PHYS490: Nuclear Physics

" WE FEEL THAT YOU JUST DON'T APPRECIATE THE IMPORTANCE OF WHAT WE DO HERE"

Advanced Nuclear Physics

Chapter 5

5. Nuclear Deformation

Non-spherical shapes

Nuclei Are Not Spherical…

- Most people think an atom's nucleus is round

- However, Aage Bohr, the son of quantum physics pioneer Niels Bohr, proved the inaccuracy of the nuclear spherical shell model

- Along with Ben R. Mottelson and James Rainwater, Bohr demonstrated that the atomic nucleus' shape varied from sphericity because of the distortions created by protons and neutrons on the outer rim of the nucleus moving around in different paths and interacting with the protons and neutrons inside of the nucleus

…Nuclei Are Not Spherical

- The previously developed model, known as the liquid-drop model, suggested that the nuclear shape was spherical because of how the nucleon-nucleon force held the protons and neutrons together, similar to how molecules behave in a drop of liquid

- Their discovery earned them the Nobel Prize in Physics in 1975 for "the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection" and has been key to our current understanding of the central building blocks of atoms

Evidence for Deformation

- The great majority of nuclei known experimentally are non-spherical
- They are deformed – mainly prolate rugby ball shaped
- Evidence:

1. Large electric quadrupole moments Q_0
2. Low-lying rotational bands ($E \propto I[I+1]$)

Nuclear Deformation

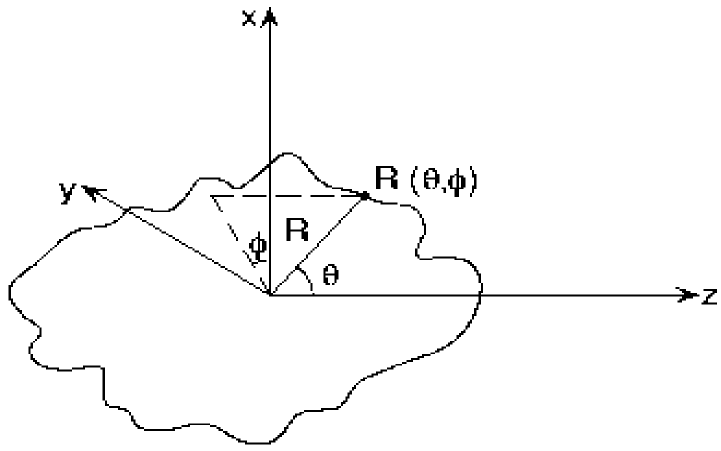
- The origin of nuclear deformation lies in the long range component of the nucleon-nucleon residual interaction: a quadrupole-quadrupole interaction gives increased binding energy for nuclei which lie between closed shells if the nucleus is deformed.

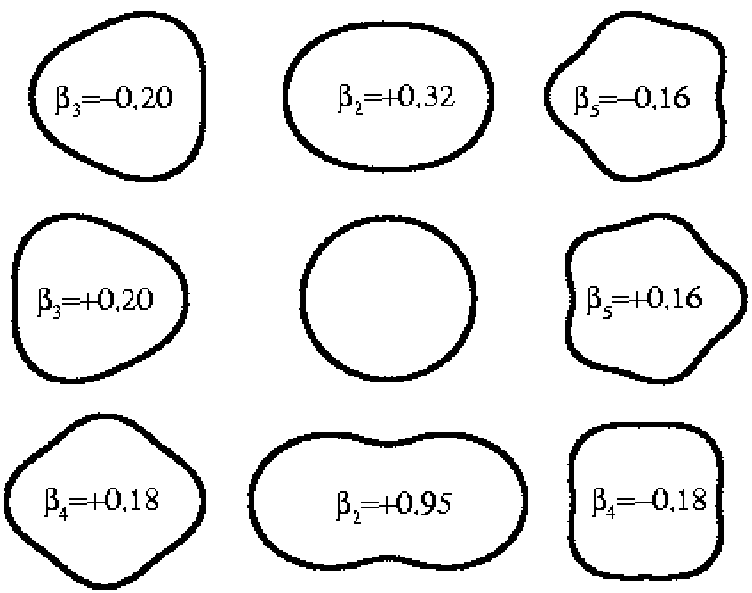
- Incomplete quantum shells – anisotropic orbits

- In contrast, the short range (pairing) component favours sphericity

- Example of spontaneous symmetry breaking (nuclear Jahn-Teller effect – cf molecules)

Simple Nuclear Shapes





- In the description of a 'drop' of nuclear matter with a sharp surface, the equipotential surface $R(\theta,\varphi)$ can be expressed as a sum over spherical harmonics $Y_{\lambda\mu}(\theta,\varphi)$:

$$\boxed{R(\theta,\varphi) = R_0[1 + \Sigma_\lambda \Sigma_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi)]}$$

- Here R_0 is the radius of a sphere and the $\alpha_{\lambda\mu}$ coefficients represent distortions from the equilibrium spherical shape

Volume Conservation

- By integrating over the shape of the nucleus, the volume for small deformation is:

$$V \approx (4\pi/3)\,[1 + 3a_{00}/\surd(4\pi)]\ R_0^3$$

- To account for the incompressibility of nuclear matter we demand volume conservation under distortions and hence set $a_{00} = 0$

- A factor $C(a_{\lambda\mu})$ may be introduced to satisfy the conservation of volume more precisely:

$$R(\theta,\varphi) = C(a_{\lambda\mu})\, R_0[1 + \Sigma_\lambda \Sigma_\mu a_{\lambda\mu}\, Y_{\lambda\mu}(\theta,\varphi)]$$

Most Important Multipoles

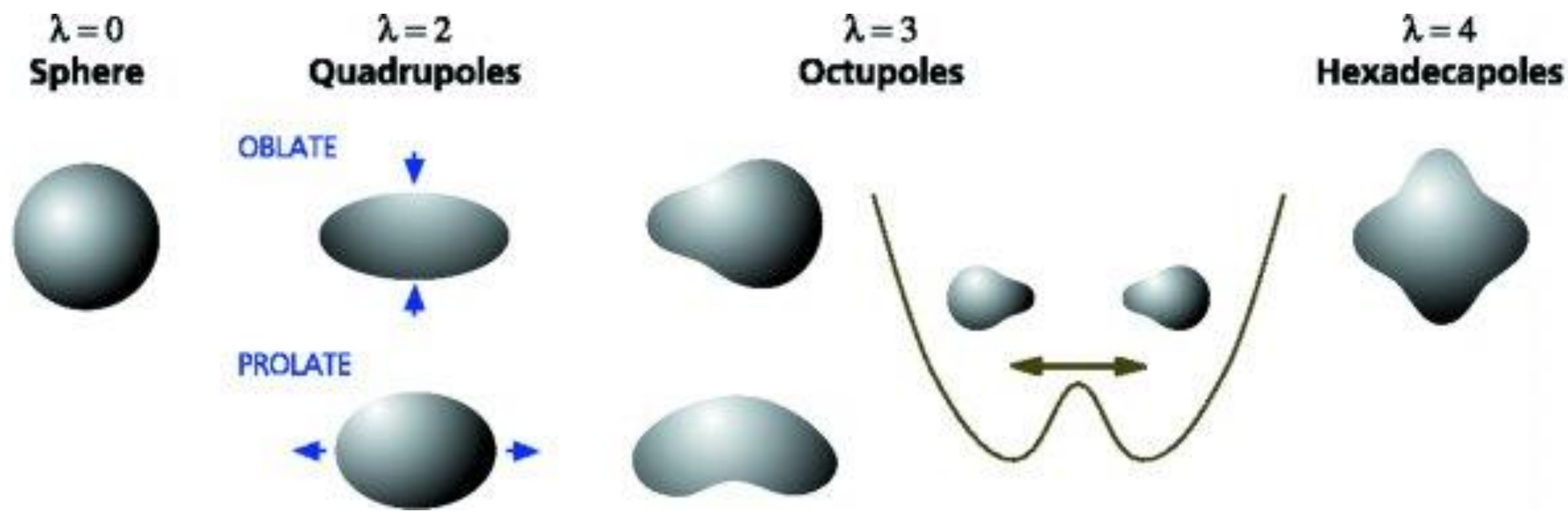
- The $\lambda = 1$ term describes the displacement of the centre of mass and therefore cannot give rise to intrinsic excitation of the nucleus – ignore !

- The $\lambda = 2$ term is the <u>most important</u> term and describes quadrupole deformation

- The $\lambda = 3$ term describes octupole shapes which can look like pears ($\mu = 0$), bananas ($\mu = 1$) and peanuts ($\mu = 2,3$)

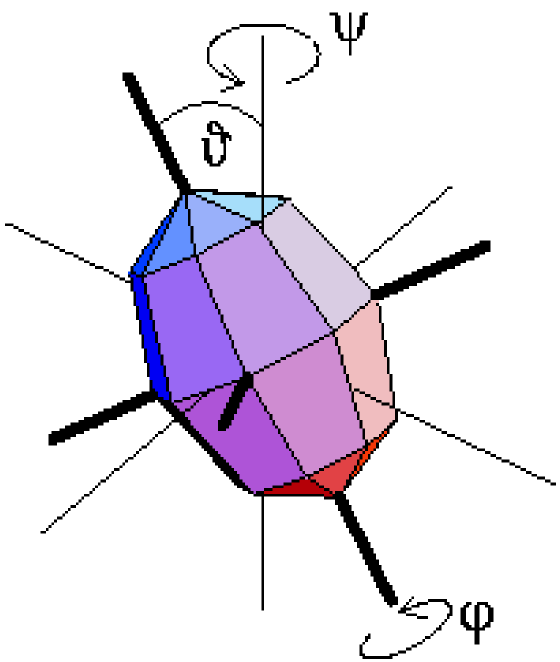
- The $\lambda = 4$ term describes hexadecapole shapes

Exotic Nuclear Shapes



- The $\lambda = 3$ term describes octupole shapes which can look like pears ($\mu = 0$), bananas ($\mu = 1$) and peanuts ($\mu = 2,3$)
- The $\lambda = 4$ term describes hexadecapole shapes
- In general most nuclei are prolate with a small additional hexadecapole deformation

Quadrupole Deformation

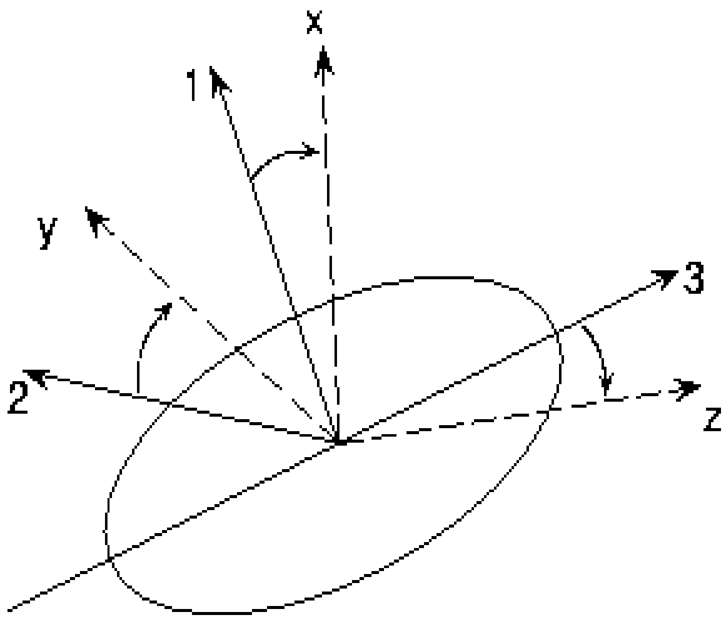


The Euler angles relate the intrinsic (nucleus) and lab frame axes

- The description of the nuclear shape simplifies if we make the principal axes of our coordinate system, i.e. (x, y, z), coincide with the nuclear axes $(1, 2, 3)$
- Then $a_{22} = a_{2-2}$, and $a_{21} = a_{2-1} = 0$
- The two independent coefficients a_{20} and a_{22}, together with the three Euler angles, then completely define the system
- The shape then simplifies to:

$$R = C\, R_0[1 + a_{20}Y_{20} + a_{22}\,(Y_{22}+Y_{2-2})]$$

Principal Axes



Intrinsic (nuclear) and laboratory frame axes

- The description of the nuclear shape simplifies if we make the principal axes of our coordinate system (x, y, z) coincide with the nuclear axes (1, 2, 3)
- For quadrupole shapes we then need only two parameters (β, γ) to describe the shape
- 'Prolate' (rugby ball): $\beta > 0$
- 'Oblate' (smartie): $\beta < 0$

β_2 and γ Parameters

- An alternative parameterisation in the system of principal axes introduces the polar coordinates (β_2, γ) through the relations:

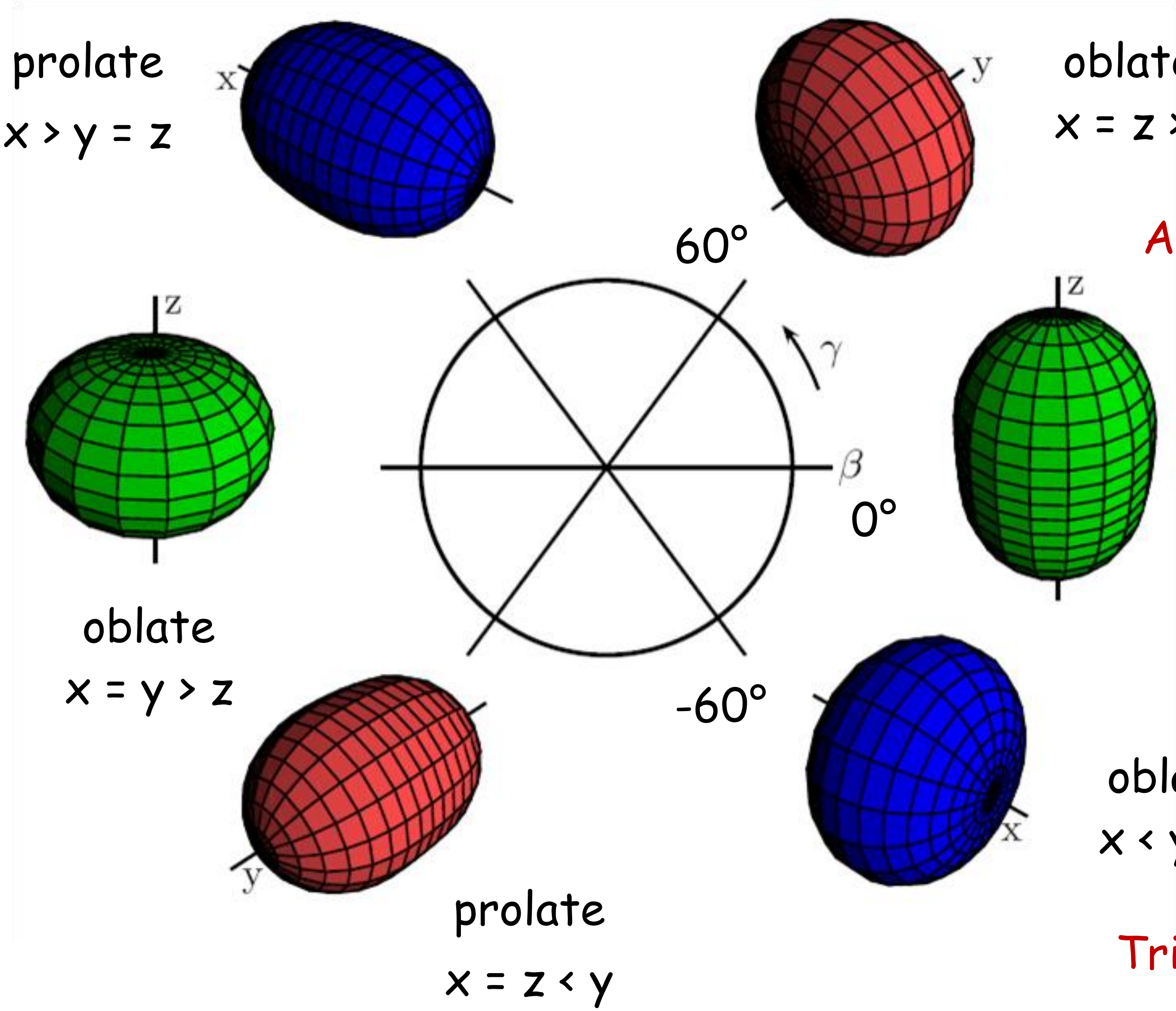
$$\boxed{\alpha_{20} = \beta_2 \cos\gamma} \quad \text{and} \quad \boxed{\alpha_{22} = -1/\sqrt{2}\,\beta_2 \sin\gamma}$$

- The parameter β_2 measures the total deformation:

$$\boxed{\beta_2^{\,2} = \sum_\mu |\alpha_{2\mu}|^2}$$

- The parameter γ measures the lengths along the principal axes For $\gamma = 0°$, the shape is prolate with the z-axis as the (long) symmetry axis

Quadrupole β and γ Parameters



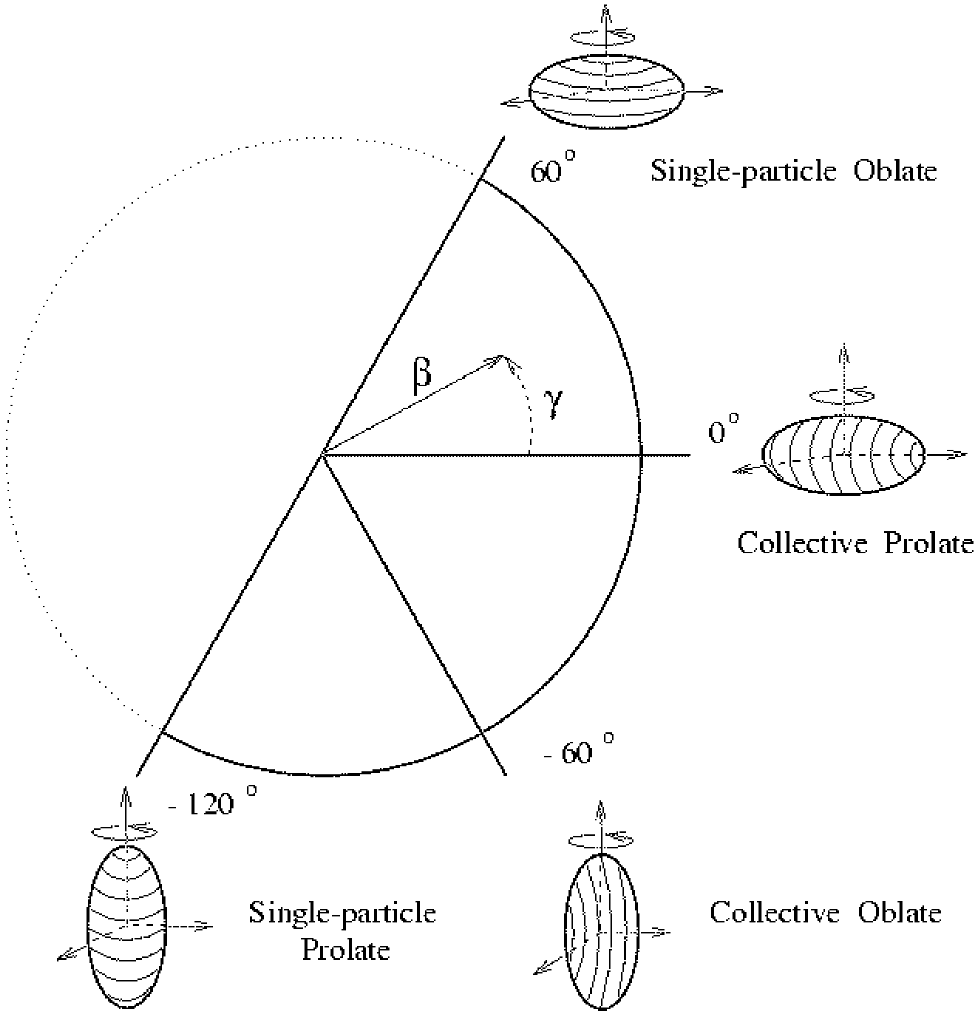
Axially symmetric shapes

γ = n 60°

Triaxial shapes : x ≠ y ≠ z

γ ≠ n 60°

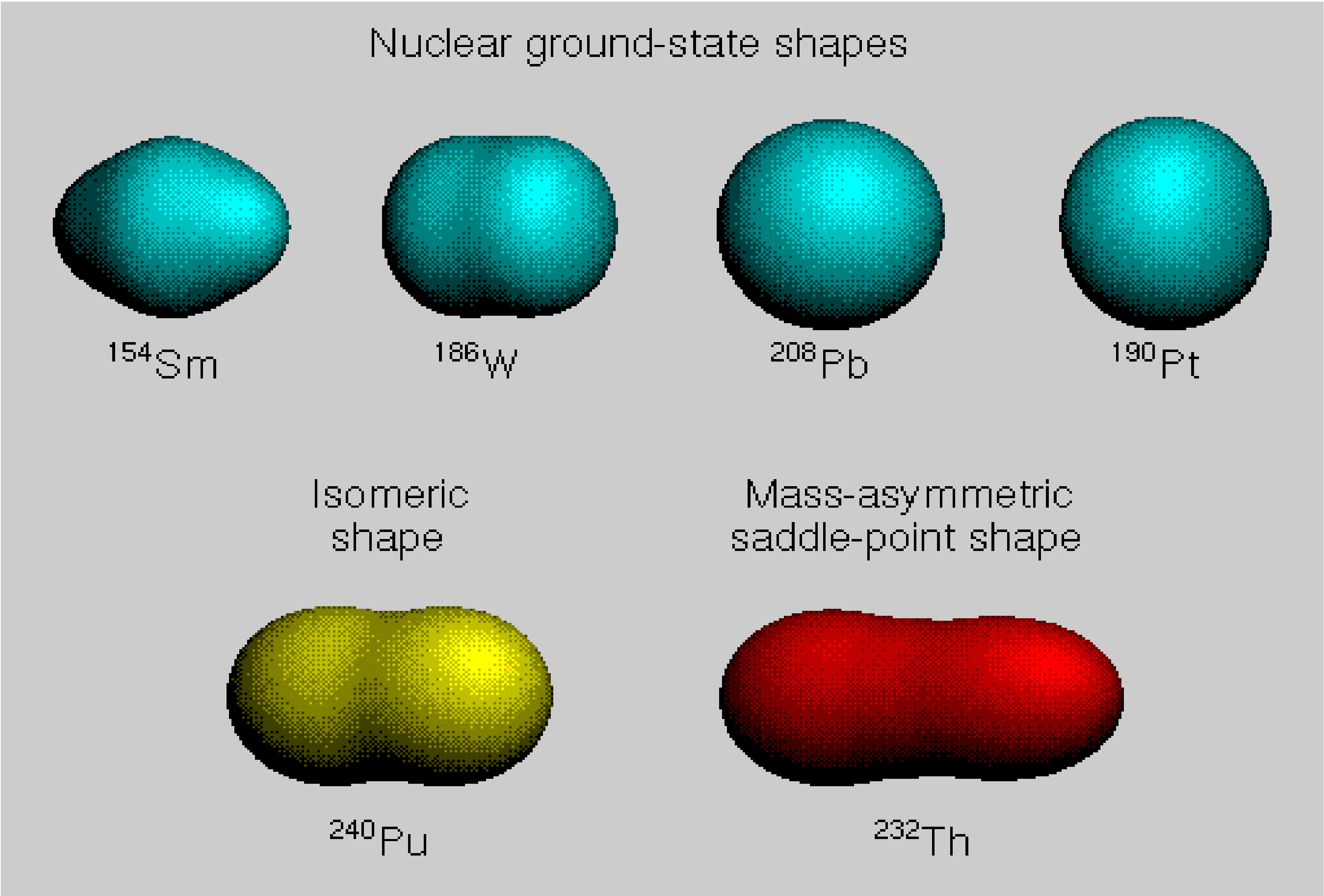
Lund Convention



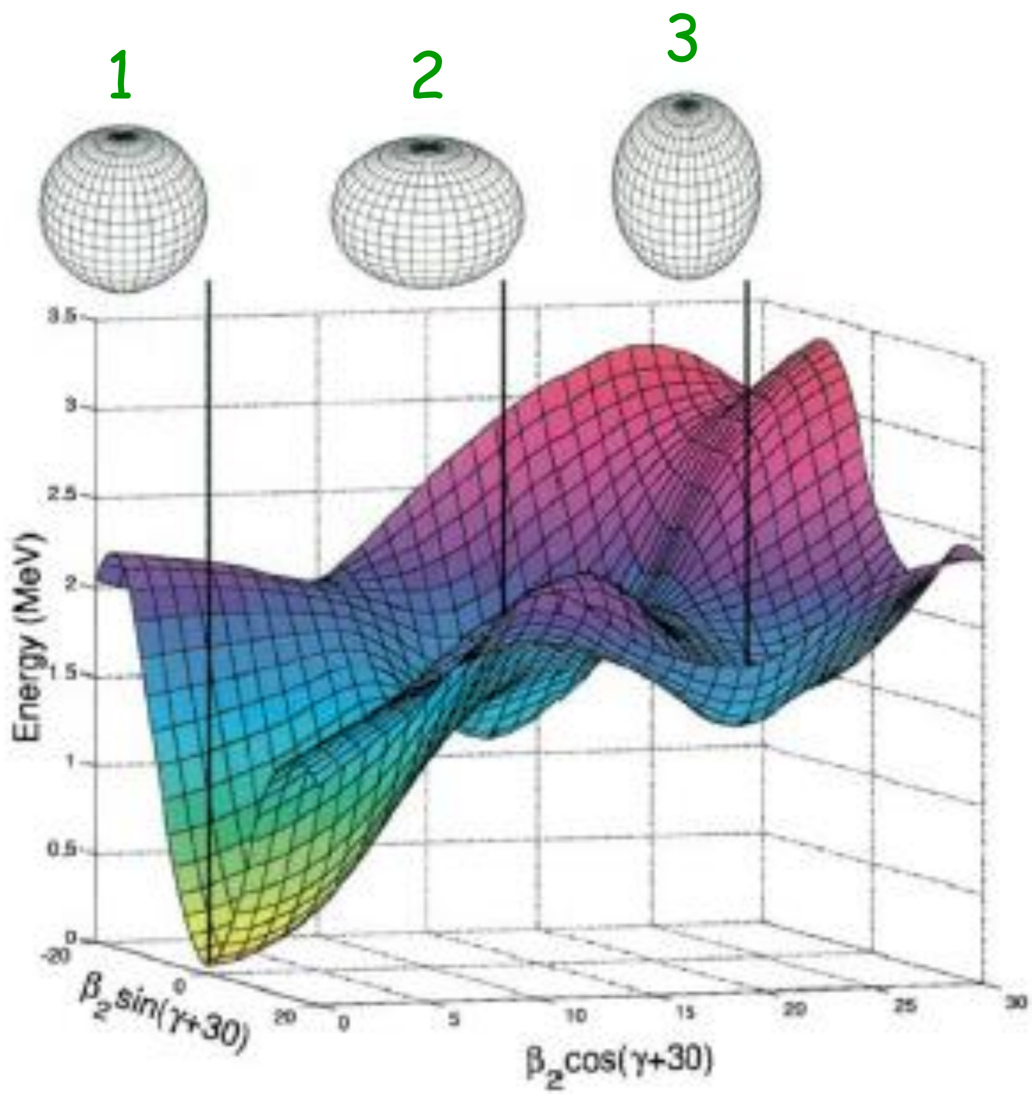
- In order to specify the triaxiality of a deformed quadrupole intrinsic shape, the range of γ values,

  $0° \leq \gamma \leq 60°$ is sufficient
- However, in order to specify a cranked system, we need three times this range, corresponding to the three principal axes about which the system can be cranked

Theoretical Deformations



Shape Coexistence

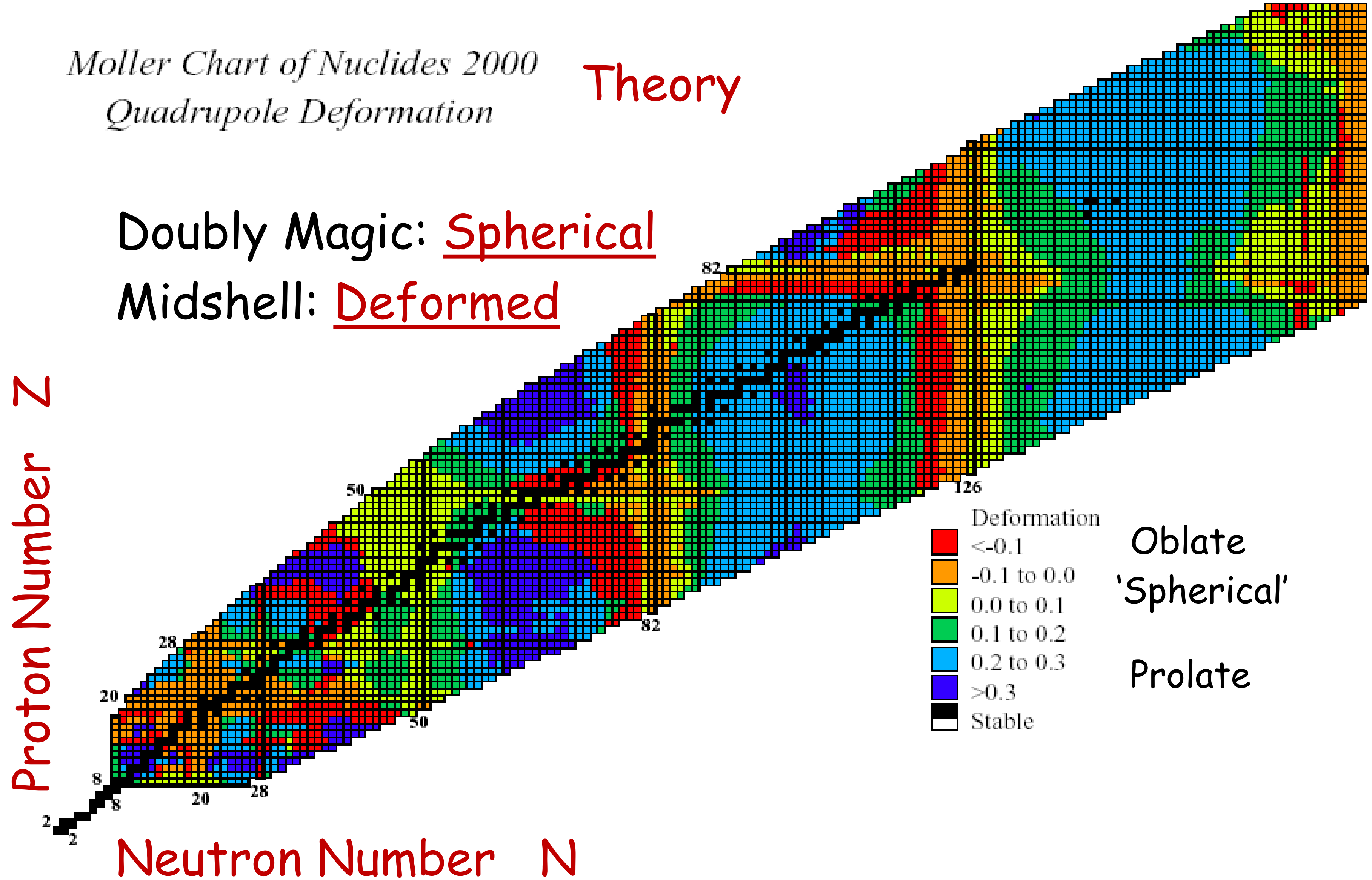


The nucleus ^{184}Pb has three low-lying 0^+ states

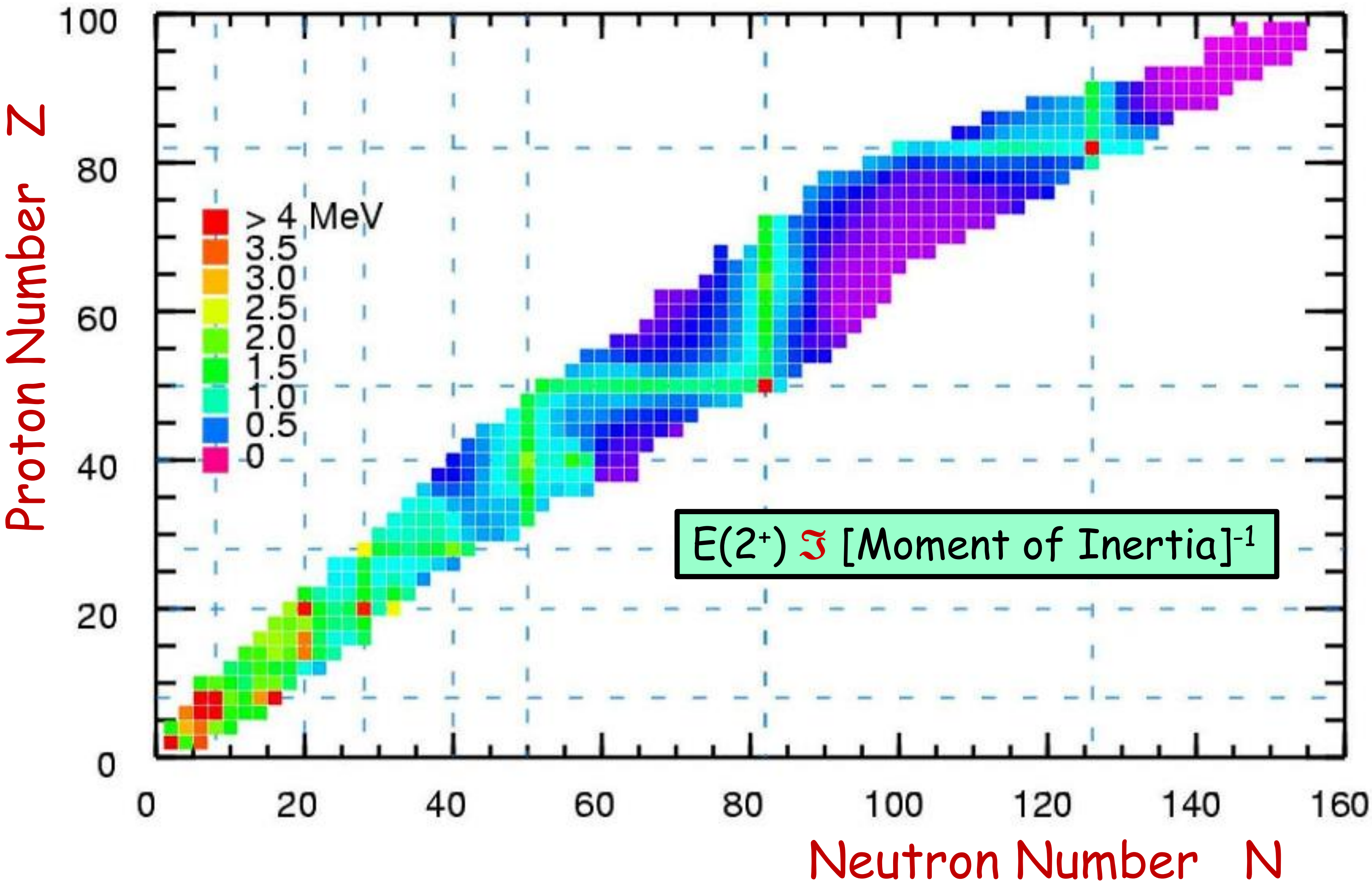
1. Spherical
2. Oblate
3. Prolate

This plot shows the calculated 'potential energy surface'

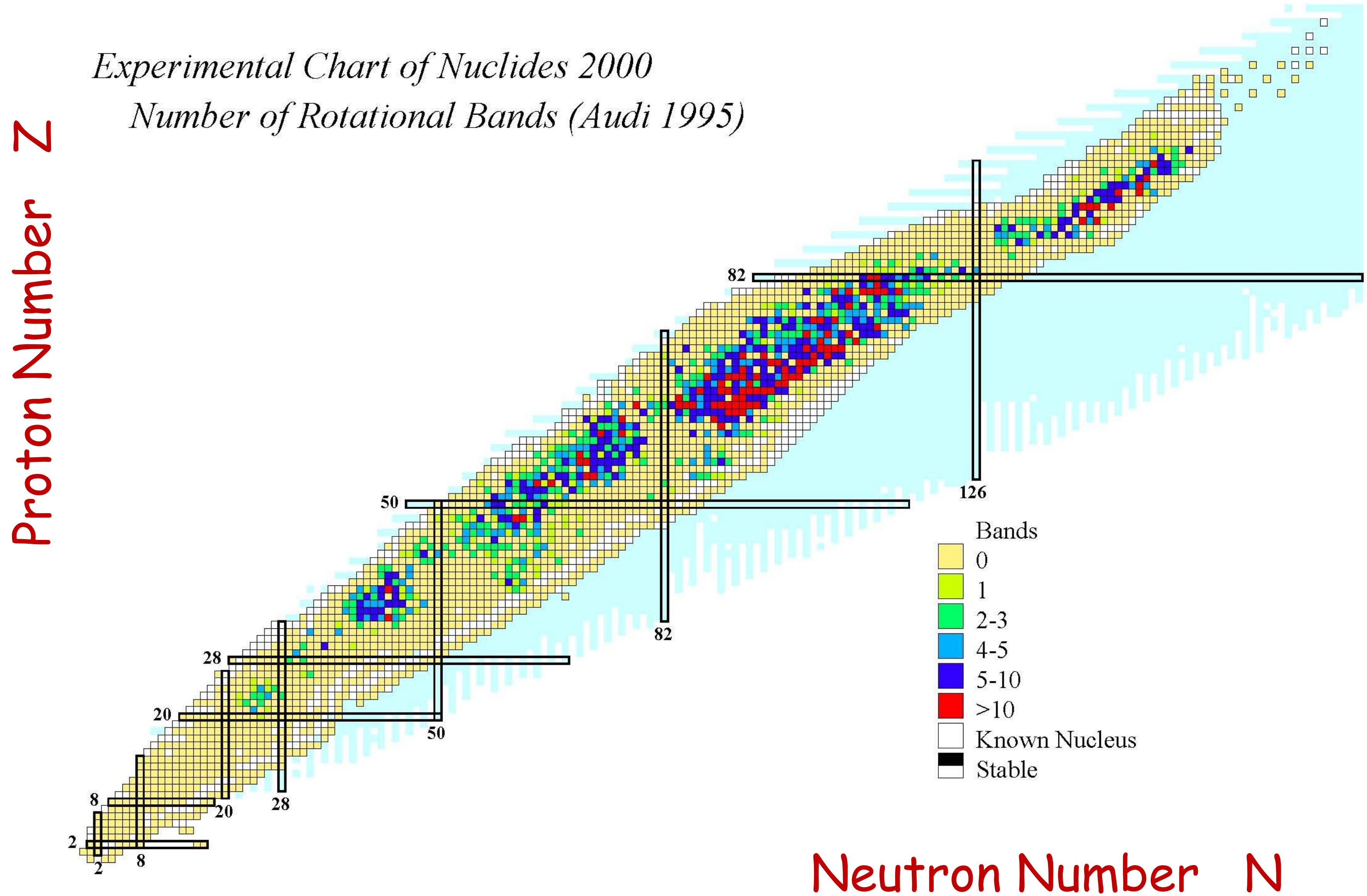
Deformation Systematics



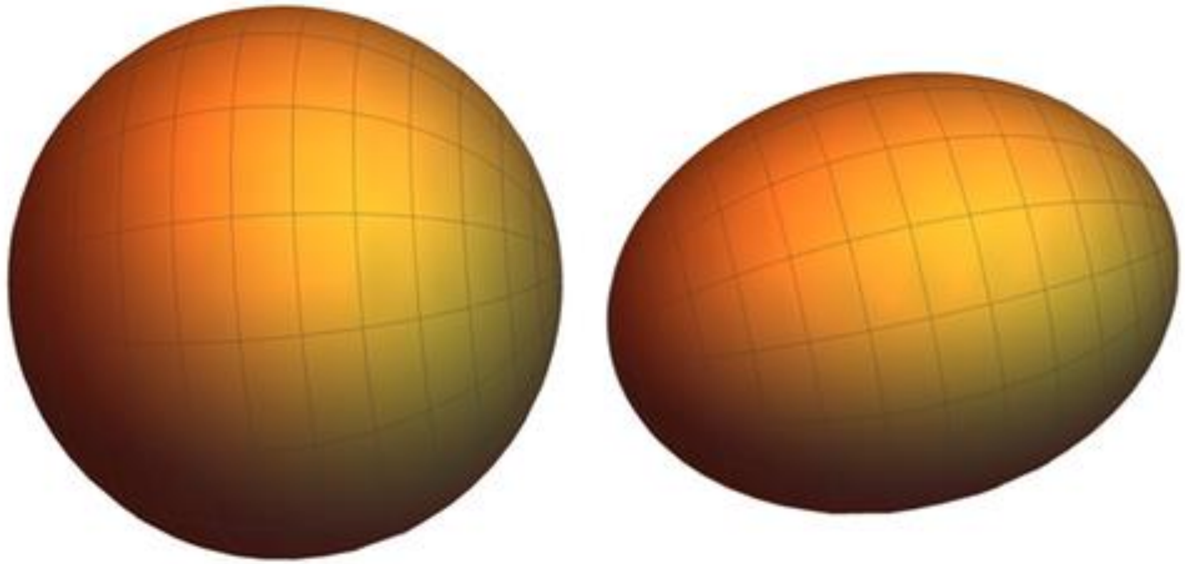
First Excited 2^+ Energies



Deformation: Rotational Bands



Rapid Onset Of Deformation



Atomic nuclei are seldom spherical. The shape of a nucleus can rapidly change when neutrons are added to an element to create a heavier isotope. Zirconium is a prime example: zirconium-96, which contains 40 protons and 56 neutrons is spherical, while zirconium-100, with just four more neutrons, is shaped more like an American football.

Nilsson Model

- In order to introduce nuclear deformation Nilsson modified the harmonic oscillator potential to become anisotropic:

$$V = \tfrac{1}{2}m[\omega_1^2x^2 + \omega_2^2y^2 + \omega_3^2z^2]$$

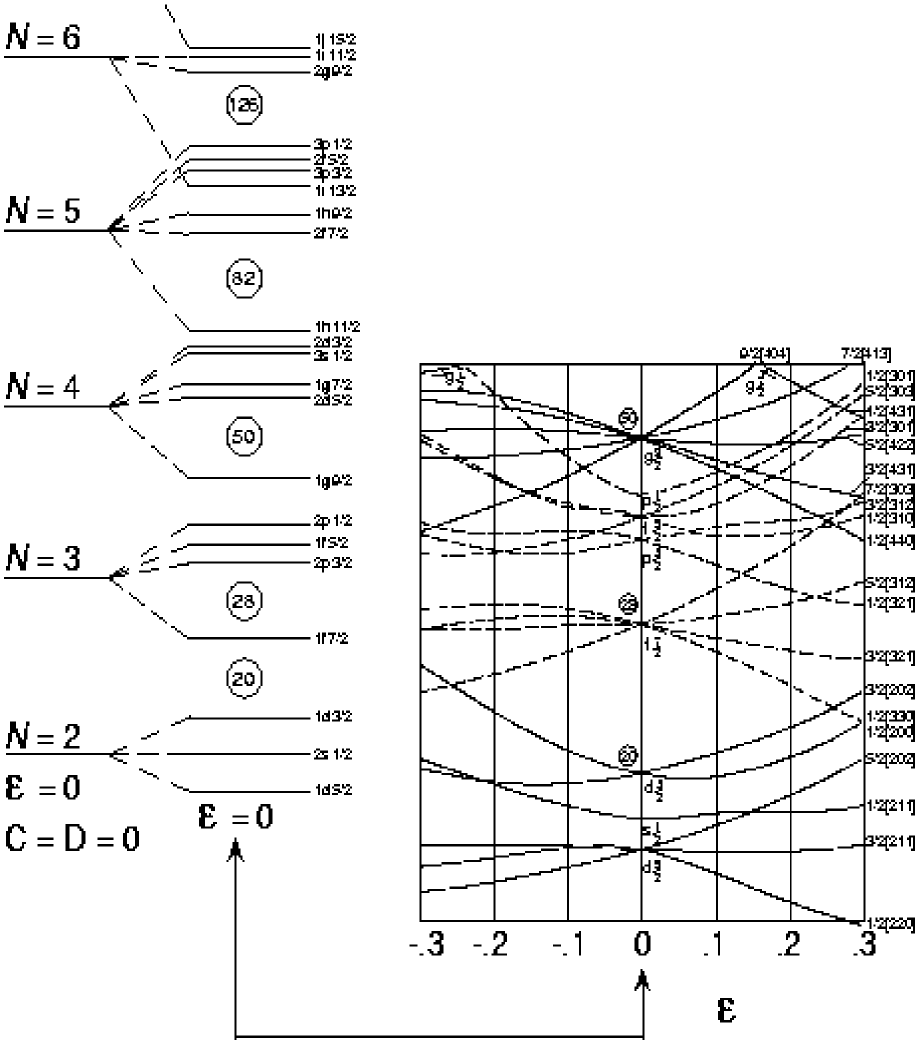
with $\omega_k R_k = \omega_0 R_0$ and $\omega_1 = \omega_2 \neq \omega_3$

- If axial symmetry is assumed ($\gamma = 0$) then the deformation is described by the parameter ε:

$$\varepsilon = (\omega_{1,2} - \omega_3) / \omega_0$$

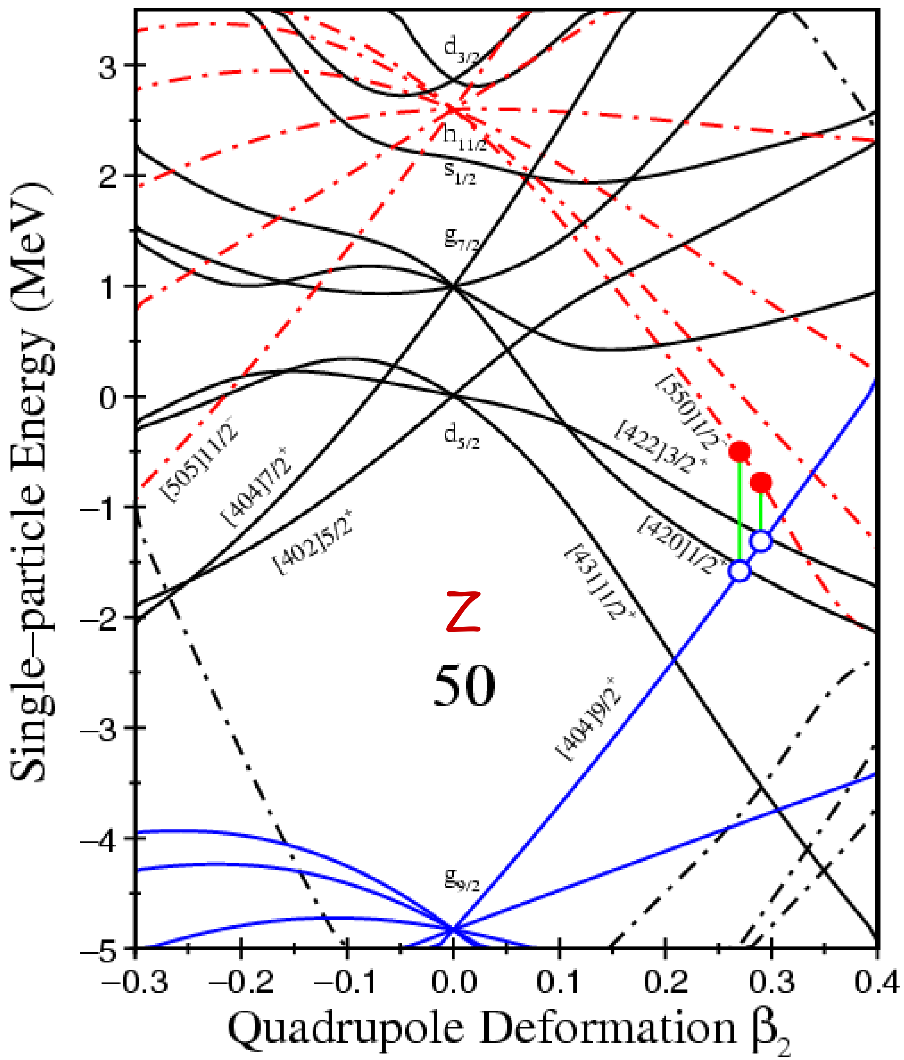
- It can be shown that $\varepsilon \sim \beta$

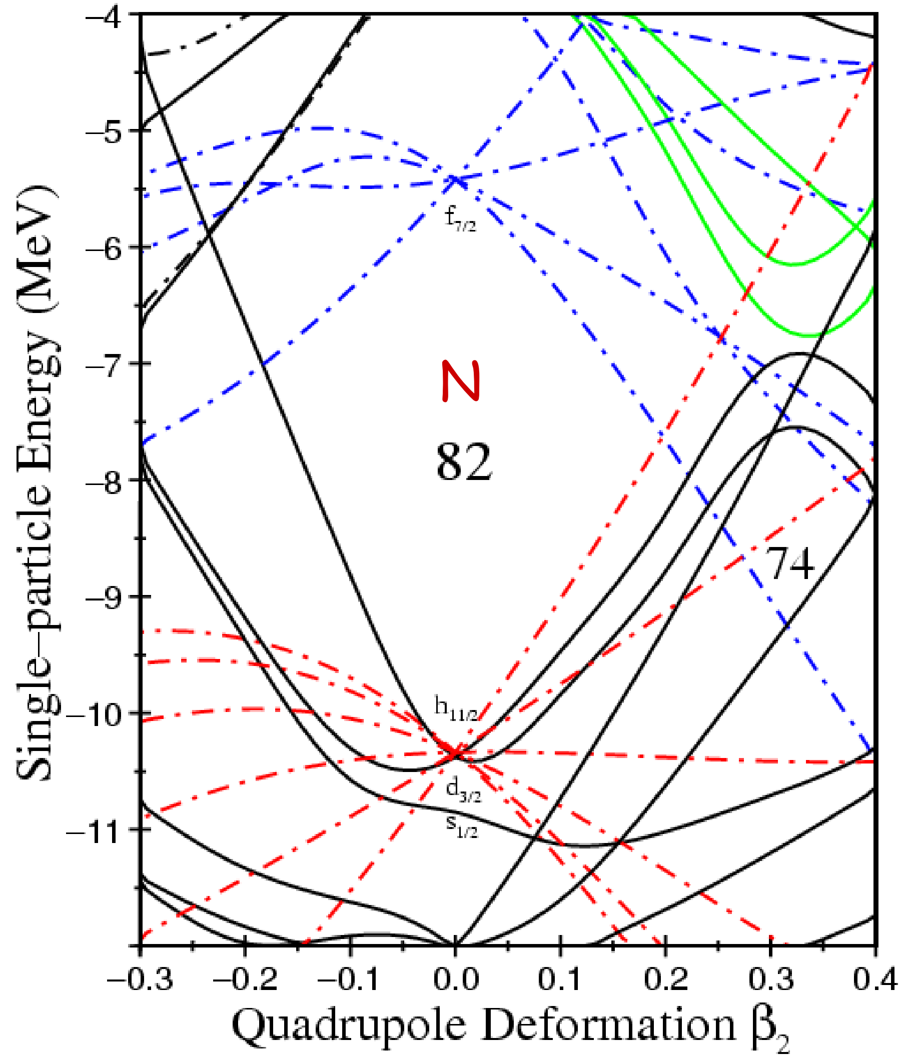
Nilsson Diagram (Energy vs. ε)



- In order to reproduce the observed nuclear behaviour $C\underline{\ell}.\underline{s}$ and $D\ell^2$ terms need to be added (C and D are constants)
- The $\underline{\ell}.\underline{s}$ term is the spin-orbit term
- The ℓ^2 term has the effect of flattening the potential to make it more realistic (like the shape of the Woods-Saxon potential)

Nilsson Single-Particle Diagrams





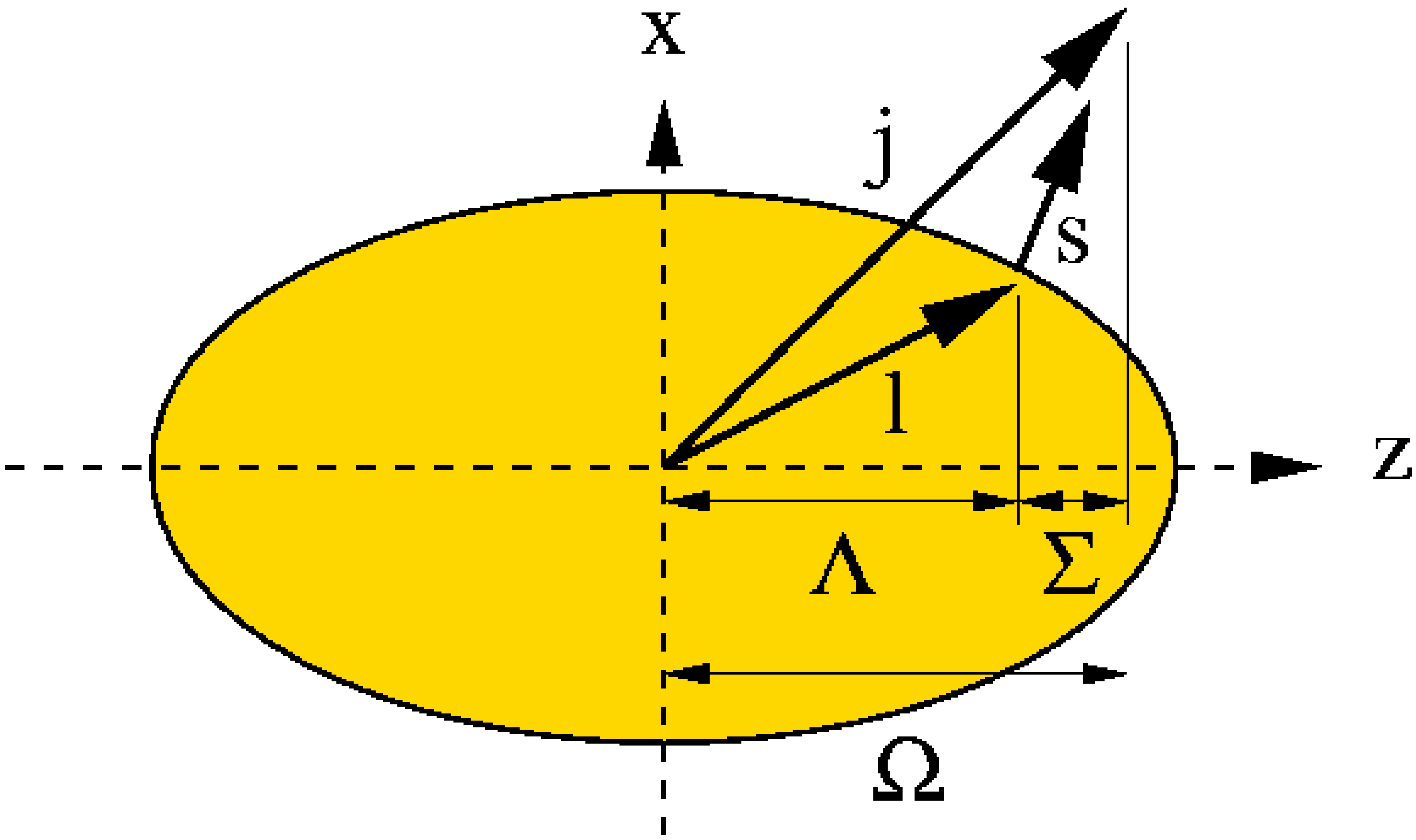
Nilsson Labels

- The energy levels are labelled by the asymptotic quantum numbers:

$$\Omega^{\pi}\ [N\ n_3\ \Lambda]$$

- 'N': $N = n_1 + n_2 + n_3$ is the oscillator quantum number
- 'n_3': n_3 is the z-axis (symmetry axis) component of N
- 'Λ': $\Lambda = \ell_z$ is the projection of ℓ onto the z-axis
- 'Ω': $\Omega = \Lambda + \Sigma$ is the projection of $j = \ell + s$ onto the z-axis
- 'π': $\pi = (-1)^N = (-1)^{\ell}$ is the parity of the state

The Λ, Σ, Ω Quantum Numbers



- Spin projections: $\Omega = \Lambda + \Sigma = \Lambda \pm \frac{1}{2}$

Asymptotic Quantum Numbers

- Because of the additional $\underline{\ell}.\underline{s}$ and ℓ^2 terms the physical quantities labelled by n_3 and Λ are not constants of the motion, but only approximately so

- These quantum numbers are called asymptotic as they only come good as $\varepsilon \to \infty$

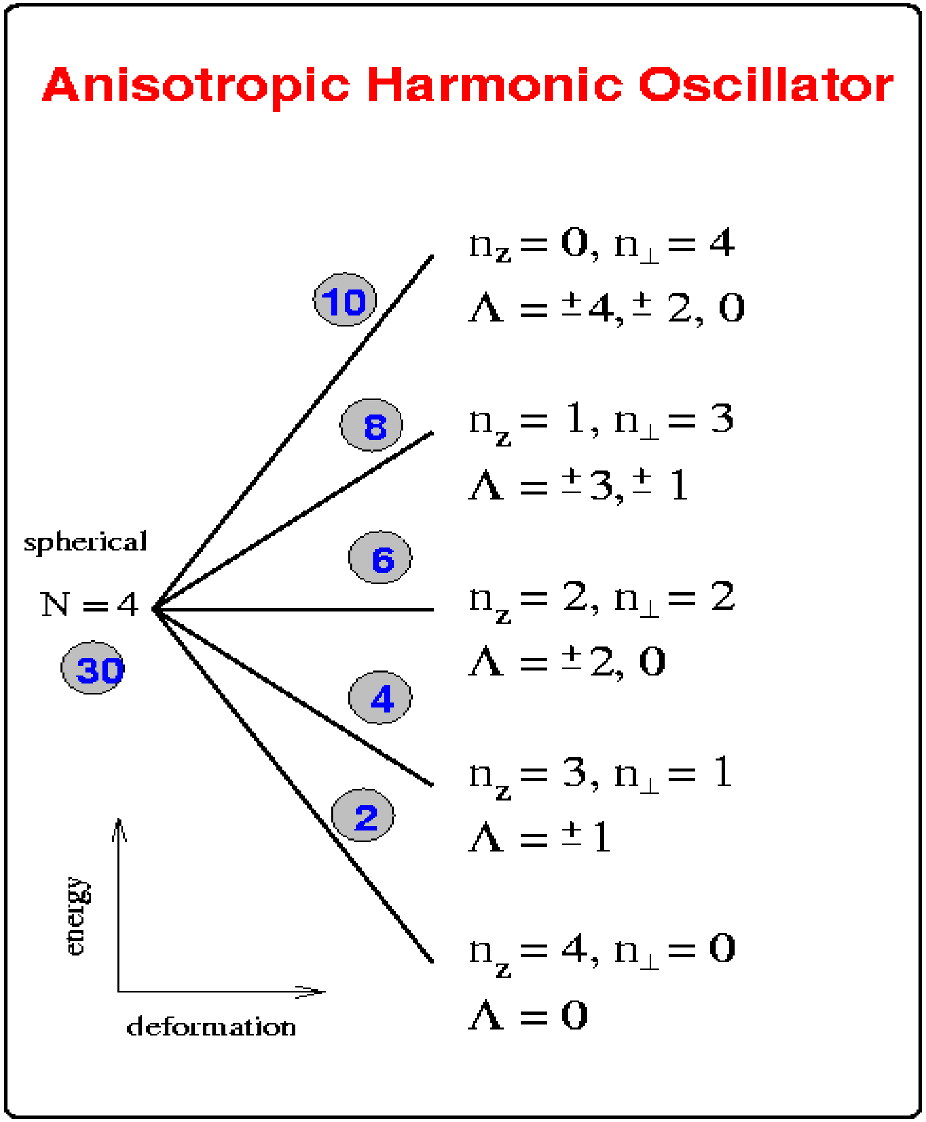
- However, the quantum numbers N, Ω and π are always good labels provided that:
    1. the nucleus is not rotating and
    2. there are no residual interactions

AHO Degeneracies

- Some of the degeneracies of the SHO are lifted
- Consider the $N = 4$ shell spherical oscillator shell which has degeneracy $(N + 1)(N + 2) = 30$ with $\ell = 4, 2, 0$.
- The onset of deformation causes these levels to split into $(N + 1)$ levels, each of degeneracy $2(n_\perp + 1)$:

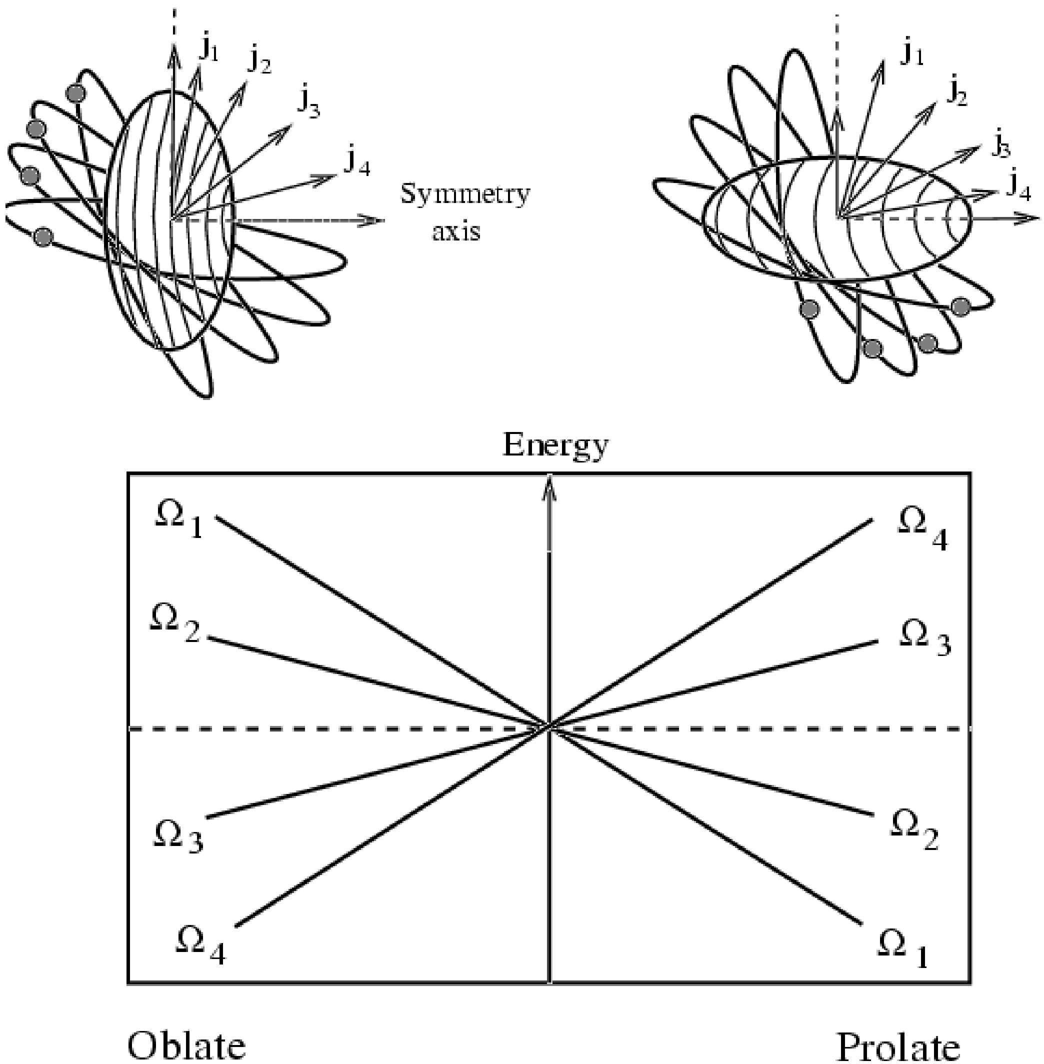
n_z	$n_\perp$	Occupation
4	0	2
3	1	4
2	2	6
1	3	8
0	4	10

Levels of the AHO



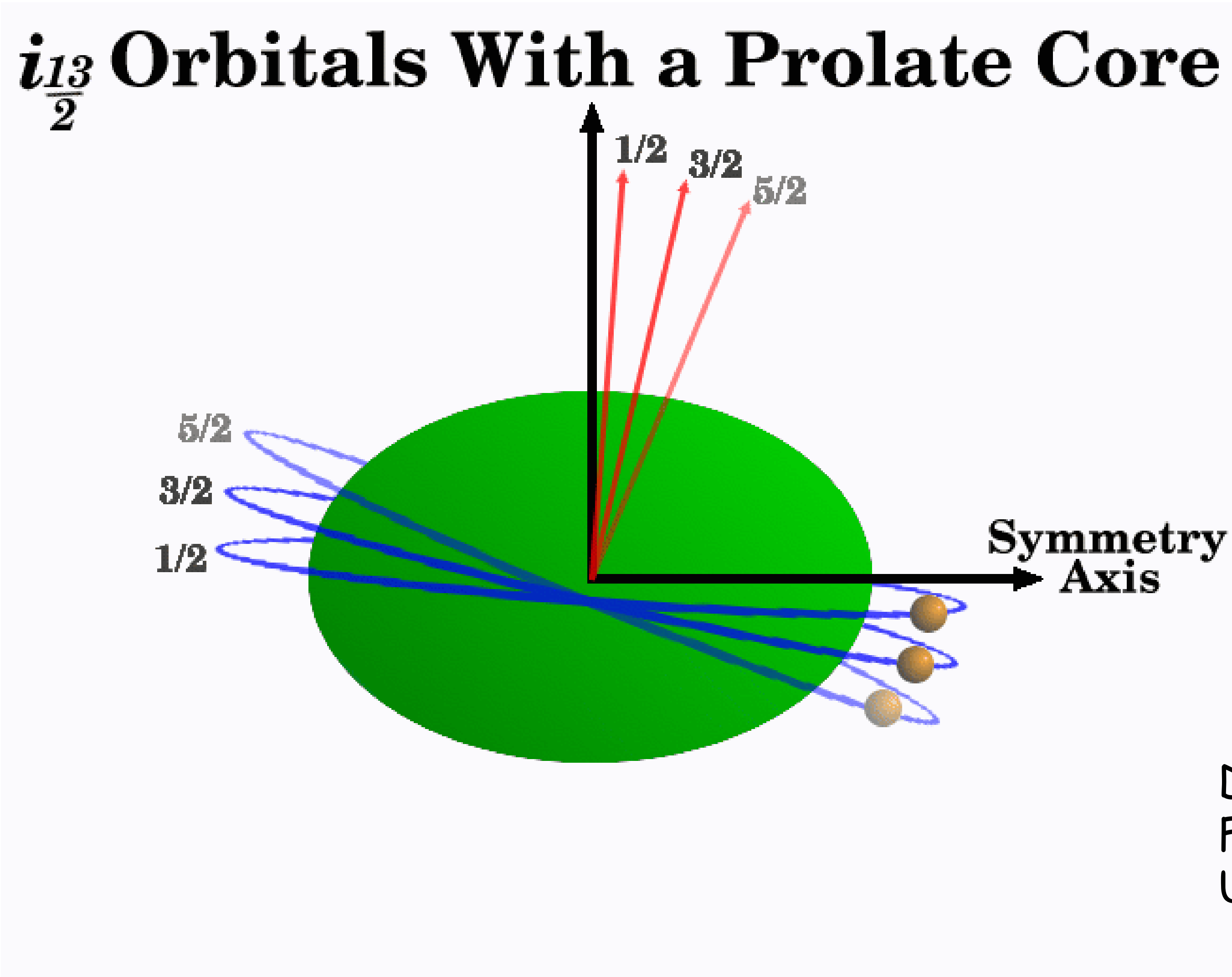
- The splitting of the N = 4 oscillator shell is shown here when deformation is introduced
- Note that levels with large n_z (and hence small $n_\perp$) are favoured

Splitting of Ω States



- Low Ω states favour prolate shapes
- High Ω states favour oblate shapes
- Note that each Ω state is now only twofold degenerate (±Ω)

Splitting of Ω States



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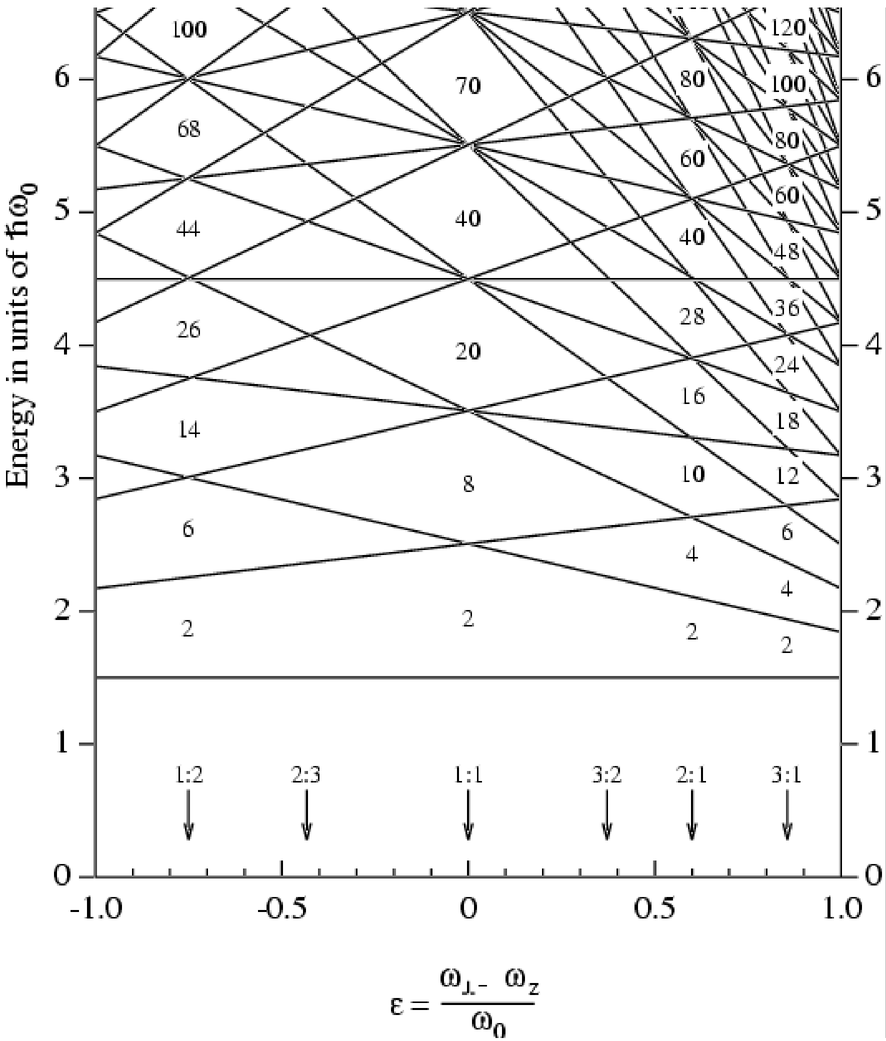
Large Deformations



- This figure ignores the $\ell.s$ and ℓ^2 terms
- Deformed shell gaps emerge when ω_3 and $\omega_{1,2}$ are in the ratio of small integers, i.e.

  $$\omega_3 / \omega_{1,2} = p/q$$

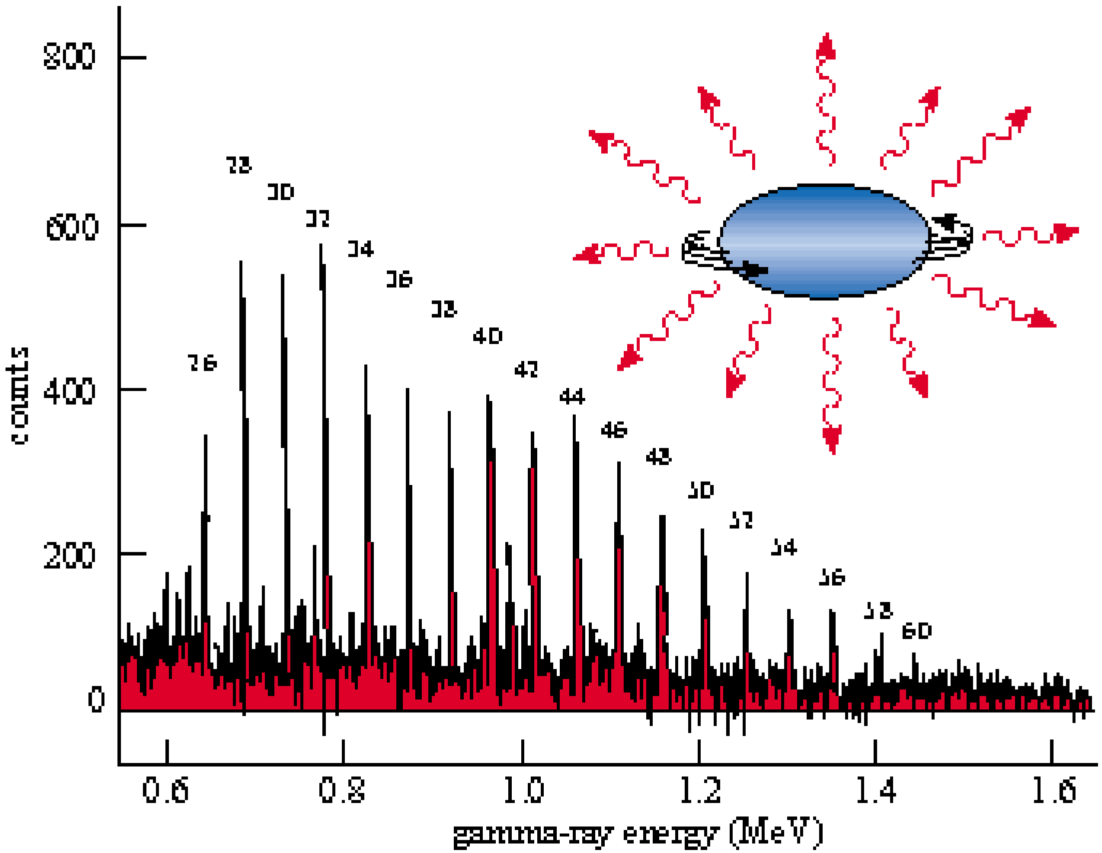
- A superdeformed shape has $p/q = \frac{1}{2}$ or

  $$R_3 : R_{1,2} = 2:1$$

- A hyperdeformed shape has

  $$R_3 : R_{1,2} = 3:1$$

Superdeformed ^{152}Dy



Original SD γ-ray spectrum from 1986 (Daresbury)

- The SD band in ^{152}Dy is a very regular structure with equally spaced gamma-ray transitions

- The spacing is relatively small, i.e. the band has a large moment of inertia (close to the rigid body value)

Summary

- Nuclei are not spherical
- Deformation and shape parameters
- Quadrupole deformation (rugby ball)
- Nilsson Model (Modified Anisotropic Oscillator)
- Superdeformation (2:1 axis ratio)

Advanced Nuclear Physics

Edward Paul