PHYS490: Nuclear Physics

" WE FEEL THAT YOU JUST DON'T APPRECIATE THE IMPORTANCE OF WHAT WE DO HERE"

Advanced Nuclear Physics

Chapter 4

4. Forms of Mean Potential

Nuclear Shell Model

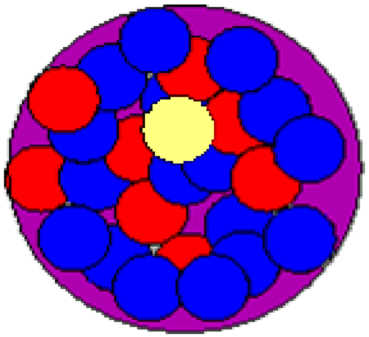
- The nuclear shell model is a model of the nucleus which uses the Pauli exclusion principle to build up the structure of the nucleus in terms of quantised energy levels

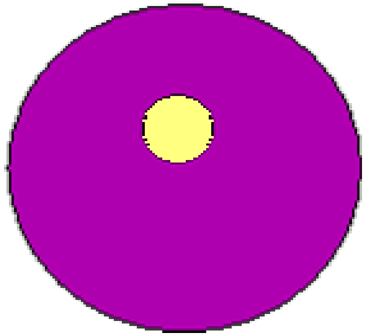
- It is analogous to the atomic shell model which describes the arrangement of electrons in an atom

- Closed (full) shells correspond to stable (more-bound) configurations and define magic numbers for protons and neutrons

Shell Model – Mean Field

N nucleons in
a nucleus

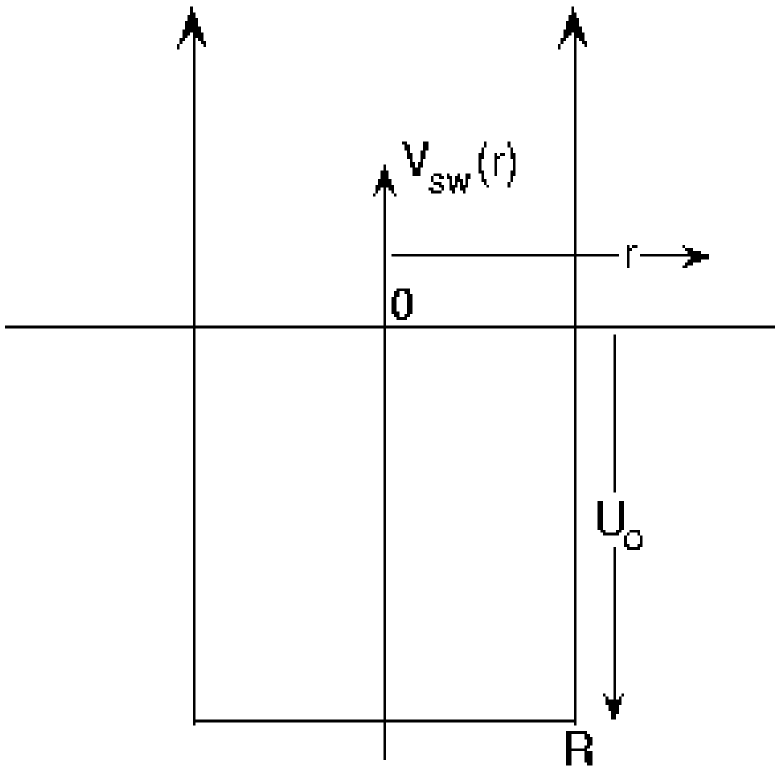




A nucleon in the
Mean Field of
N-1 nucleons

- Assumption – ignore detailed two-body interactions
- Each particle moves in a state independent of the other particles
- The Mean Field is the average smoothed-out interaction with all the other particles
- An individual nucleon only experiences a central force

Square Well Potential



Infinite square well potential

- Simplest form of potential
- Since we have a spherically symmetric potential we can separate the solutions into angular and radial parts

- Radial solutions are Bessel functions which satisfy the boundary condition $R_{n\ell}(R) = 0$
- The eigenenergies are labelled by n and ℓ:

$$E_{n\ell} = (\hbar^2/2mR^2)\xi_{n\ell}^2 - U_0$$

Square Well Quantum Numbers

- 'n' is the principal quantum number
  (number of nodes in wavefunction)

- 'ℓ' is the orbital angular momentum
  ($j = \ell \pm \frac{1}{2}$ is the total particle angular momentum)

- The energies depend simply and monotonically on n and ℓ

Properties of the Solutions

- Higher n : higher energy (more kinetic energy)
- Higher ℓ : higher energy (larger radius, less bound)
- The lowest state is : $1s_{1/2}$ ($n = 1$, $\ell = 0$) - explains ground state of the deuteron: $L = \ell_1 + \ell_2 = 0$
- Note that two orbits can have similar energies if one has larger n and smaller ℓ, or vice versa

Square Well Labels

- The levels are labelled by n and ℓ ('s' = 0, 'p' = 1, 'd' = 2, 'f' = 3, 'g' = 4, 'h' = 5, 'i' = 6, 'j' = 7, 'k' = 8)
- Each level has 2ℓ + 1 substates
- The first few levels (different from H atom):

Level	Occupation	Total
1s	2	2
1p	6	8
1d	10	18
2s	2	20
1f	14	34
2p	6	40

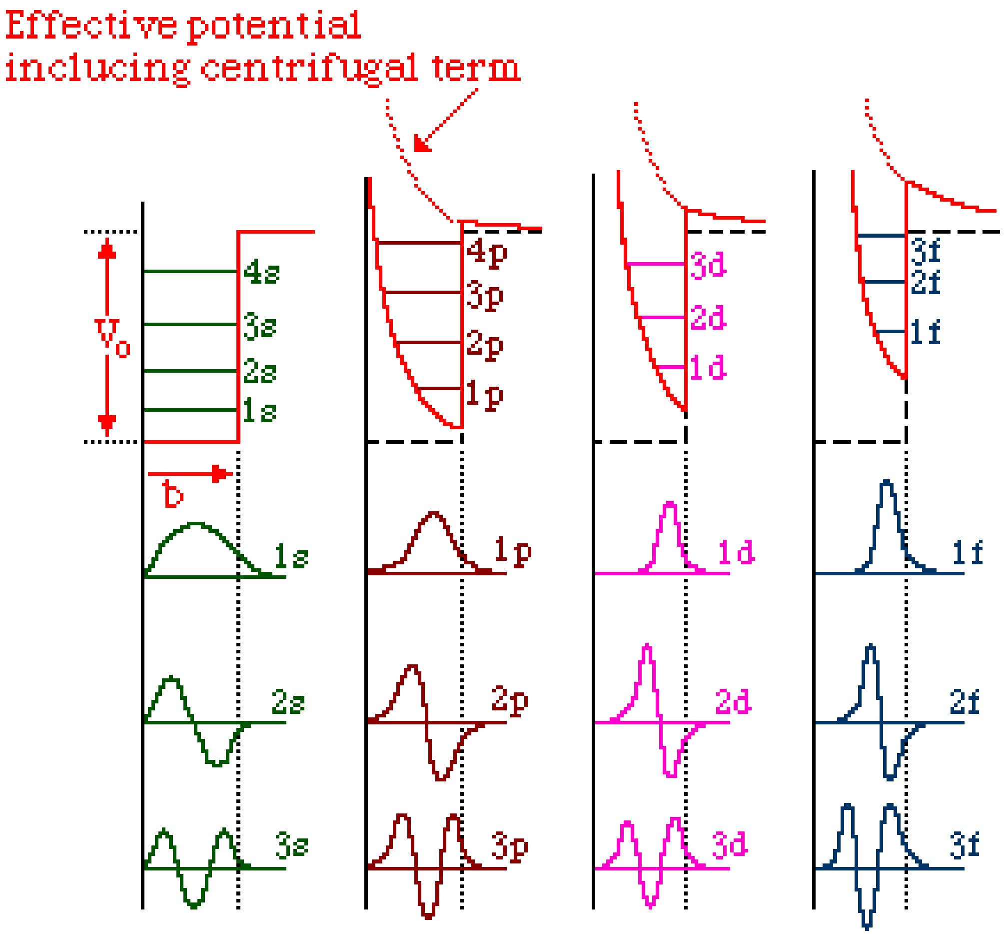
Spectroscopic Notation

- The notation of s (ℓ = 0), p (ℓ = 1), d (ℓ = 2), f (ℓ = 3), states originates from atomic spectroscopy – fine structure of electronic orbits in the hydrogen atom

sharp
principal
diffuse
fundamental

- Then it becomes simply alphabetical, i.e. g (ℓ = 4), h (ℓ = 5), i (ℓ = 6), j (ℓ = 7), k (ℓ = 8)

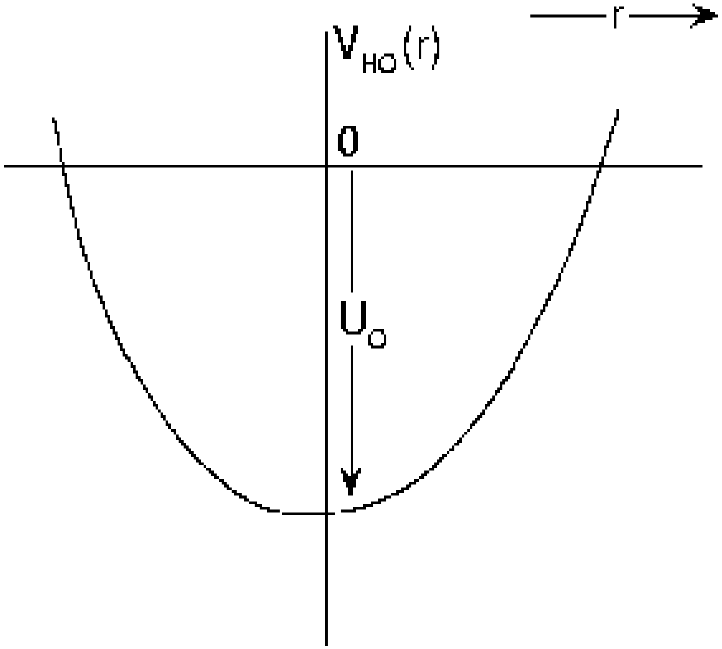
Square Well Wavefunctions



Radial wavefunctions R_{nl}

- For ℓ ≠ 0 there is an effective centrifugal barrier which modifies the shape of the potential
- Low n high ℓ states are moved towards the nuclear surface
- e.g. compare 1s and 1f states

Harmonic Oscillator potential



Simple harmonic oscillator potential

- Easy to handle analytically
- Form of potential:

  $$V_{HO}(r) = -U_0 + \tfrac{1}{2}mr^2\omega^2$$

- Solutions are Laguerre polynomials
- Eigenenergies may again be labelled by n and ℓ :

  $$E_{n\ell} = (2n + \ell + \tfrac{1}{2})\,\hbar\omega - U_0$$

Harmonic Oscillator potential

- Eigenenergies can also be labelled by the oscillator quantum number N:

  $$E_N = (N + 3/2)\,\hbar\omega - U_0$$

- For each N there are degenerate levels with n and ℓ that satisfy:

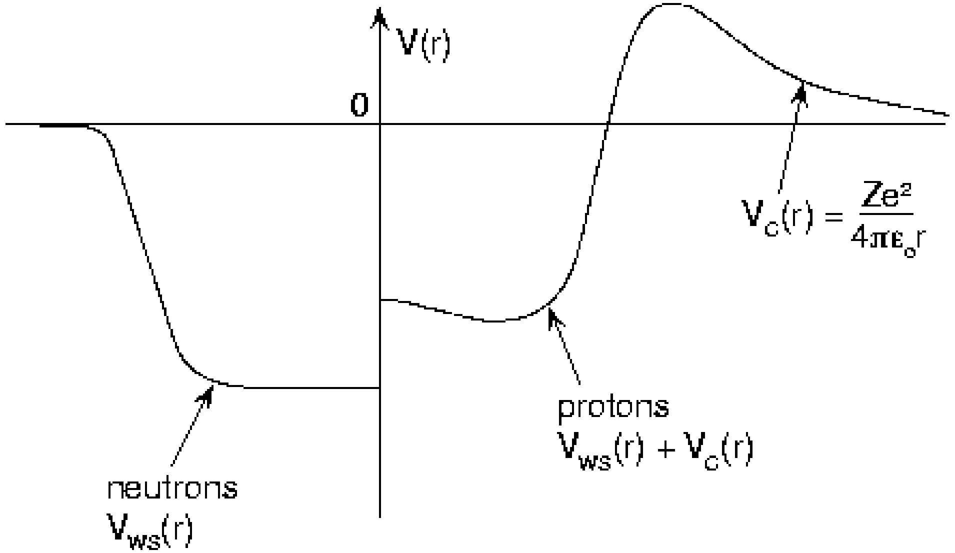
  $$2(n-1) + \ell = N, \quad N \geq 0, \quad 0 \leq \ell \leq N$$

- Even N contains only ℓ even states; odd N, odd ℓ
- The degeneracy condition is:

  $\Delta\ell = 2$ and $\Delta n = 1$ (e.g. $N = 4$ 3s, 2d, 1g orbits)

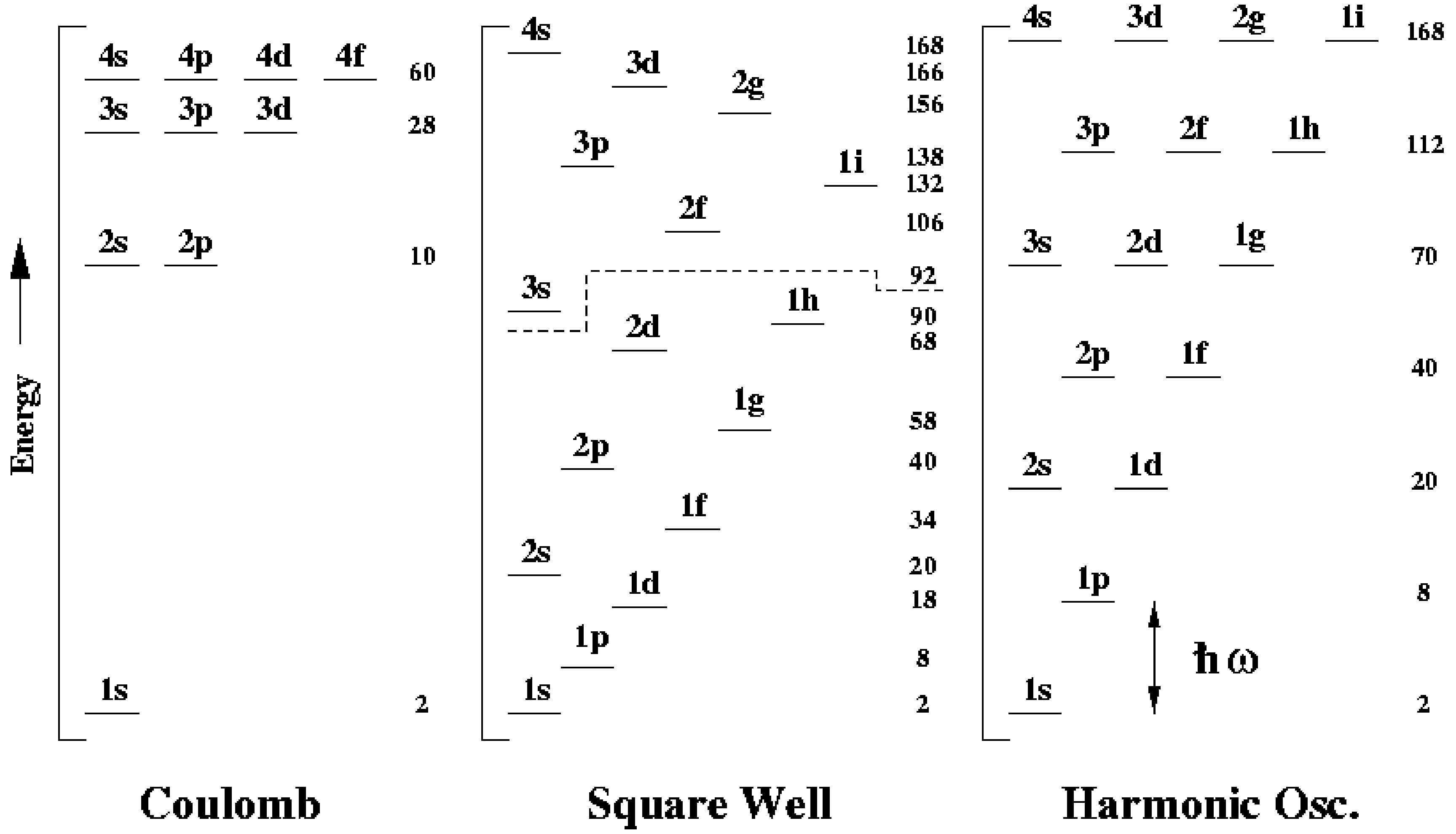
- It is the fundamental reason for shell structure, i.e. clustering of levels
- The parity of each oscillator shell is: $(-1)^N = (-1)^\ell$

Woods-Saxon Potential



- Usually finite potential forms are used such that $V(r) \rightarrow 0$ if $r \gg 0$
- The Woods-Saxon potential is considered to be the most realistic nuclear potential
- For protons a Coulomb potential $V_C(r)$ is added

(Wrong) Magic Numbers



Spin-Orbit Coupling

- In order to account for the correct nucleon numbers at which the higher shell closures occur, a spin-orbit term is added – Mayer, Haxel, Jensen, Suess (1948)
- For the modified harmonic oscillator:

$$V_{HO}(r) = -U_0 + \tfrac{1}{2}mr^2\omega^2 - 2/\hbar^2 \alpha \underline{\ell}.\underline{s}$$

- Since:

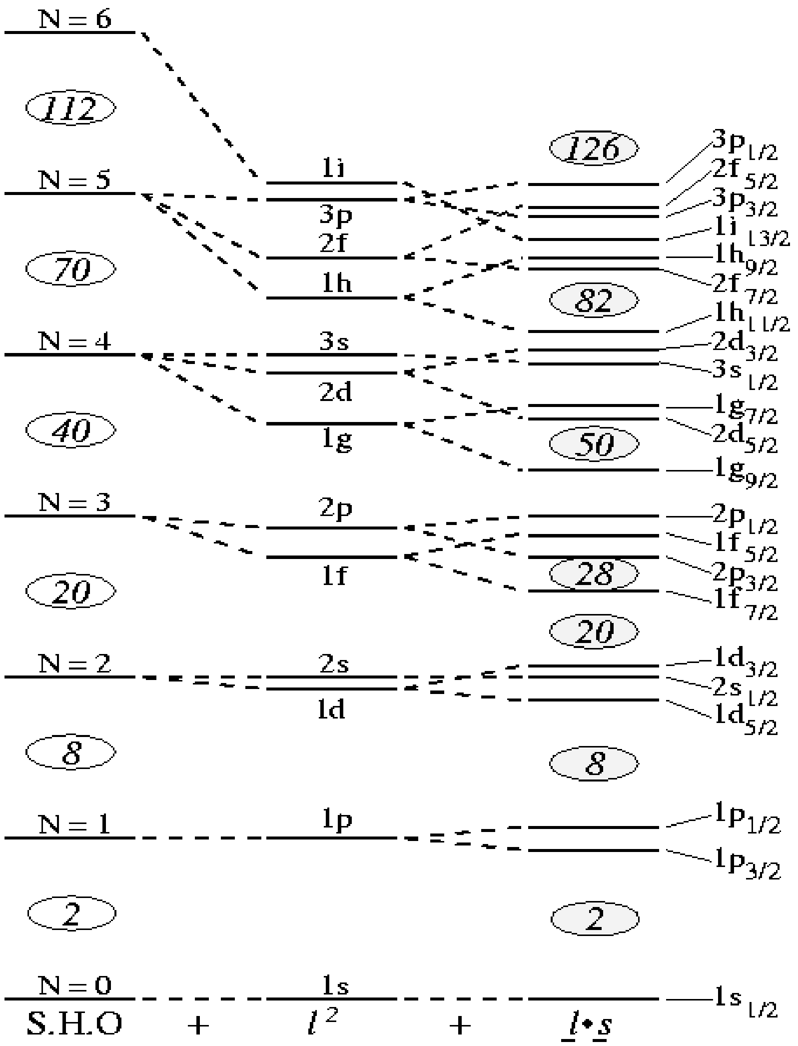
$$\underline{\ell}.\underline{s} = \tfrac{1}{2}\hbar^2[j(j+1) - \ell(\ell+1) - \tfrac{3}{4}]$$

the energy is modified by $-\alpha\ell$ if $j = \ell + \frac{1}{2}$

and by $+\alpha(\ell+1)$ if $j = \ell - \frac{1}{2}$

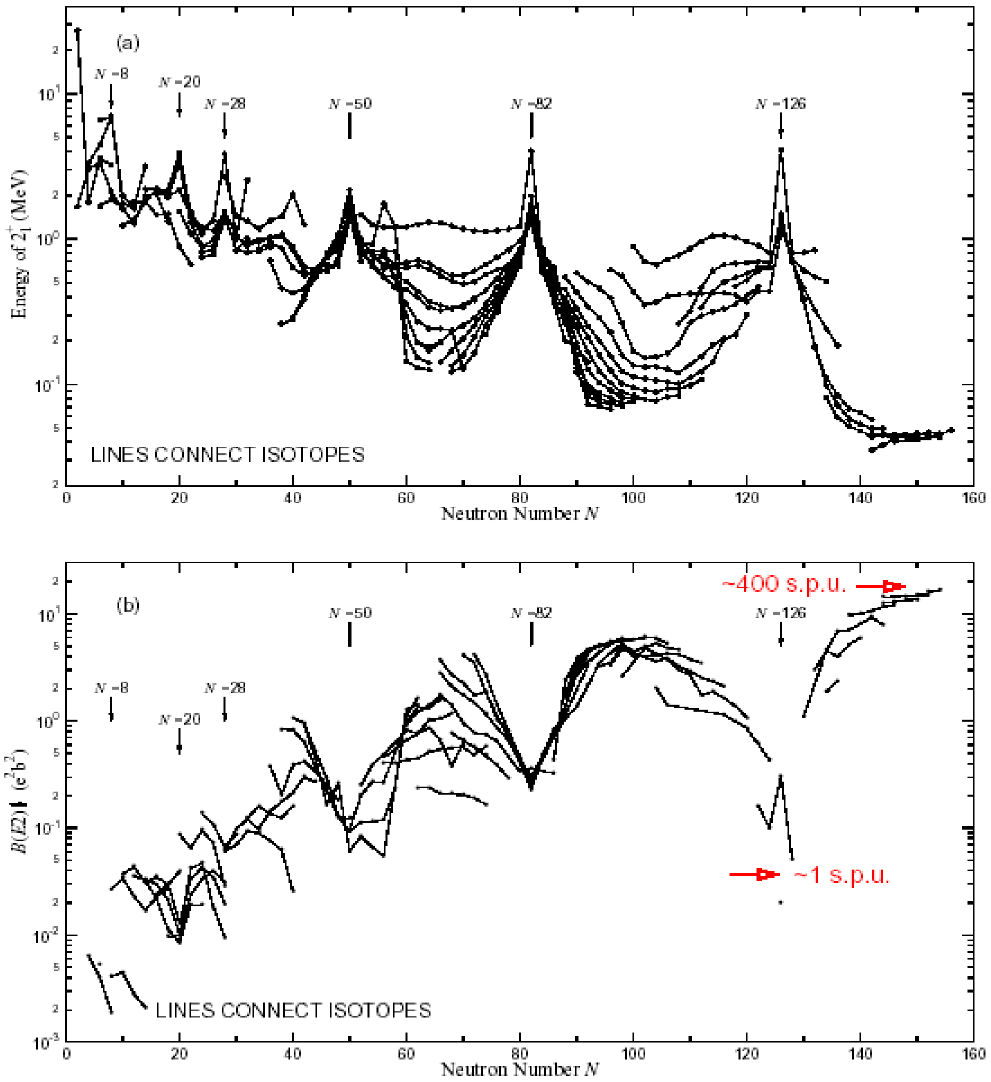
- Note: $j = \ell + \frac{1}{2}$ levels are <u>lowered</u> in energy relative to $j = \ell - \frac{1}{2}$ levels (opposite to the atomic case!)

Predicted Shell Structure



- The harmonic oscillator shells are shown to the left in this diagram
- In the middle, an ℓ^2 term is added to make the potential more realistic ('modified oscillator')
- A spin orbit term $\underline{\ell}.\underline{s}$ is added to the right with its strength (fitted to experiment) adjusted to obtain the correct nuclear magic numbers

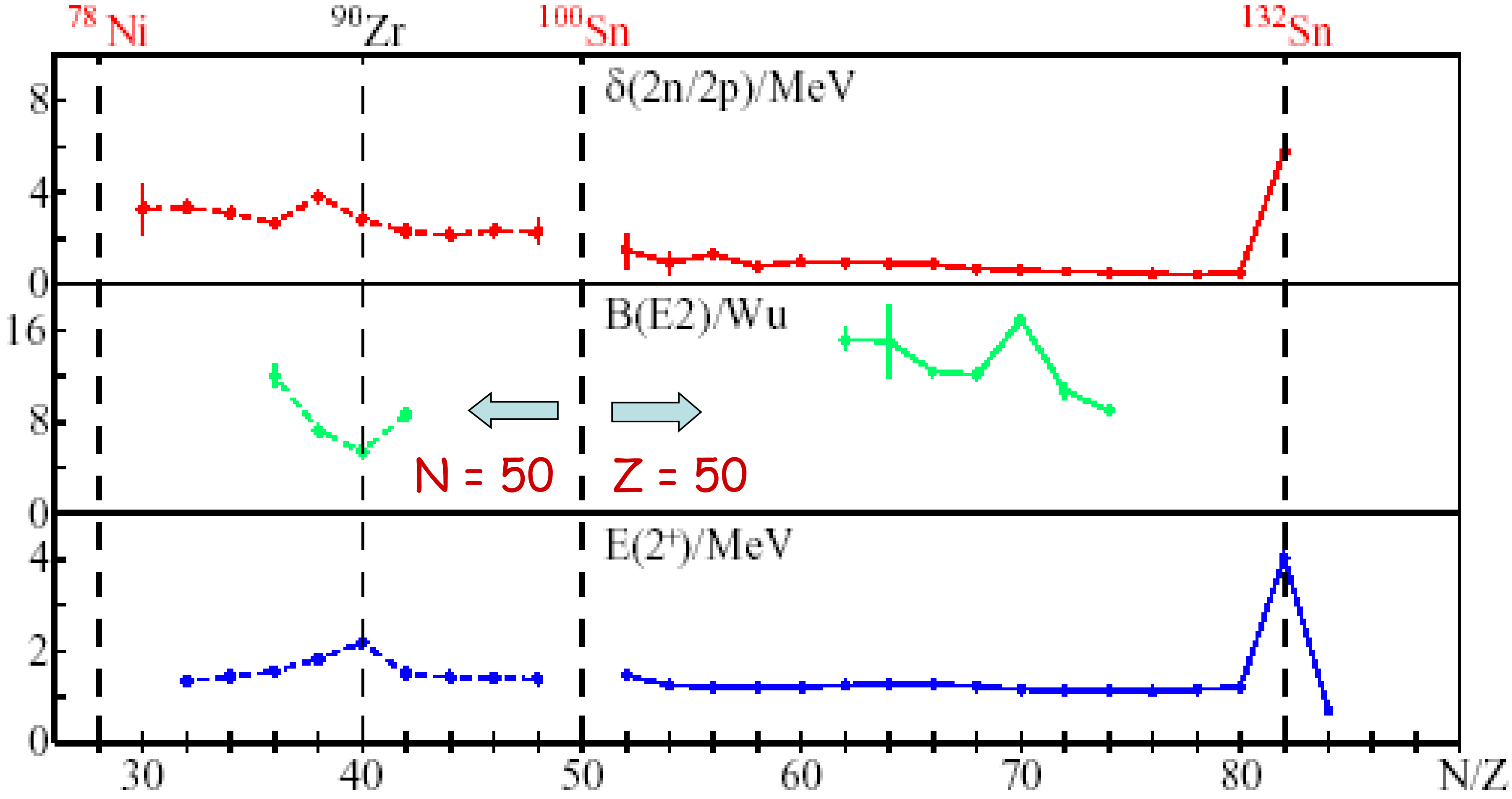
Experimental Shell Effects



S. Raman et al., Atomic Data & Nuclear Data Tables 78, 1

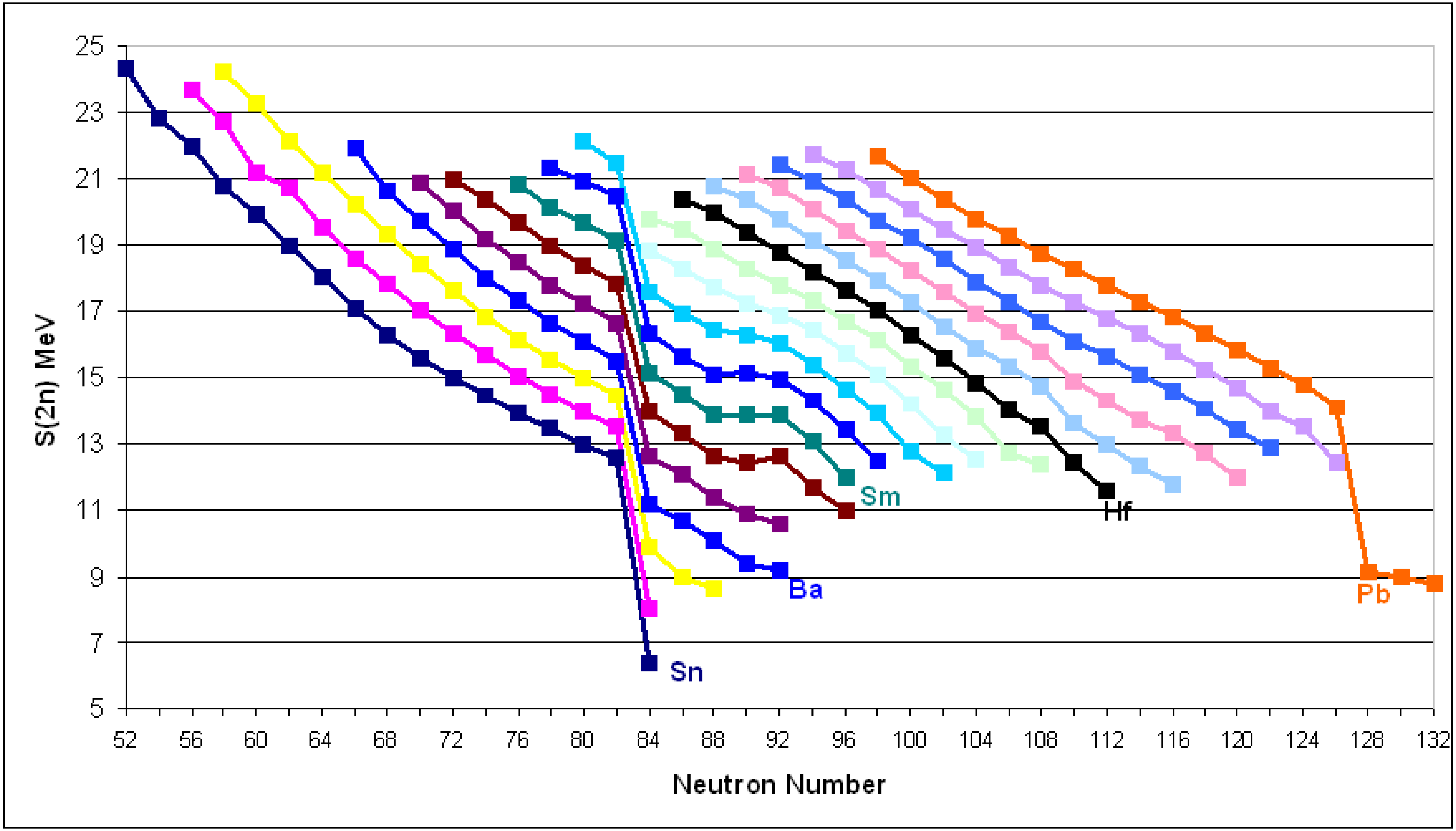
- The energies of the first excited 2^+ states in nuclei peak at the magic numbers of protons or neutrons
- 'B(E2)' values ($\propto 1/\tau$ where τ is the mean lifetime) of the 2^+ states reach a minimum at the magic numbers
- 'Magic' nuclei are spherical and the least collective

Systematics Near Z(N) = 50



- ^{100}Sn (Z=N=50) and ^{132}Sn (N=82) are doubly magic nuclei

Neutron Separation Energies

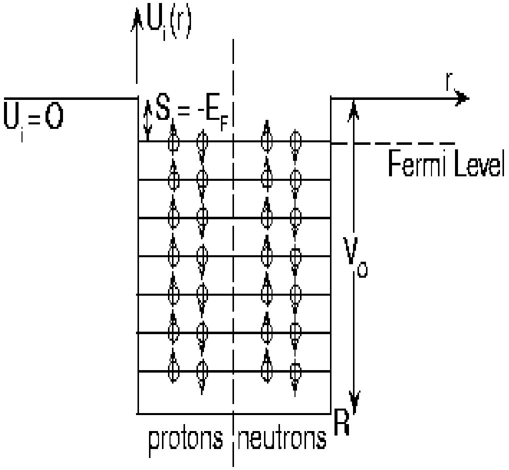


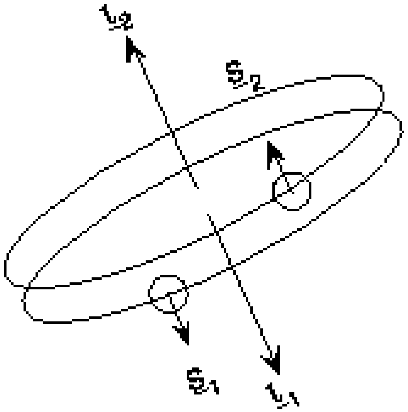
Residual Interactions

- The residual interaction v between nucleons is the difference between the actual two-nucleon potential V_α experienced by a nucleon in a state α and the average potential
- Matrix elements of v, $\langle\alpha|v|\beta\rangle$ are only appreciable near the Fermi Surface
- The interaction v is a two-body operator because it changes the state of two nucleons. It can be treated in a number of ways:

1. from the free two-nucleon potential (difficult!)
2. as a free parameter (fit to experimental data)
3. parameterised using physical intuition

Quadrupole + Pairing Interaction





Monopole pairing
$I^\pi = 0^+$

- If we assume that the interaction takes place near the Fermi surface, i.e. near $r = R$ then $V_\ell(r_i,r_j) \propto V_\ell(R)$
- The quadrupole-quadrupole ($\ell = 2$) interaction is the most important correction to a spherical field, and is relatively long range
- The pairing interaction (left) is the important short range component. It leads to greater binding between nucleons if their angular momenta are coupled to zero spin, with maximum spatial overlap

Hartree Fock Method

- The philosophy here is that the nuclear potential is self-consistent

1. We calculate the nucleon distribution (density) from the net potential
2. Then we evaluate the net potential from the nucleon-nucleon interaction
3. Then we iterate
4. The potential is self-consistent if the one with which we end up is the same as the one we start with

Summary

- Nuclear mean field
- Shell structure
- Magic numbers (shell closures)
- Spin-orbit force (parallel/antiparallel coupling)