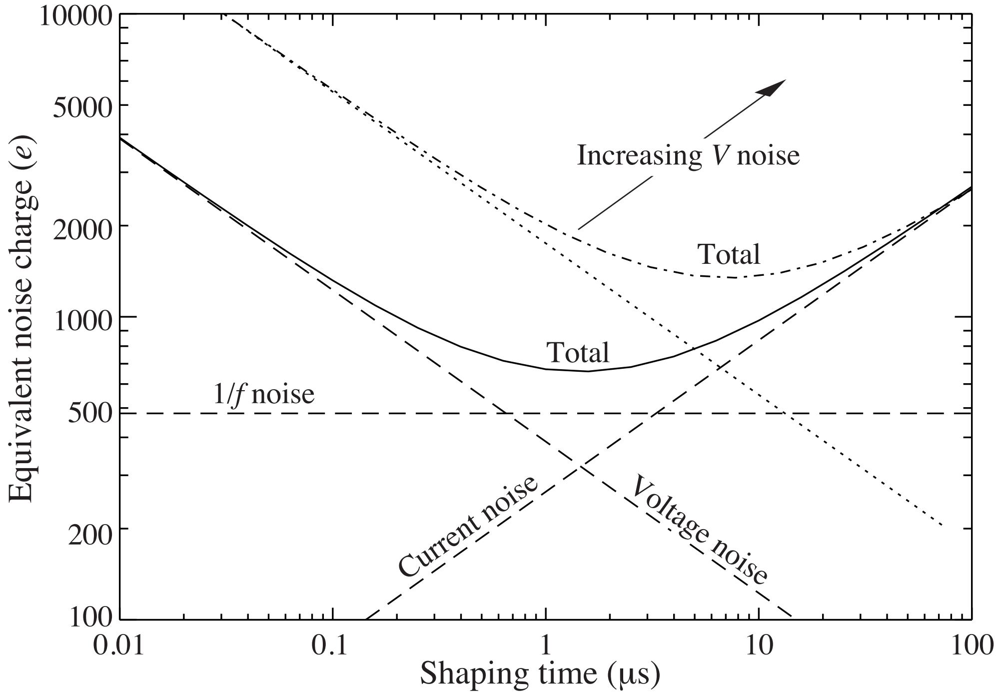
H. Spieler – "Semiconductor detector systems"

- Analysis of noise using an CR-RC shaper (time constant T_s):

$$Q_n^2 = \left(2eI_d + \frac{4kT}{R_b} + i_{na}^2\right) F_i T_S + \left(4kTR_s + e_{na}^2\right) F_v \frac{C_d^2}{T_S} + F_{vf} A_f C_d^2$$

$$F_i = F_v = 0.9$$
$$F_{vf} = 4$$

- = current noise + voltage noise + 1/f noise



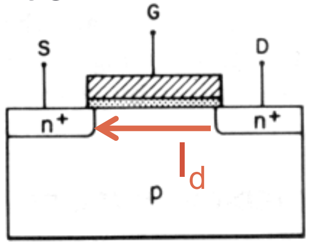
AGATA: normal = 6us shaping

If leakage current increases, optimum shaping time moves to left

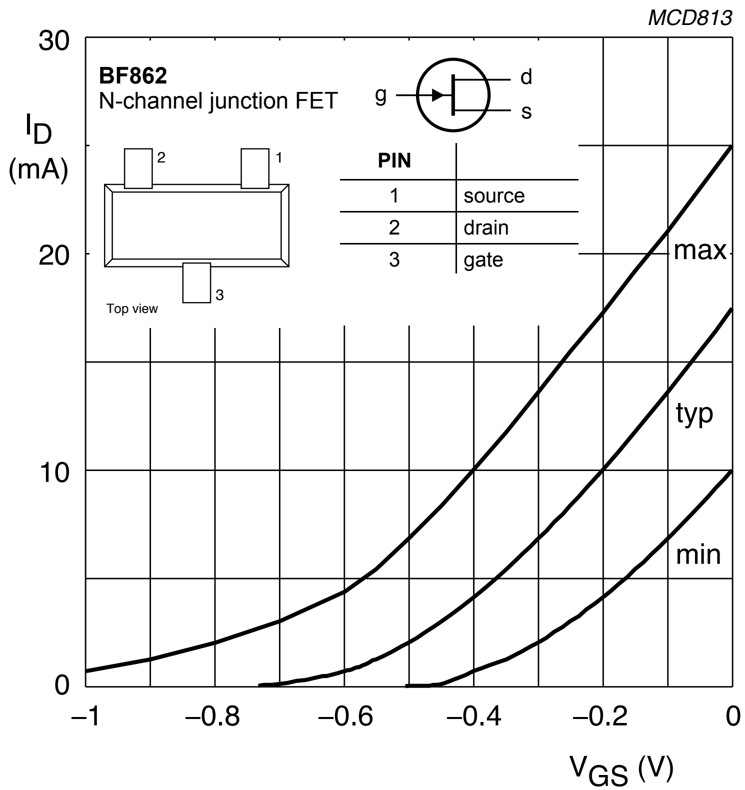
The Field Effect Transistor

- Surface effect conduction under control of gate potential:

$$I_d = K_0 . \frac{W}{L} . (Vgs - Vt)^2$$

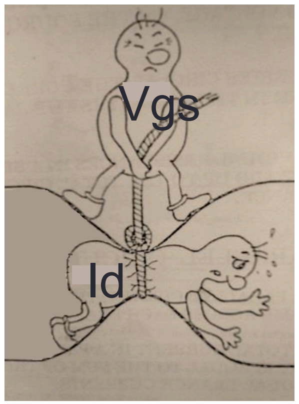


- W/L=transistor aspect ratio
- Ko=f(mobility, gate capacitance, T, …)
- Vt=f(dopant, gate capacitance, fixed charges

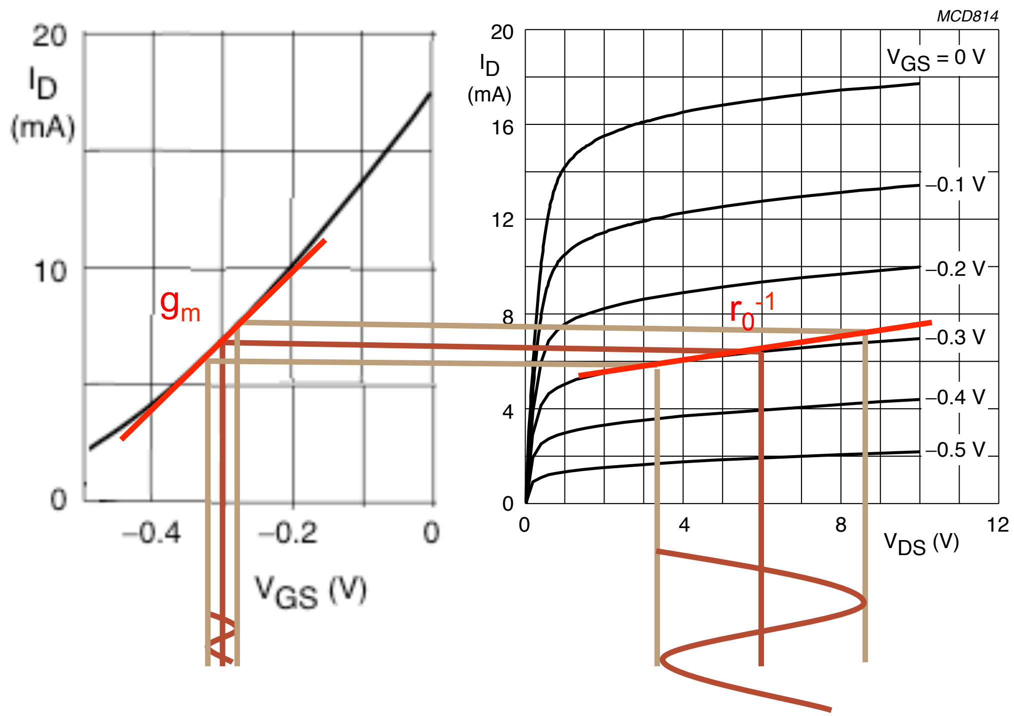


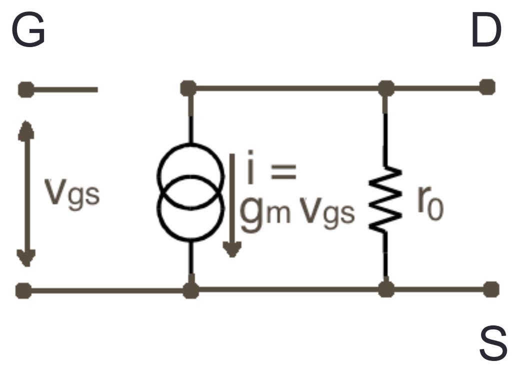
- Transconductance "g_m"

$$gm = \frac{\Delta I_d}{\Delta Vgs} = 2\sqrt{K_o . \frac{W}{L} . I_d}$$



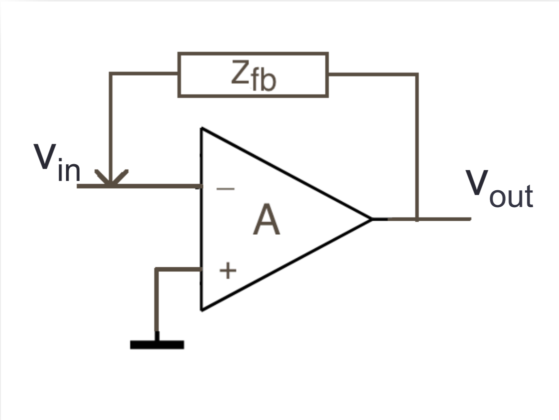
The FET small signal equivalent





- $g_m \approx 45$ mS (BF862)
- r_0 = 50k – 1M, often omitted
- Sometimes also C_{gs}

Realistic Preamplifiers Model



$$v_{out} = -Av_{in}$$
$$i_{in} = (v_{in} - v_{out})/Z_{FB}$$

Or
$$v_{out} = \frac{-AZ_{FB}}{1+A} i_{in}$$

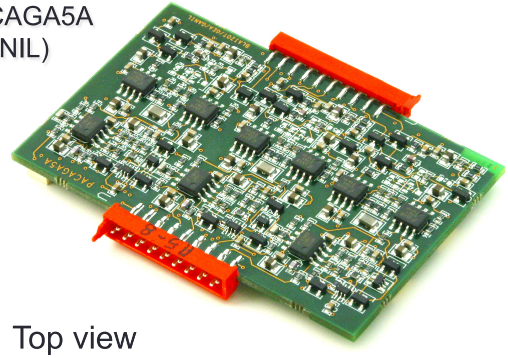
for ideal CSP: $Z_{FB} = 1/sC_{FB}$

Ideal Opamps

- Open loop gain A is very high ($10^5 \sim \infty$)
  negative Feed-back prevents saturation
- If output is not saturated, then difference between "+" and "-" must be nihil
  if "+" is ground, then "-" is a **virtual ground**
- Input impedance is very high
  no current flows into preamp
- We use preamps with only the inverting input
  this allows to reduce the noise by $\sqrt{2}$

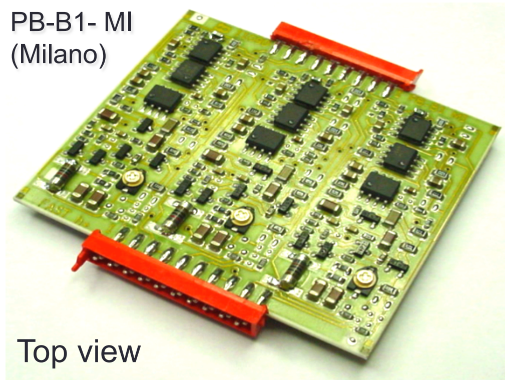
Developed preamplifiers

PACAGA5A
(GANIL)



Top view

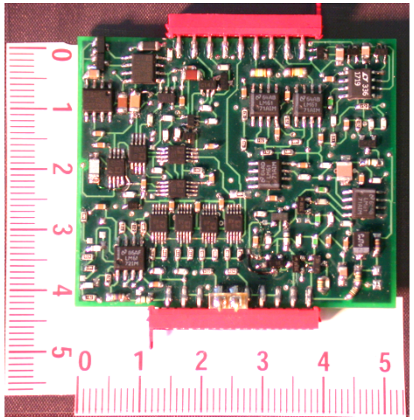
PB-B1- MI
(Milano)

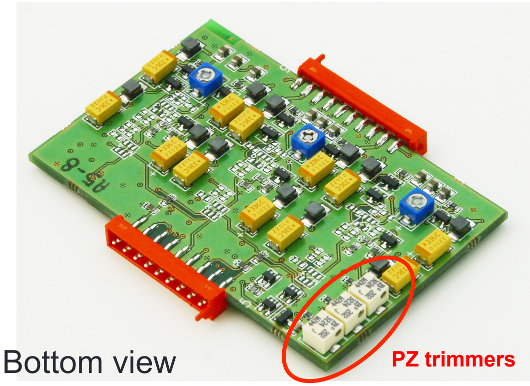


Top view

AGATA_
core-pulser
(Koeln)

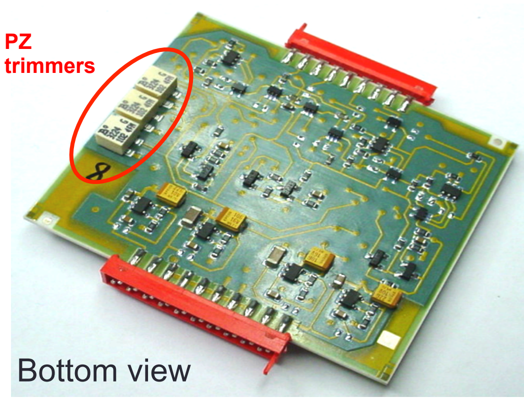
1 Channel
version





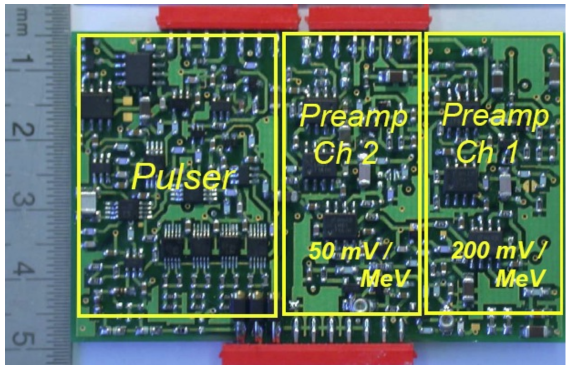
Bottom view

PZ
trimmers



Bottom view

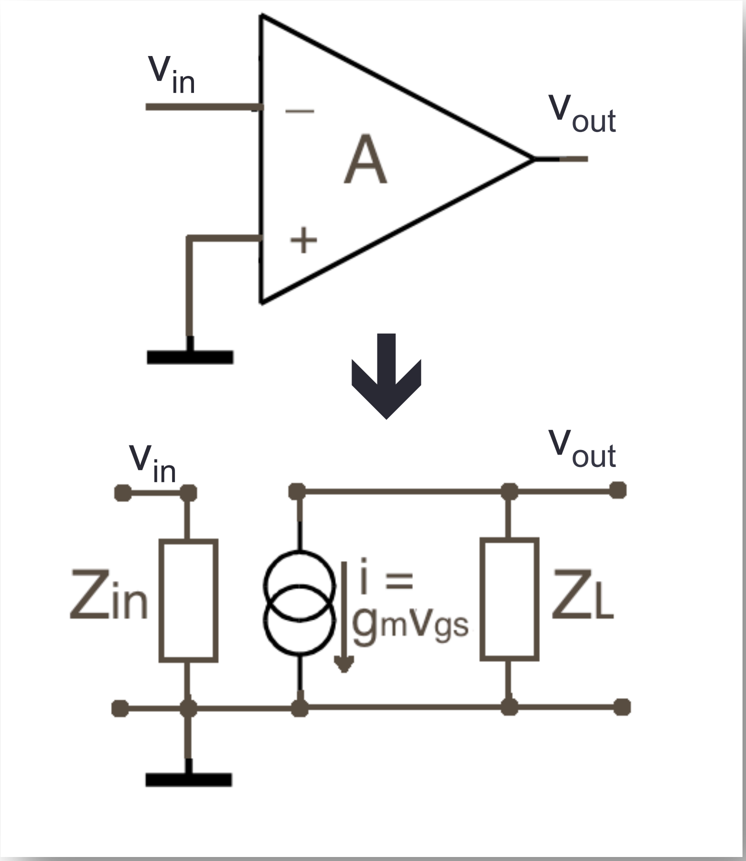
New version: “Dual Core“



Triple segment preamp on FR4 substrate	Triple segment preamp on alumina substrate	Core preamplifier & built-in pulser on FR4 substrate
Mod. “PACAGA5A” – GANIL	**Mod. “PB-B1 MI” – Milano**	**Mod. “AGATA core-pulser” – Koeln**
B. Cahan et al.	A. Pullia et al.	G. Pascovici et al.

A. Pullia, G. Pascovici, B. Cahan, D. Weisshaar, C. Boiano, R. Bassini, M. Petcu, F. Zocca, “**The AGATA charge-sensitive preamplifiers with built-in pulser and active-reset device**”, Proc. Nucl. Sci. Symp., October 16-22, 2004, Rome, Italy

Realistic Preamplifiers Model



More realistic preamps

- FET small equivalent with load impedance
  $Z_L = R_L \parallel C_L$ ($Z_{in} = C_{gs}$)

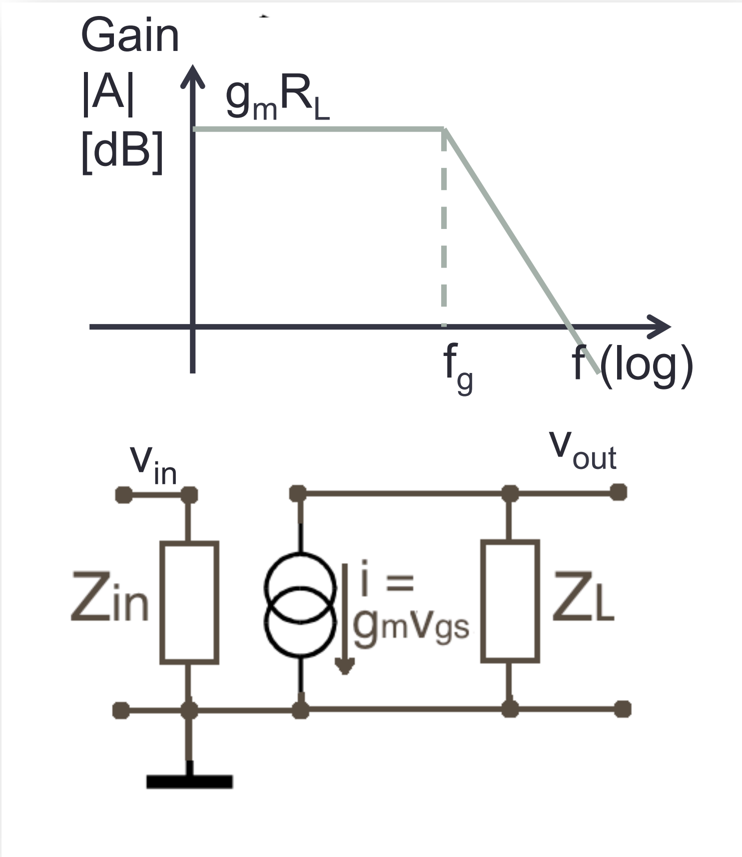
- Open loop gain is now frequency dependent
  Signals will have realistic rise times.

$$A(s) = \frac{v_{out}}{v_{in}} = -g_m Z_L$$

$$= -g_m \frac{1}{1/R_l + sC_L} = \frac{-g_m R_L}{1 + sC_L R_L}$$

- Cut off frequency $f_g = \frac{1}{2\pi R_L C_L}$
  limited by Nyquist theorem

Realistic Preamplifiers Model



More realistic preamps

- FET small equivalent with load impedance
  $Z_L = R_L \,//\, C_L$ $(Z_{in} = 1/sC_{gs})$
- Open loop gain is now frequency dependent
  Signals will have realistic rise times.

$$A(s) = \frac{v_{out}}{v_{in}} = -g_m Z_L$$

$$= -g_m \frac{1}{1/R_l + sC_L} = \frac{-g_m R_L}{1 + sC_L R_L}$$

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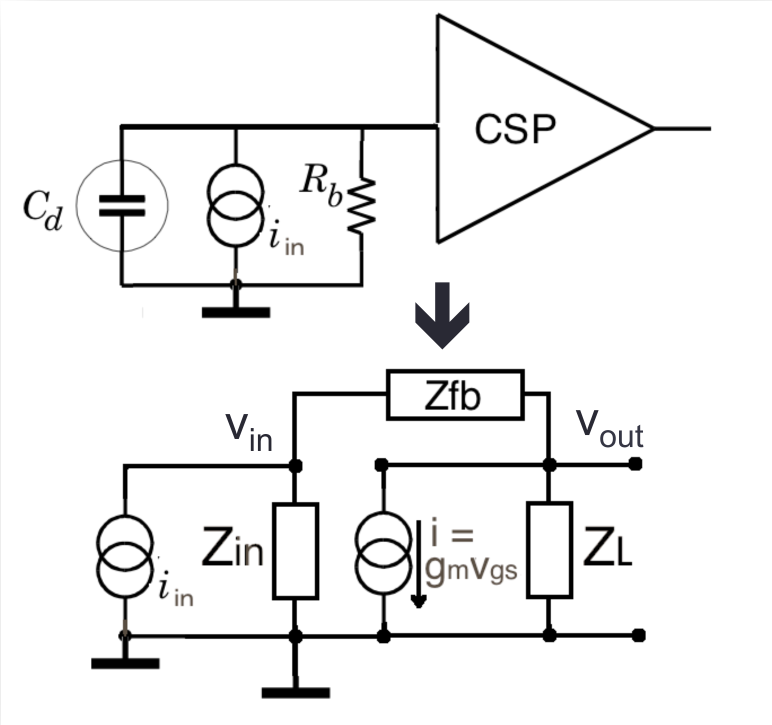
Realistic Preamplifiers Model



Charge Sensitive Preamp

- i_{in} = current from Ramo Theorem
- Z_{in} = C_d // C_{gs} // R_b ~ 50pF // 1G
- Z_L = R_L // C_L
- Z_{fb} = C_{fb} // R_{fb} =1pF // 1G

Nodal network:

$$i_L = i_{FB} + g_m v_{in} \quad \texttt{or}$$

$$-\frac{v_{out}}{Z_L} = \frac{v_{out} - v_{in}}{Z_{FB}} + g_m v_{in}$$

$$\approx \frac{v_{out}}{Z_{FB}} + g_m v_{in} \quad \text{(Virtual ground)}$$

$$i_{in} = \frac{v_{out} - v_{in}}{Z_{FB}} + \frac{v_{in}}{Z_{in}}$$

Realistic Preamplifiers Model

SOLUTION: (Compare with ideal integrator)

$$\frac{v_{out}}{i_{in}} = -\frac{-Z_{FB} \cdot A_{eff}}{1 + A_{eff}} \approx -Z_{FB} = \frac{R_{fb}}{1 + sR_{FB}C_{FB}}$$

Looking this function up in the list yields:

$$v_{out}(t) = i_{in}(t) \star \frac{1}{C_{FB}} \cdot \left(e^{-t/\tau}\right) \qquad \texttt{with} \qquad \tau = R_{FB}C_{FB} = 1ms$$

The output voltage is a convolution with a near-step function.

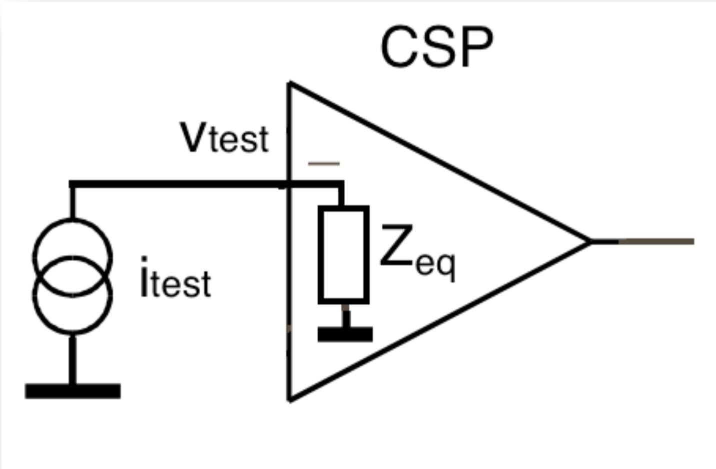
$$A_{eff} = \frac{g_m}{Z_{FB}} \cdot [Z_{FB}//Z_L] \cdot [Z_{FB}//Z_{in}]$$

For the approximation, this "effective" open loop gain should be still large

Another result, of importance for dynamic input impedance calculations – see next slide:

$$\frac{v_{out}}{v_{in}} = -g_m \left[Z_{FB}//Z_L\right]$$

Dynamic input impedance of CSP



- Apply a test current to the CSP, from previous results:

$$v_{out} = -Z_{FB}\ i_{test}$$
$$v_{out} = -g_m\ [Z_{FB}//Z_L]\ v_{test}$$

- The dynamic input impedance is:

$$Z_{eq} = \frac{v_{test}}{i_{test}} = \frac{Z_{FB}}{g_m\ [Z_{FB}//Z_L]}$$

or

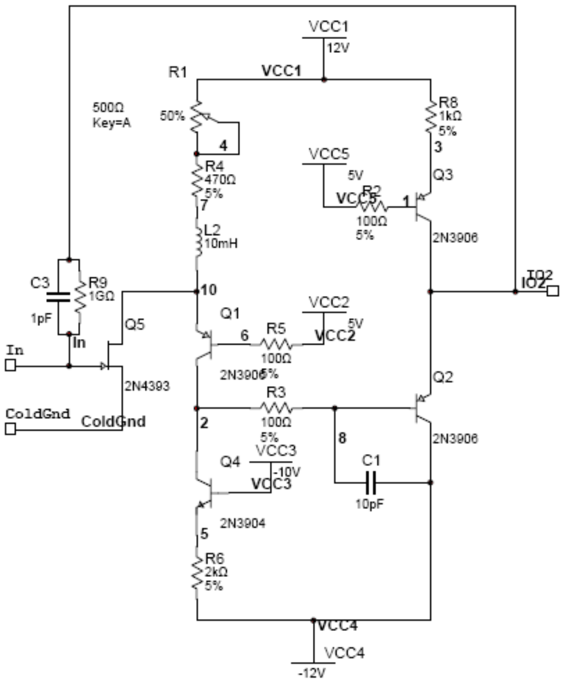
$$Z_{eq} = \frac{Z_{FB}}{g_m R_L} + \frac{C_{FB} + C_L}{g_m C_{FB}}$$

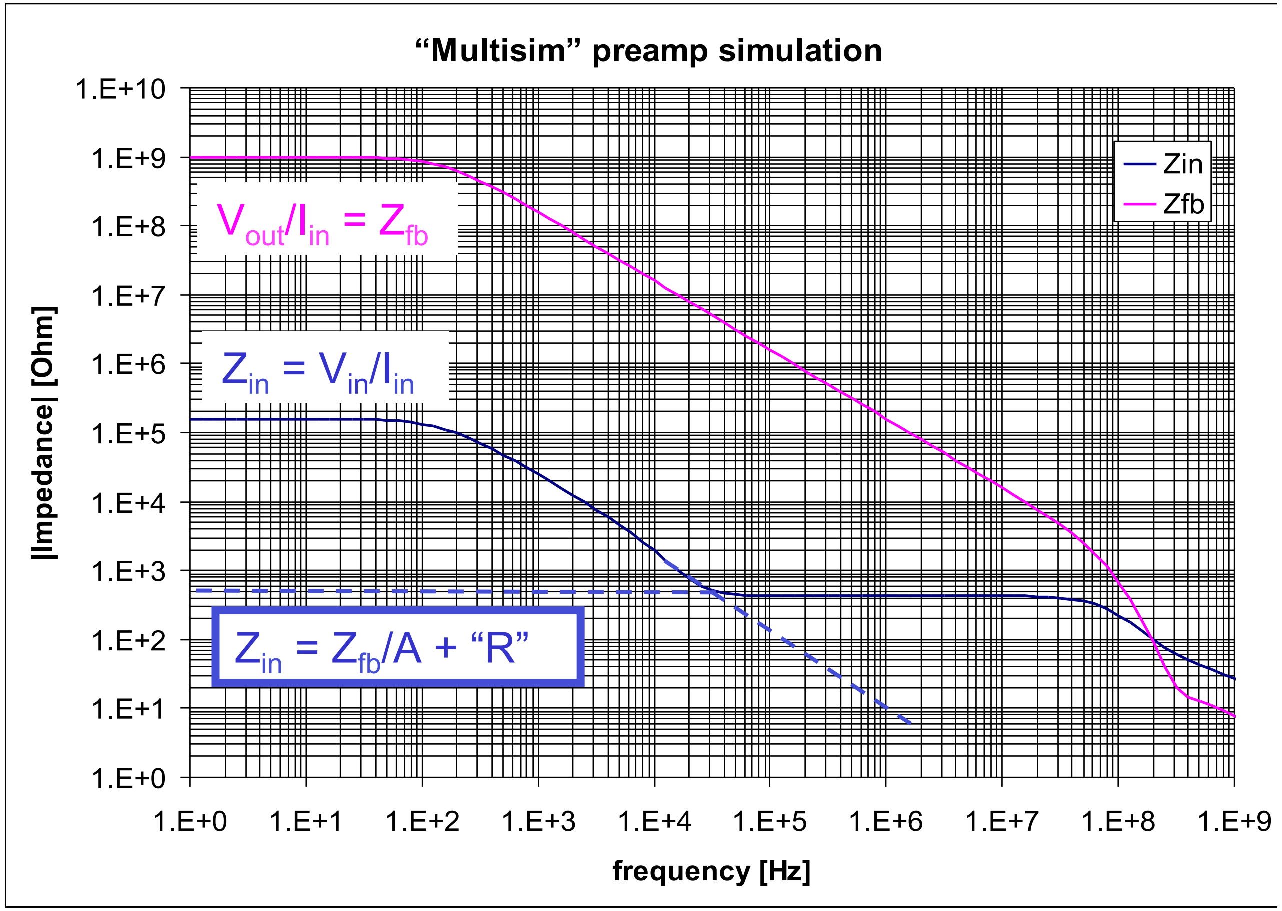
Zeq = Miller equiv. + noiseless R_{eq}

$$Z_{eq} \approx \frac{1}{sAC_{fb}} + R_{eq}$$

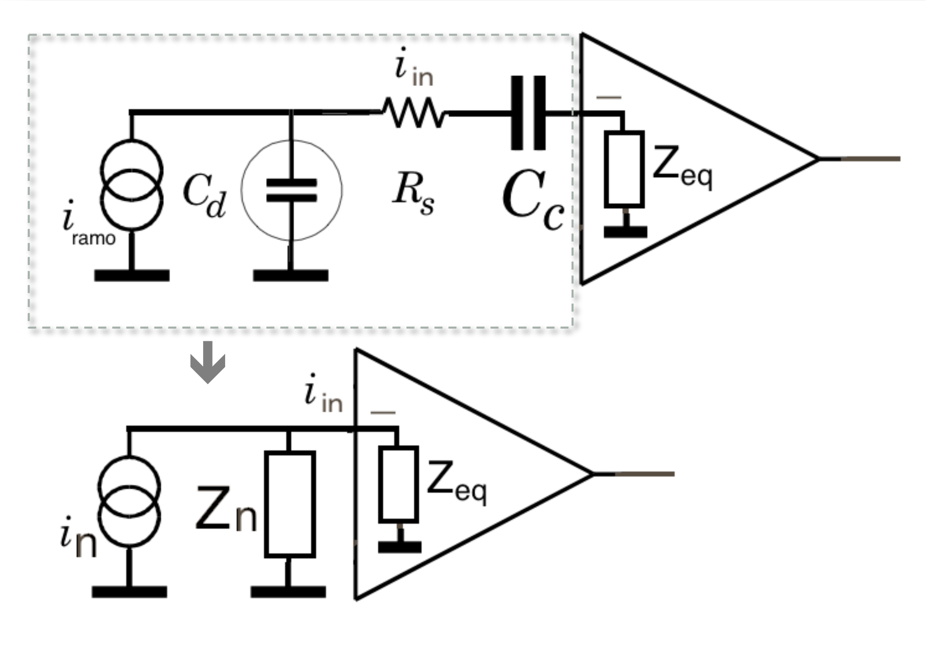
A numerical simulation (Multisim)

Model "HeKo"
(Miniball)





Preamp rise time



- Consider an undepleted detector with coupling capacitor C_c
- Calculation simplifies by using a Norton equivalent i_n, Z_n

$$Z_n = R_s + 1/sC_d + 1/sC_c$$

$$i_n = \frac{1/sC_d}{Z_n} i_{ramo}$$

- In the Norton equivalent scheme:

$$v_{out} = -Z_{FB} \cdot i_{in}$$

$$= -Z_{FB} \cdot \frac{Z_n}{Z_n + Z_{eq}} \cdot i_n$$

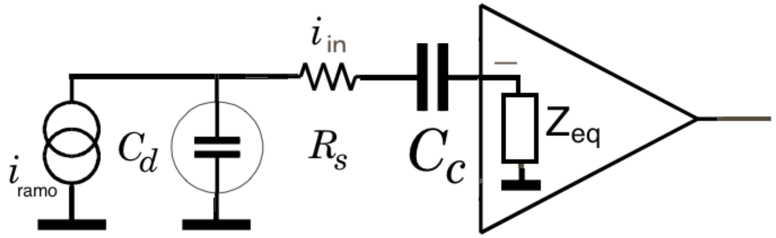
- The current divider term equals:

$$v_{out} = -Z_{FB} \cdot \frac{1/sC_d}{Z_n + Z_{eq}} i_{ramo}$$

$$= -Z_{FB} \cdot \boxed{\frac{1}{1 + \frac{C_d}{C_c} + \frac{Cd}{AC_{FB}}} \cdot \frac{1}{1 + sC_d(R_s + R_{eq})}} i_{ramo}$$

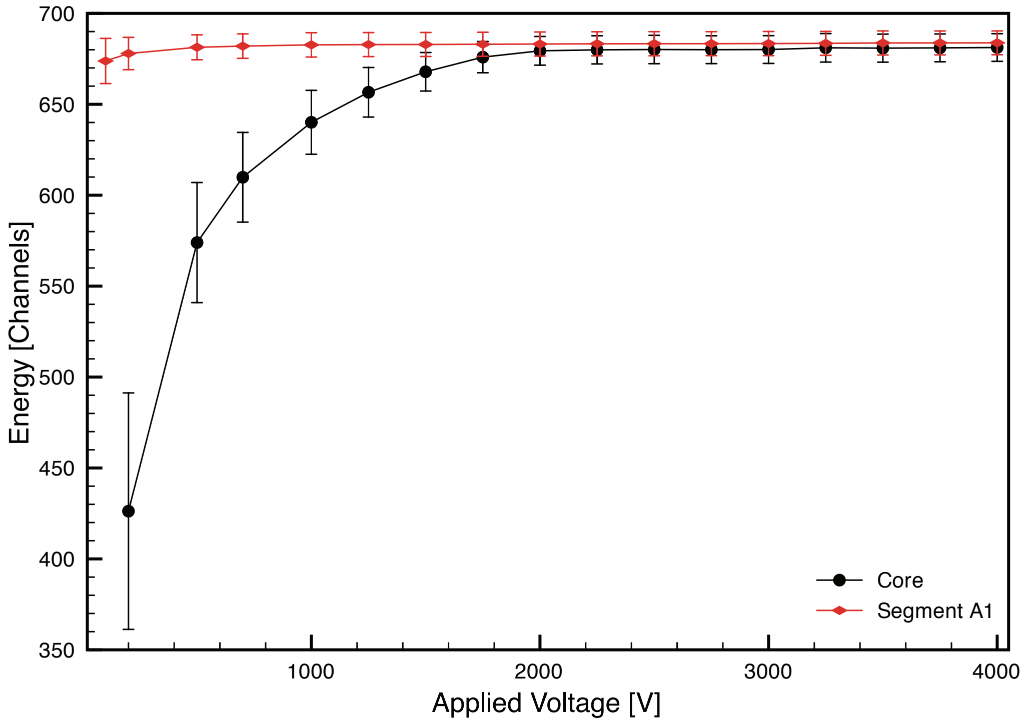
Undepleted detector = amplitude drop and rise time increase

Preamp rise time



$$v_{out} \propto \frac{1}{1 + \frac{C_d}{C_c} + \frac{Cd}{AC_{FB}}} \cdot \frac{1}{1 + sC_d(R_s + R_{eq})}$$

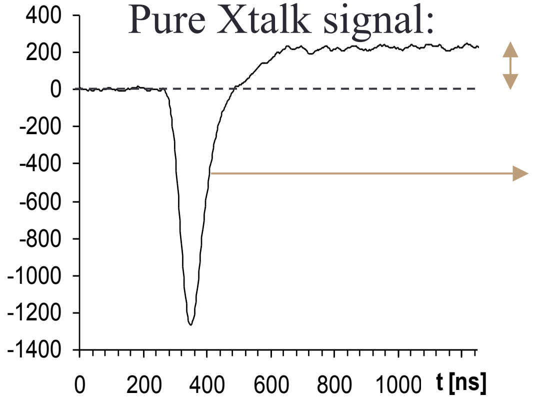
Birkenbach, et al. NIM A Vol.40, Iss. 1 (2011), p.176



- Variation of Am peak position with bias voltage
- Segments shift less due to DC coupling
- Allows determination of depletion voltage and impurity concentration

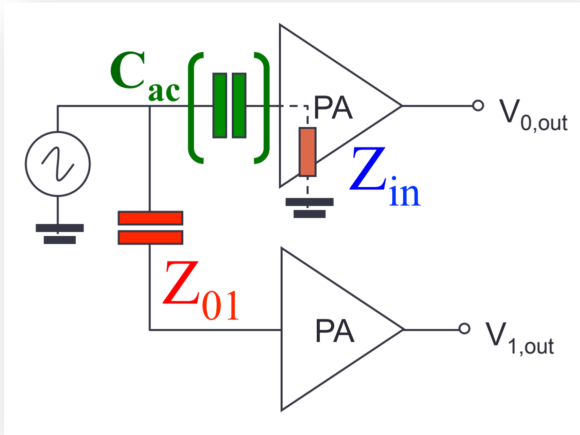
- Error bars in graph correspond to FWHM of the energy peak
- Error bars increase with increased capacity and serial resistivity

Origin of cross talk



Proportional Xtalk (50μs decay) → Energy

Differential Xtalk (only during risetime) → PSA



With $Z_{in} = 1/sAC_{fb} + (1/sC_{ac}) + R_{eq}$

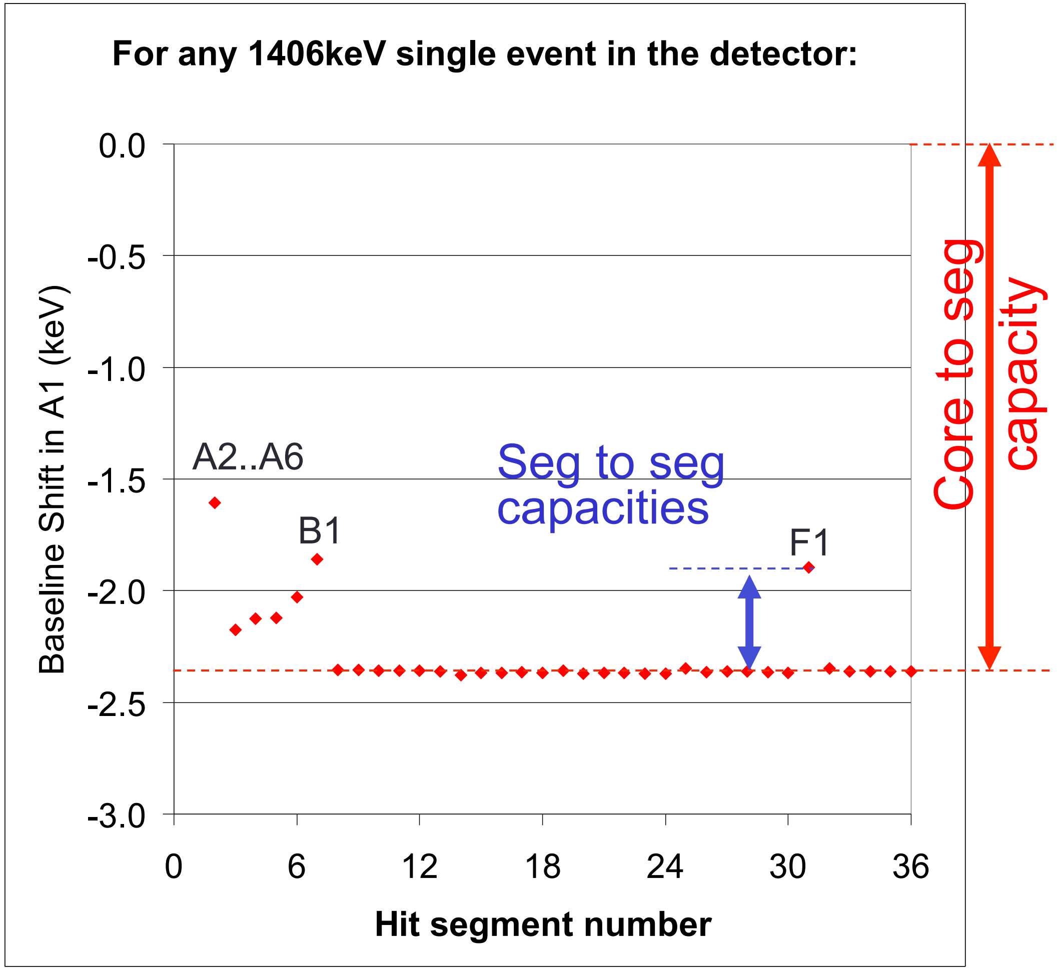
$$\text{Xtalk} = Z_{in} / (Z_{01}+Z_{in}) \sim Z_{in} / Z_{01}$$

$$\sim \underbrace{C_{01}/AC_{fb} + (C_{01}/C_{ac})}_{} + \underbrace{s \,.\, R_{eq}\, C_{01}}_{}$$

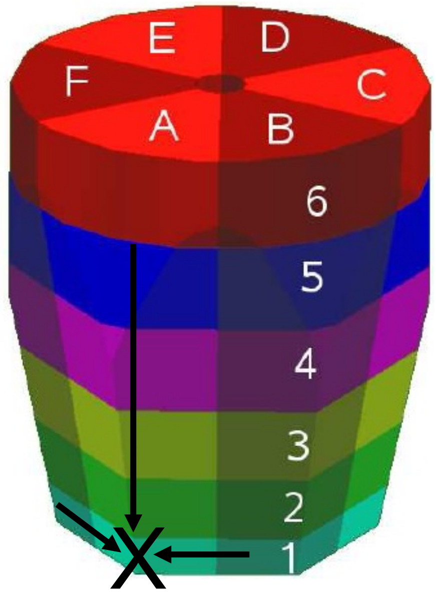
$$= \quad \text{Proportional} \quad + \text{Differential Xtalk}$$

Proportional and Differential Xtalk are related by **same C_{01}**

Proportional Cross talk



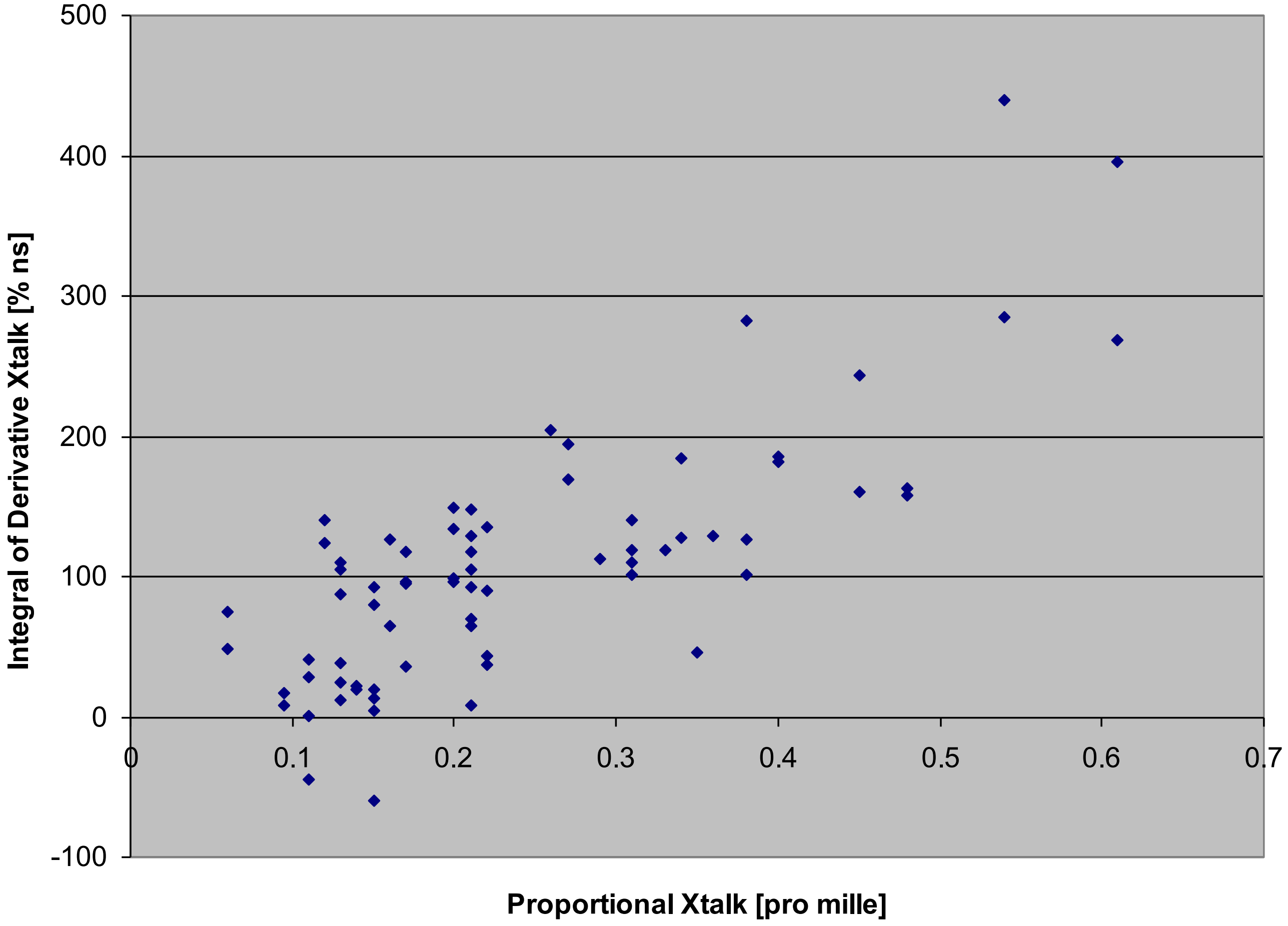
Segment labeling:



Sectors: A...F

Rings: 1...6

Derivative cross talk is correlated



Summary

G. Pascovici – "Front-End Electronics for Large Arrays of HP-Ge", MSU, Sept. 2006

- AC equivalent schemes is all we need
- Charge sensitive preamp:

$$v_{out} = -Z_{FB}\, i_{in}$$

- CSP's dynamic input impedace Z_{eq} is source of all evil
- You should now be able to understand
- Ramo's extended theorem yields crosstalk

Specs	*IKP-Cologne (a) (FET_BF862)*
Sensitivity (*mV / MeV*)	~ *100 mV/MeV* (*differential*)
Resolution (*Cd= 0pF; cold FET*)	~ *600 eV*
Slope (+ *eV/ pF*) *[Cd]*	< *10 eV / pF* (cold FET)
Rise time *) (*Cd= 0pF*); **[Amplit.]*	< *10 ns* (cold FET)
Slope (+ *ns/ pF*) *[Cd]*	~ *0.2 ns* (~ 20 ns / 45 pF)
U(out) @ *[100 Ohm]* / *Power* *[mW]*	~ 2.0V*/ ~290 mW (*LM-6171; *AD-8057*)
Saturation of the 1st stage @	*equiv.* ~ *90 MeV* (*@ ~20 mW_jFET*)
Open Loop Gain	> 80,000 - 100,000

References

Rivista Del Nuovo Cimento
Vol 9 nr 1 (1986)