

# PHYS490: Nuclear Physics



" WE FEEL THAT YOU JUST DON'T APPRECIATE  
THE IMPORTANCE OF WHAT WE DO HERE"

# Advanced Nuclear Physics

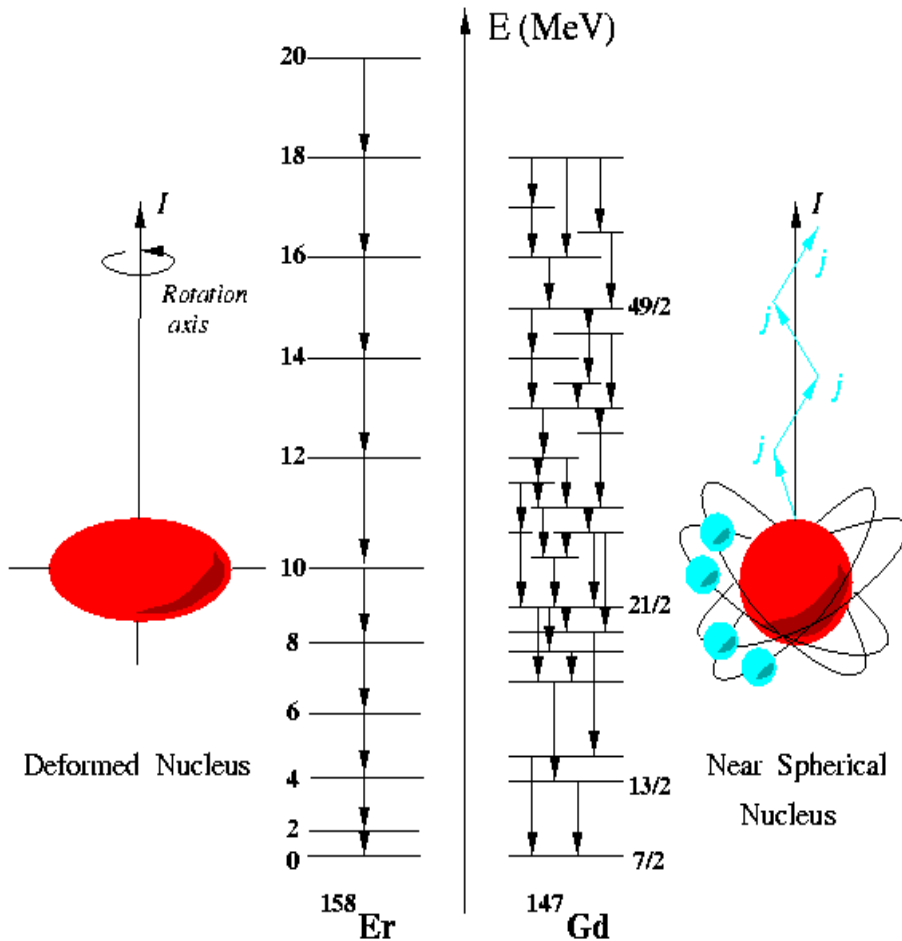
## Chapter 9

# 9. Nuclei at Extremes of Spin

# High-Spin States

- As the nucleus is rotated to states of higher and higher angular momentum, or **spin  $I$** , it tries to assume the configuration which has the lowest rotational energy
- The spin  **$I$**  is made up of a collective part  **$R$**  and a contribution  **$J$**  arising from single particles
- The energy can be minimised by reducing  **$R$**  or by increasing the nuclear moment of inertia
- The pairing is broken by the effect of rapid rotation

# Generation of Angular Momentum



- There are two basic ways of generating high-spin states in a nucleus
1. Collective (in-phase) motions of the nucleons: vibrations, rotations etc
  2. Single-particle effects: pair breaking, particle-hole excitations. The individual spins of a few nucleons  $j_i$  generate the total nuclear spin

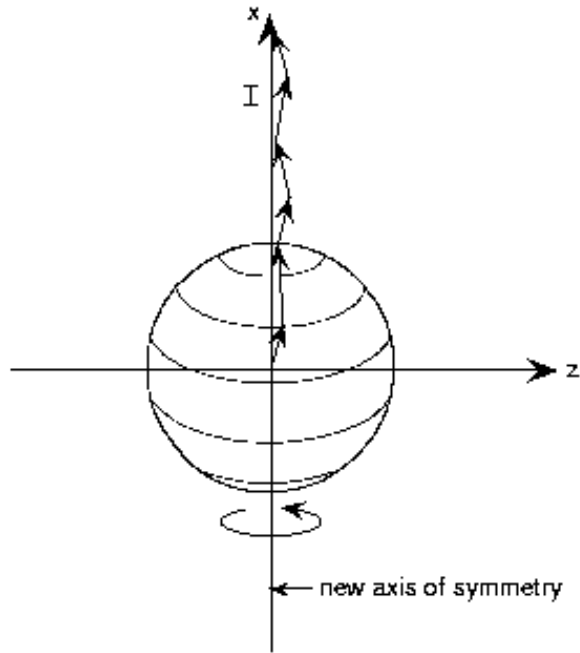
# High $I_x$ Bands

- In backbending the value of  $R$  (collective spin) is reduced by breaking a single pair of nucleons and aligning their individual angular momenta  $j$  with the  $x$ -axis, i.e.

$$I_x = \sum j_x + R$$

- The quantity  $I_x$  is approximately a good quantum number and hence a given nuclear state can be described by a single value of  $I_x$
- The alignment of broken pairs becomes easier if
  1. the particle  $j$  is high but its projection  $\Omega$  small
  2. the Coriolis force is large: small  $\mathfrak{I}$  and high  $\omega$

# Aligned Particles

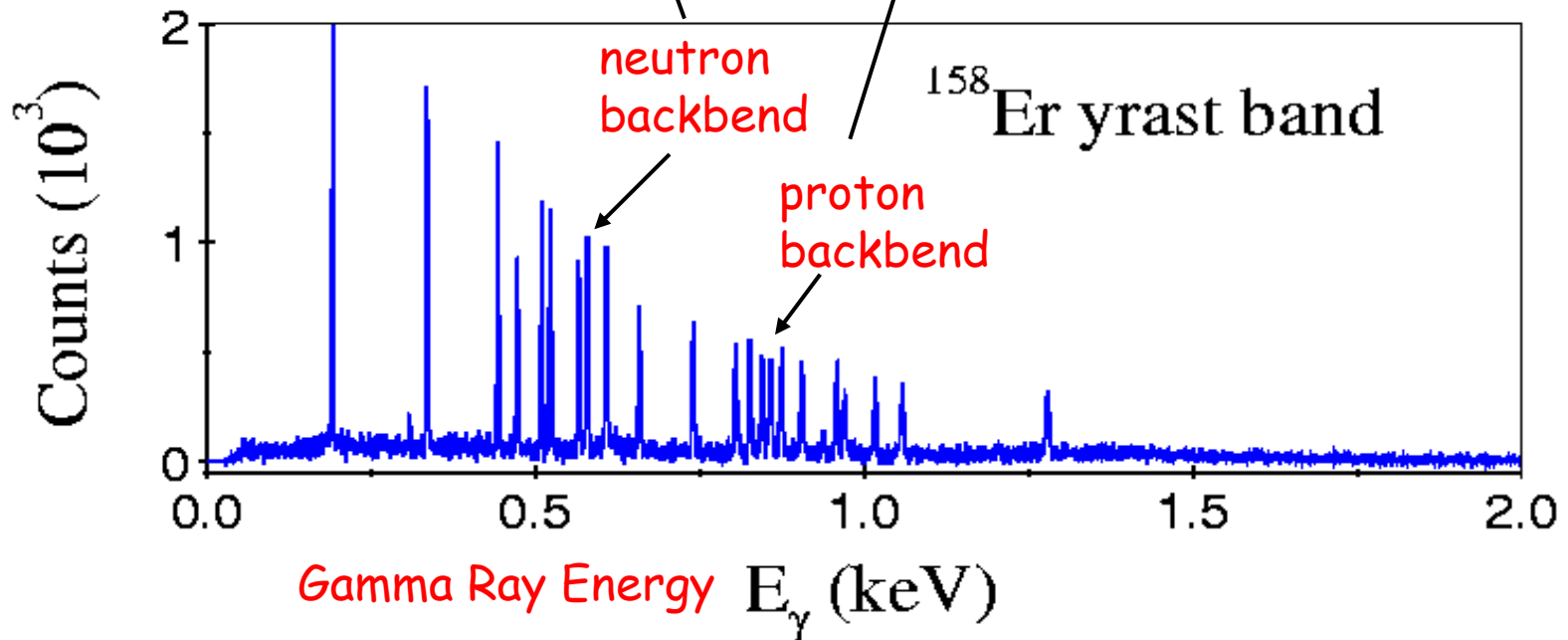
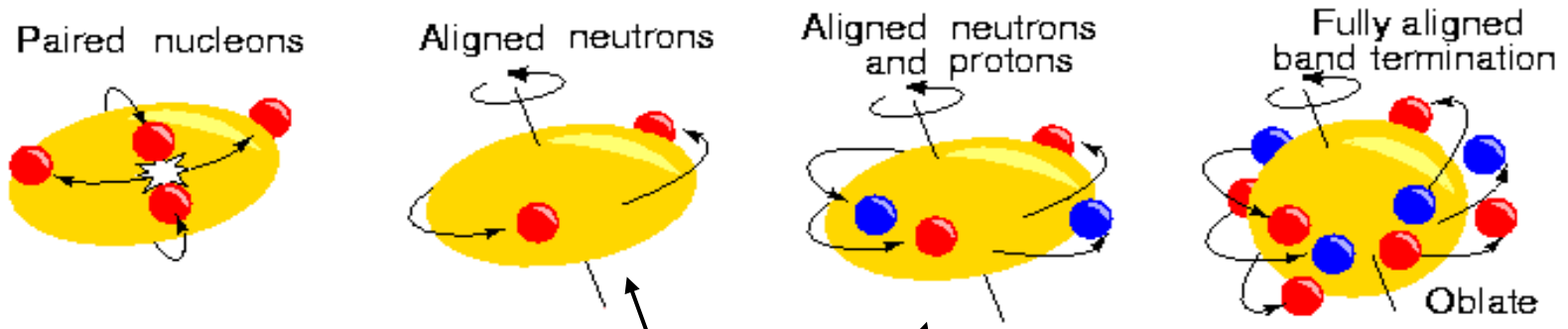


$$I = \sum j_i$$

$$R = 0$$

- Alignment effects should be prominent for nuclei with a few nucleons outside a closed shell, e.g.  $^{158}\text{Er}$  with 8 neutrons above the  $N = 82$  closed shell
- If we continue to rotate faster and faster then more of the valence pairs break and align
- Eventually all the particles outside the closed shell (spherical) core align
- These move in equatorial orbits giving the nucleus an **oblate** appearance

# Band Termination



# Band Termination in $^{158}\text{Er}$

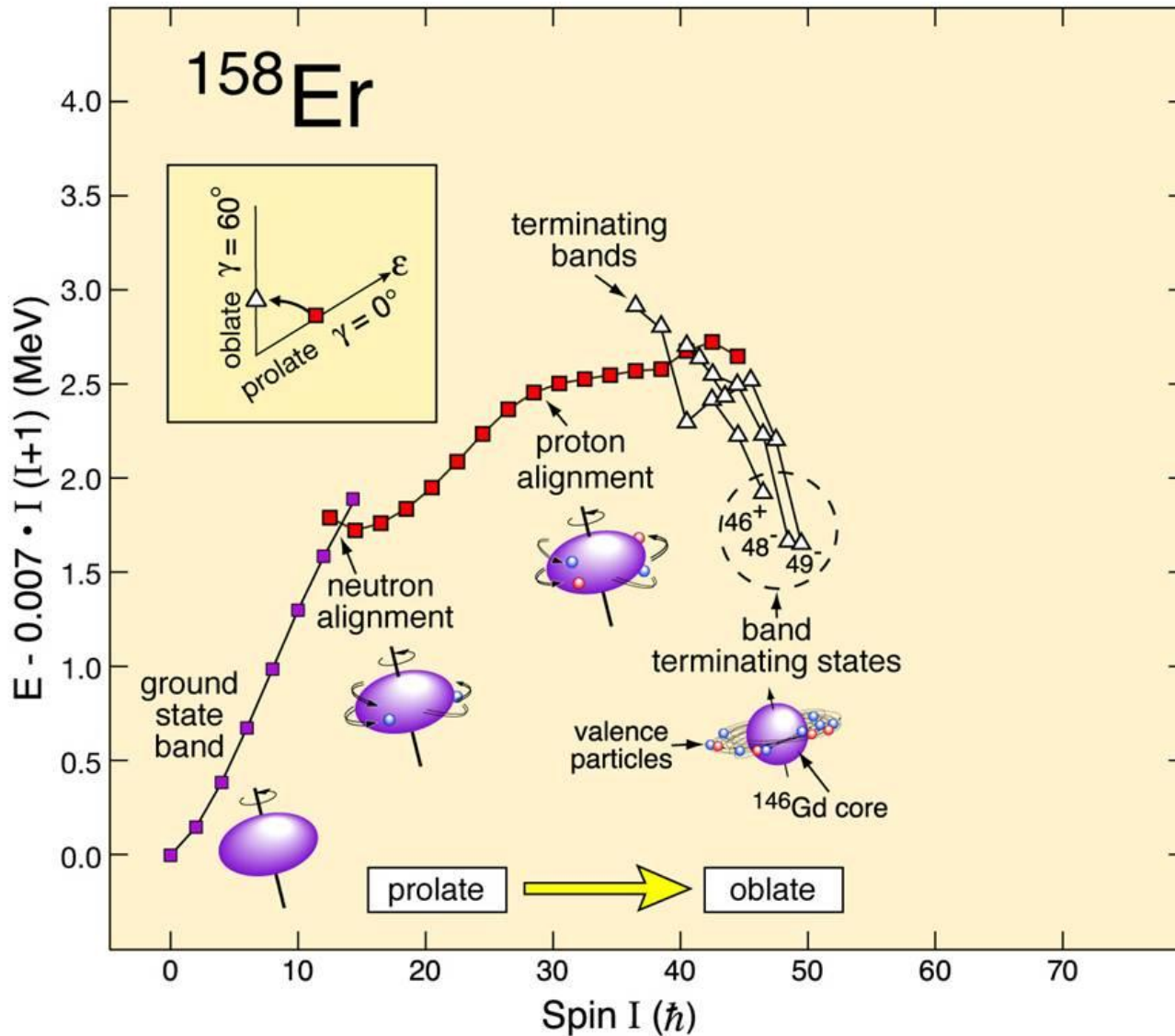
- When we align the  $np$  protons and  $nn$  neutrons outside the closed shell the total spin is:

$$I = \sum_i^{np} j_i(p) + \sum_i^{nn} j_i(n)$$

and the rotational band is said to 'terminate'

- At termination  $^{158}\text{Er}$  can be thought of as a spherical  $^{146}\text{Gd}$  core plus 4 protons and 8 neutrons generating a total spin  $46\hbar$
- The configuration is:  
$$\pi(h_{11/2})^4 \otimes \nu(i_{13/2})^2(h_{9/2})^3(f_{7/2})^3$$
- The terminating spin value of 46 is generated as:  
$$(11/2+9/2+7/2+5/2) + (13/2+11/2) + (9/2+7/2+5/2) + (7/2+5/2+3/2)$$

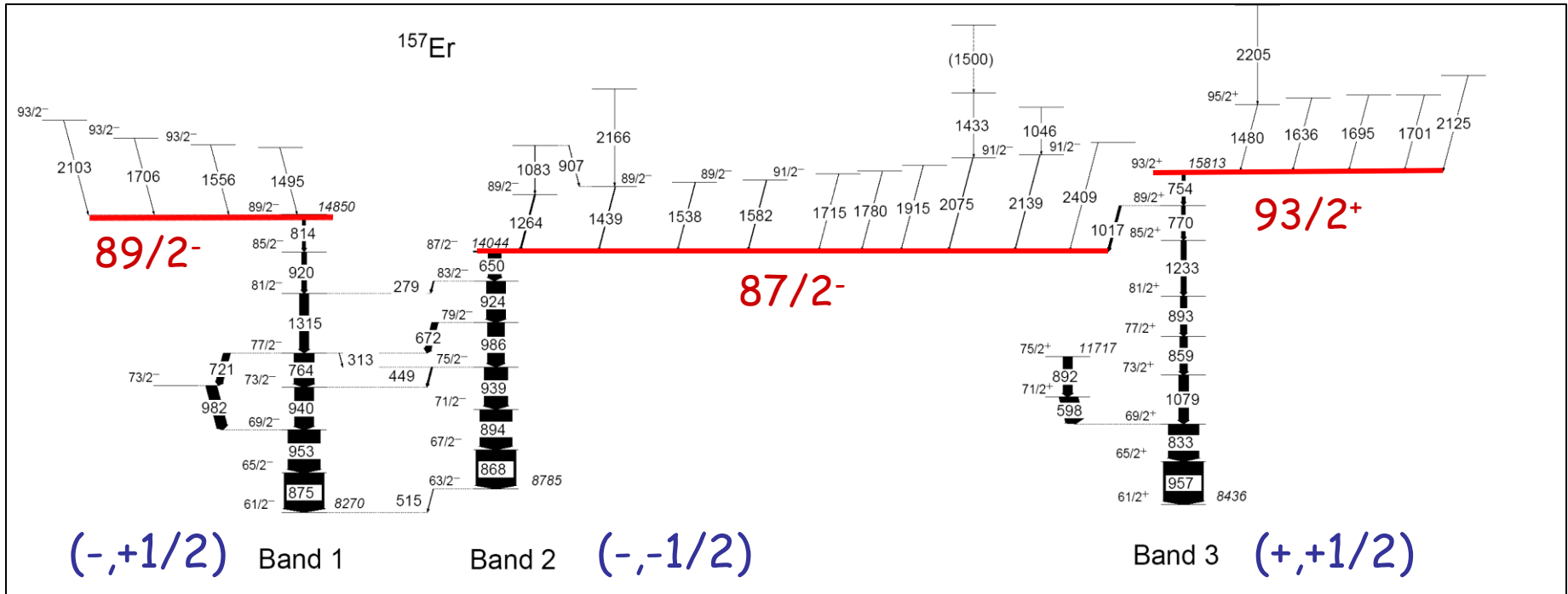
# Terminating States in $^{158}\text{Er}$



# Beyond Band Termination

- How does the nucleus generate states of yet higher spin?
- Energetically expensive **core-breaking** excitations
- These excitations may allow the nucleus to adopt a **deformed** (triaxial) shape
- Discrete energy levels in  $^{158}\text{Er}$  reach a record **75h**!

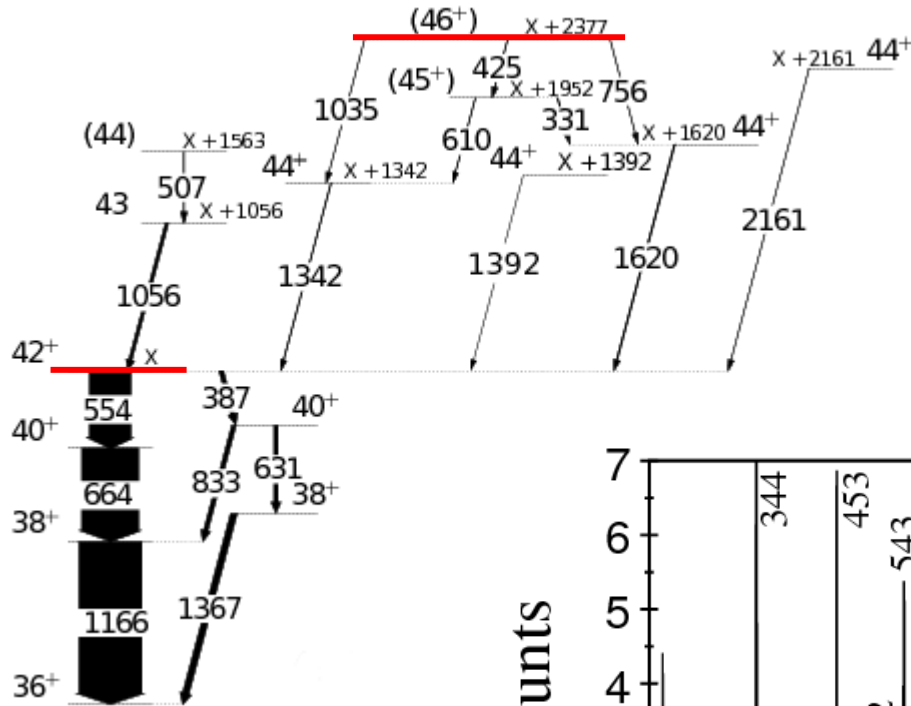
# Beyond Termination: $^{157}\text{Er}$



- Many **high-energy** dipole and quadrupole gamma-ray transitions feed the three terminating states in  $^{157}\text{Er}$
- Core breaking: **1p-1h** and **2p-2h** excitations across  $Z = 64$

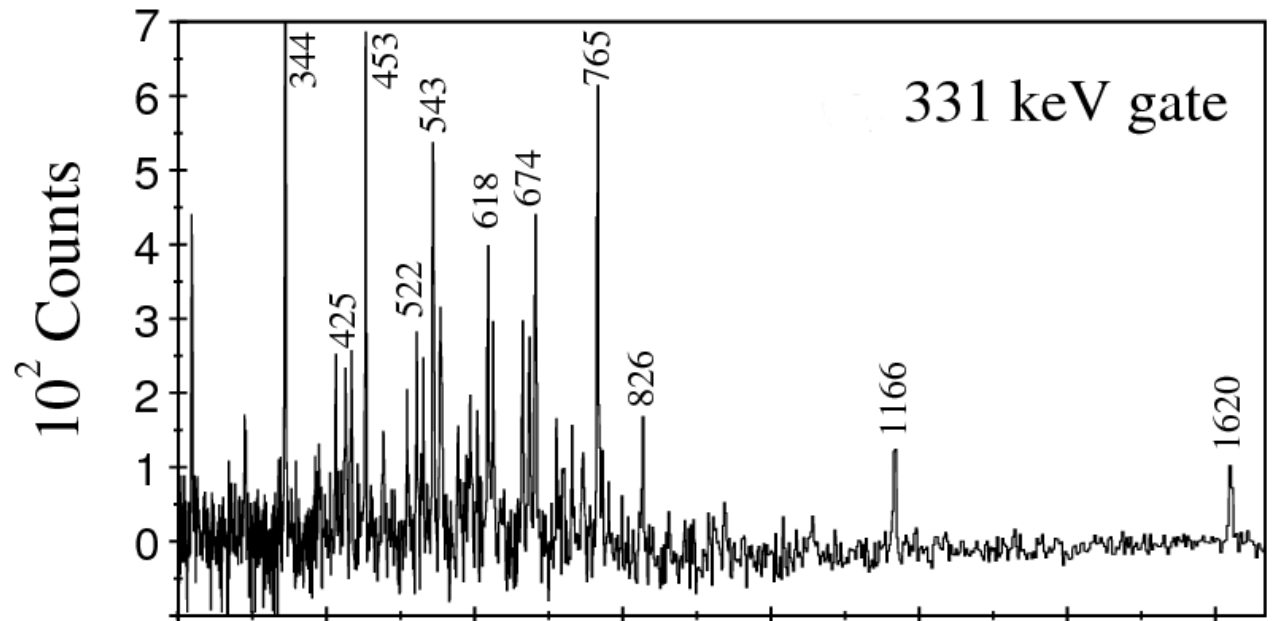
A.O. Evans et al., PRL **92**, 252502 (2004)

# 46<sup>+</sup> State in <sup>156</sup>Er



42<sup>+</sup> - valence particles

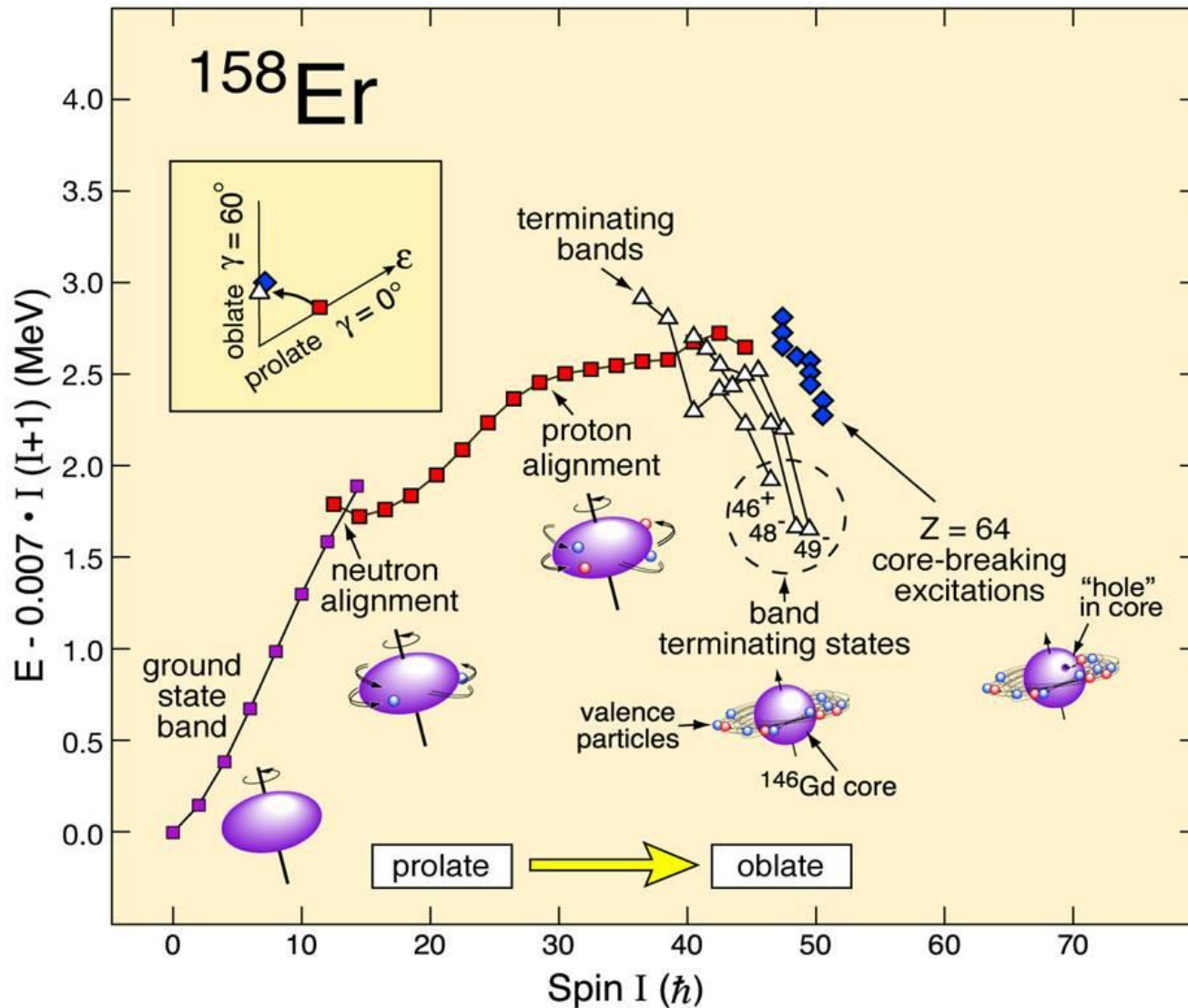
46<sup>+</sup> - plus  $\pi 1p-1h$



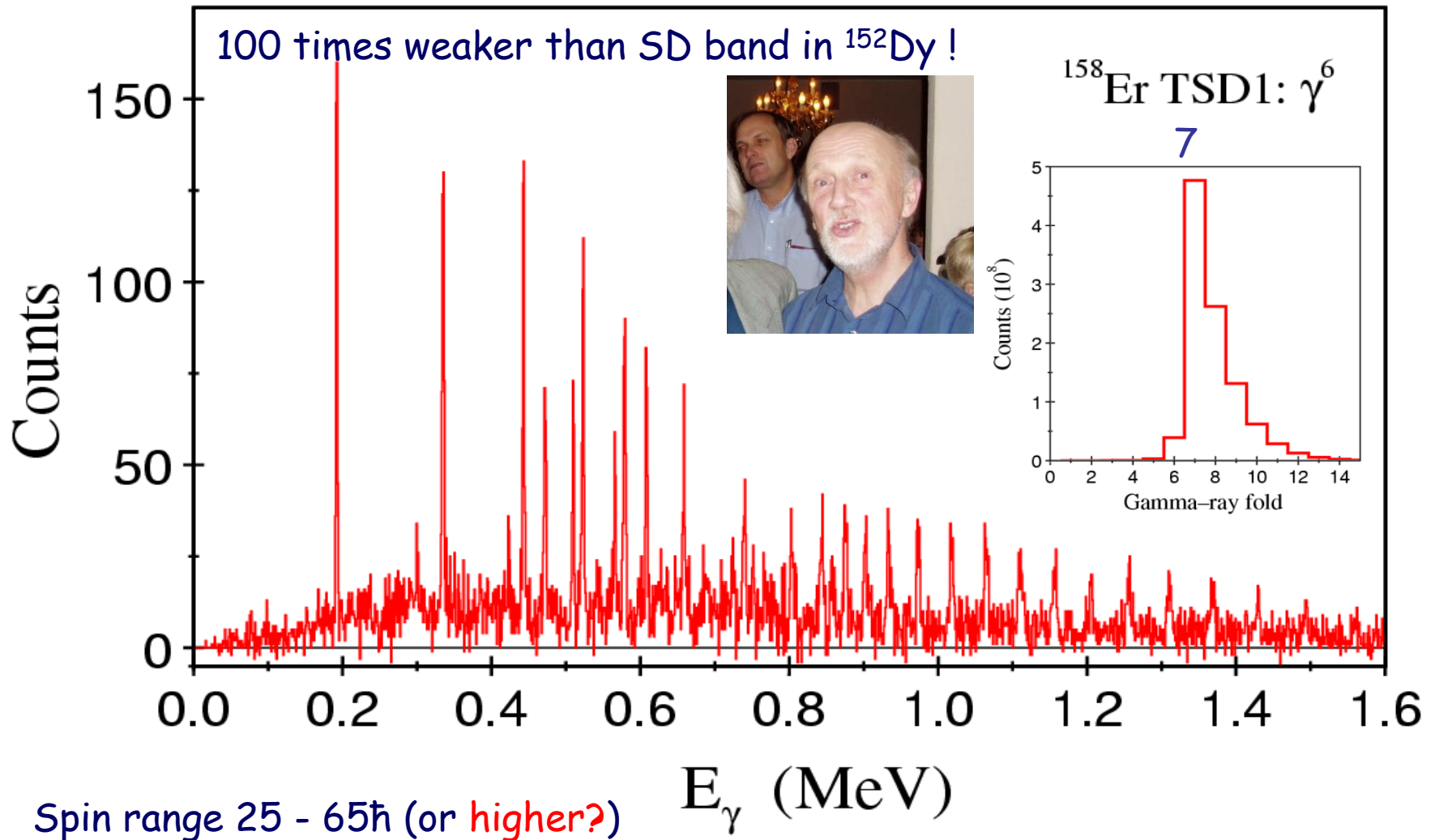
J.M. Rees, Thesis (2013)



# Feeding States

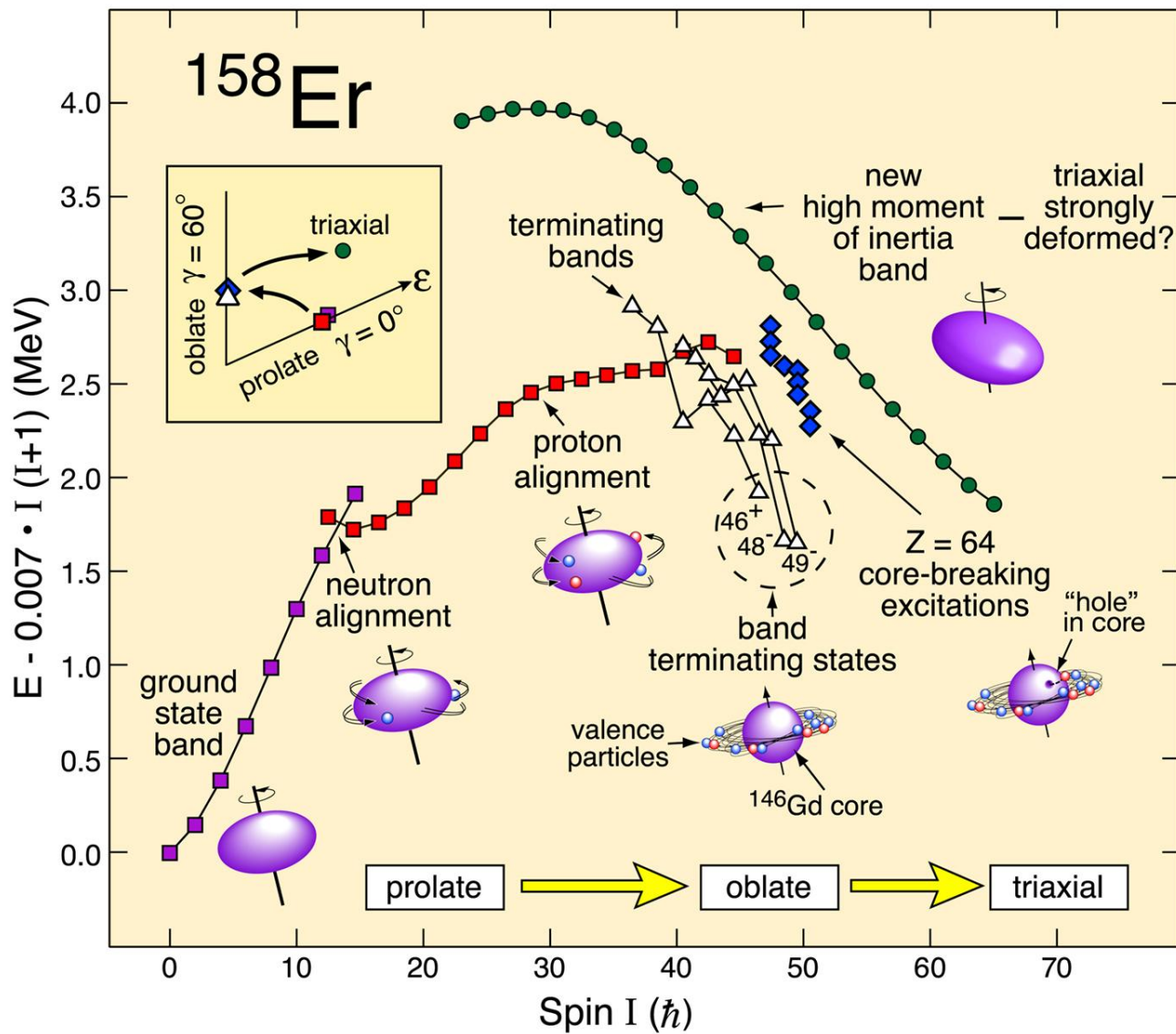


# Ultrahigh-Spin Bands in $^{157,158}\text{Er}$

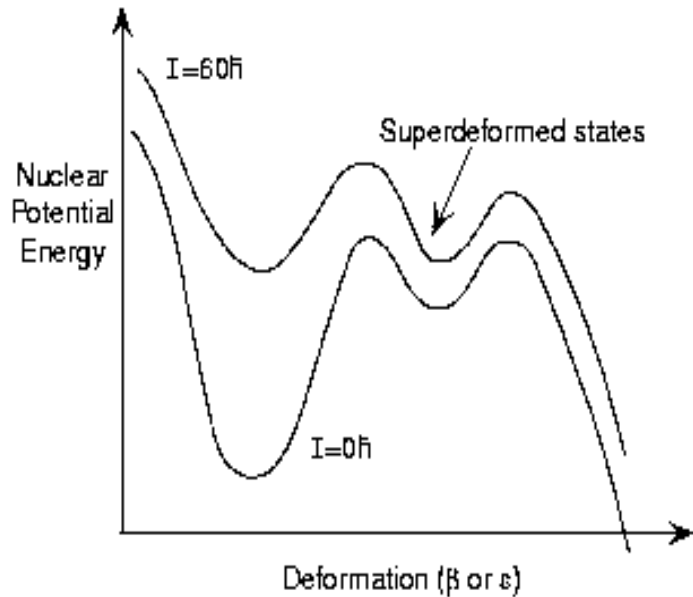


E.S. Paul, P.J. Twin et al., PRL **98**, 012501 (2007)

# Structural Evolution in $^{158}\text{Er}$



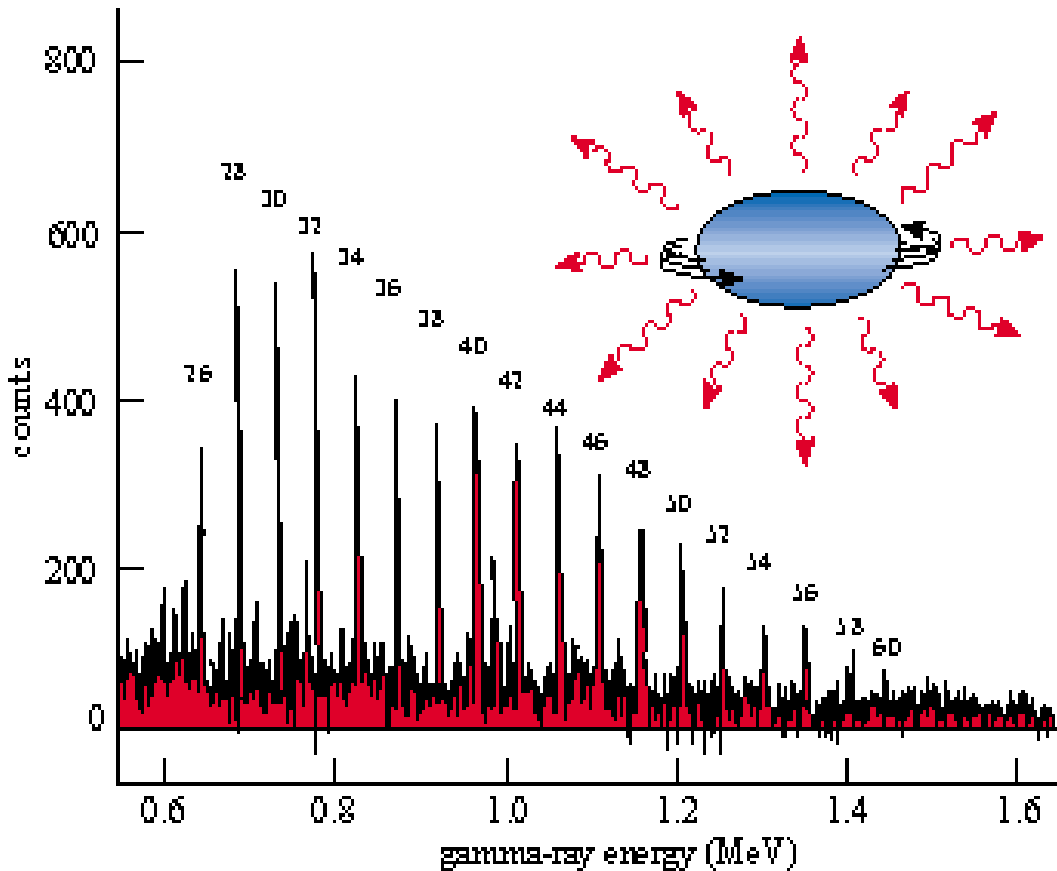
# Superdeformation



Nuclear potential at low and high spin

- Shell effects can give large energy corrections for large values of prolate deformation, e.g. when the major/minor axis ratio is **2:1**
- The smooth liquid-drop contribution to the total nuclear energy includes the **rotational energy**, which can be substantially reduced at high spin by **increasing the moment of inertia**
- At sufficiently high spin a **secondary** minimum can become energetically favourable

# Superdeformed Band in $^{152}\text{Dy}$



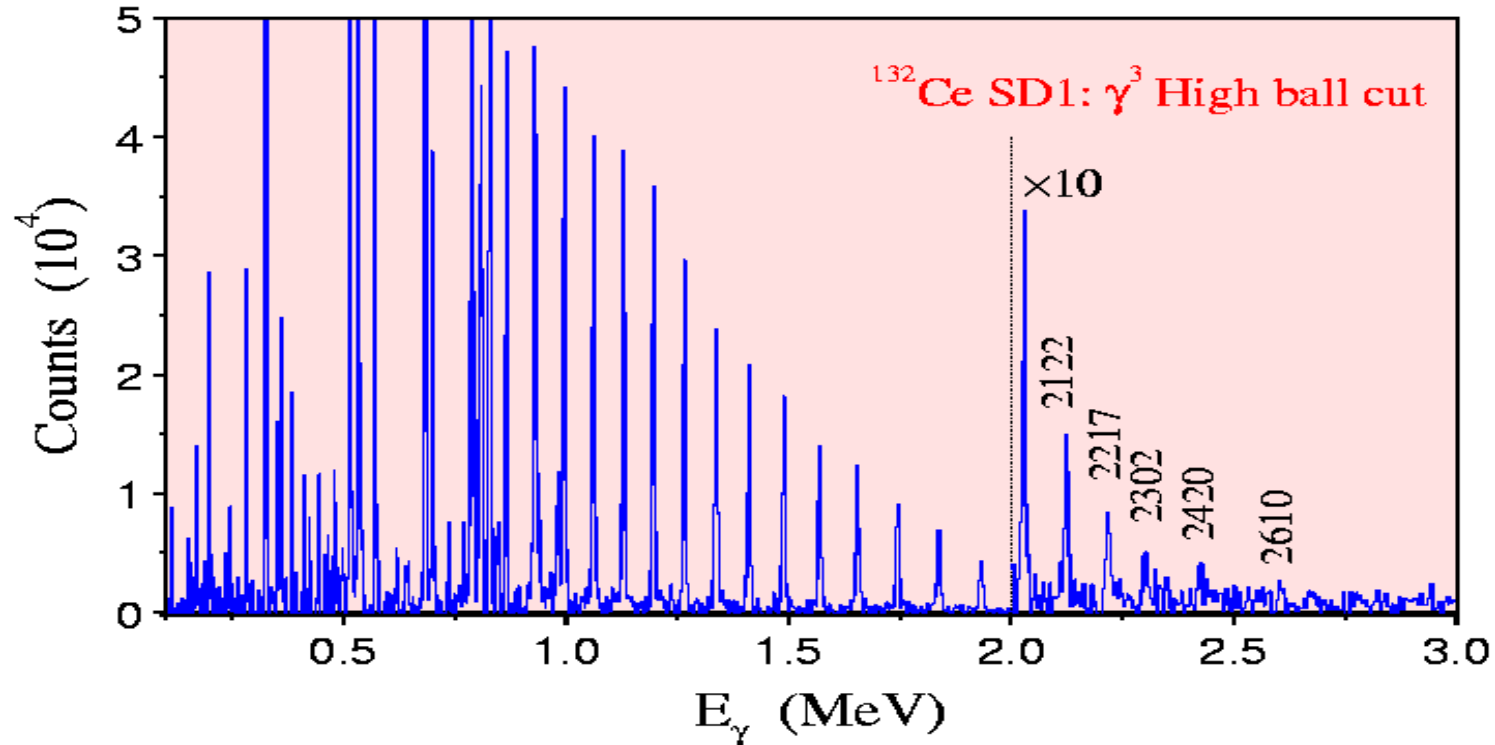
$\beta_2 \sim 0.6$ , 2:1 axis ratio

- The experimental signature of these superdeformed (SD) shapes is a very regular sequence of equally spaced  $\gamma$  rays
- In  $^{152}\text{Dy}$  the (first) SD band spans a spin range 20 - 60  $\hbar$
- Nowadays multiple SD bands are known in this and other nuclei

# Some Big Numbers

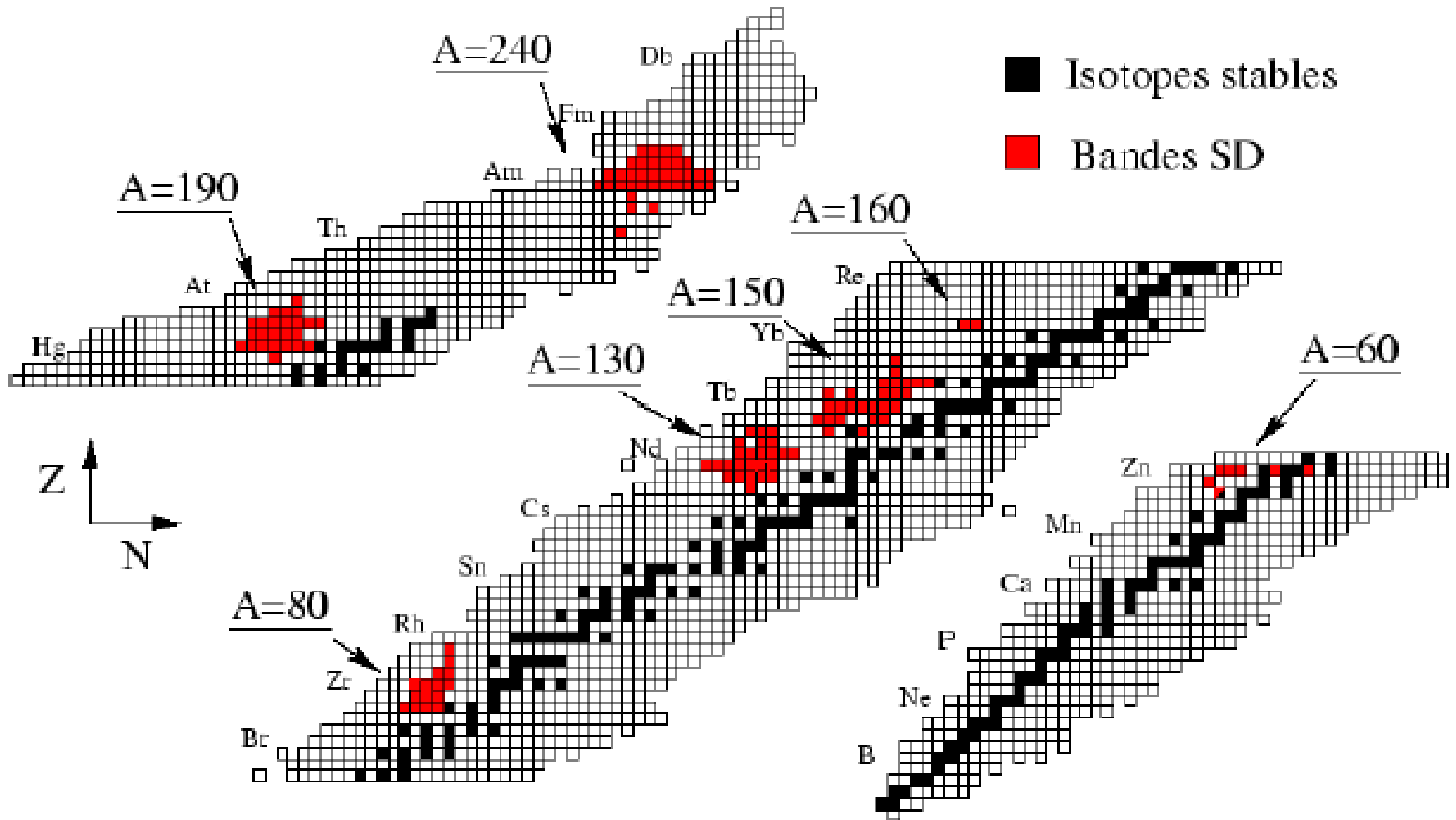
- The **SD** band of  $^{152}\text{Dy}$  emits  $\sim 20$  gamma rays in  $\sim 10^{-13}$  s. The total energy released is:  
 $\Sigma E_\gamma \sim 20 \text{ MeV}$  ( $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ )
- The power is:  $(3.2 \times 10^{-12} \text{ J}) / (10^{-13} \text{ s}) = 32 \text{ W} !$
- The rotational frequency is:  $\hbar\omega \sim 500 \text{ keV}$ , so  
 $\omega \sim 8 \times 10^{20} \text{ radians/sec} \rightarrow 10^{20} \text{ Hz}$  or  $10^7$  rotations in  $10^{-13} \text{ s} \rightarrow$  same as number of days in 30,000 years !
- The decay of the **SD** band passes through a long-lived **isomeric** level ( $86 \text{ ns}$ )  $\rightarrow \sim 5 \times 10^{12}$  rotations  $\rightarrow$  same as number of days since the Big Bang !

# Superdeformation in $^{132}\text{Ce}$

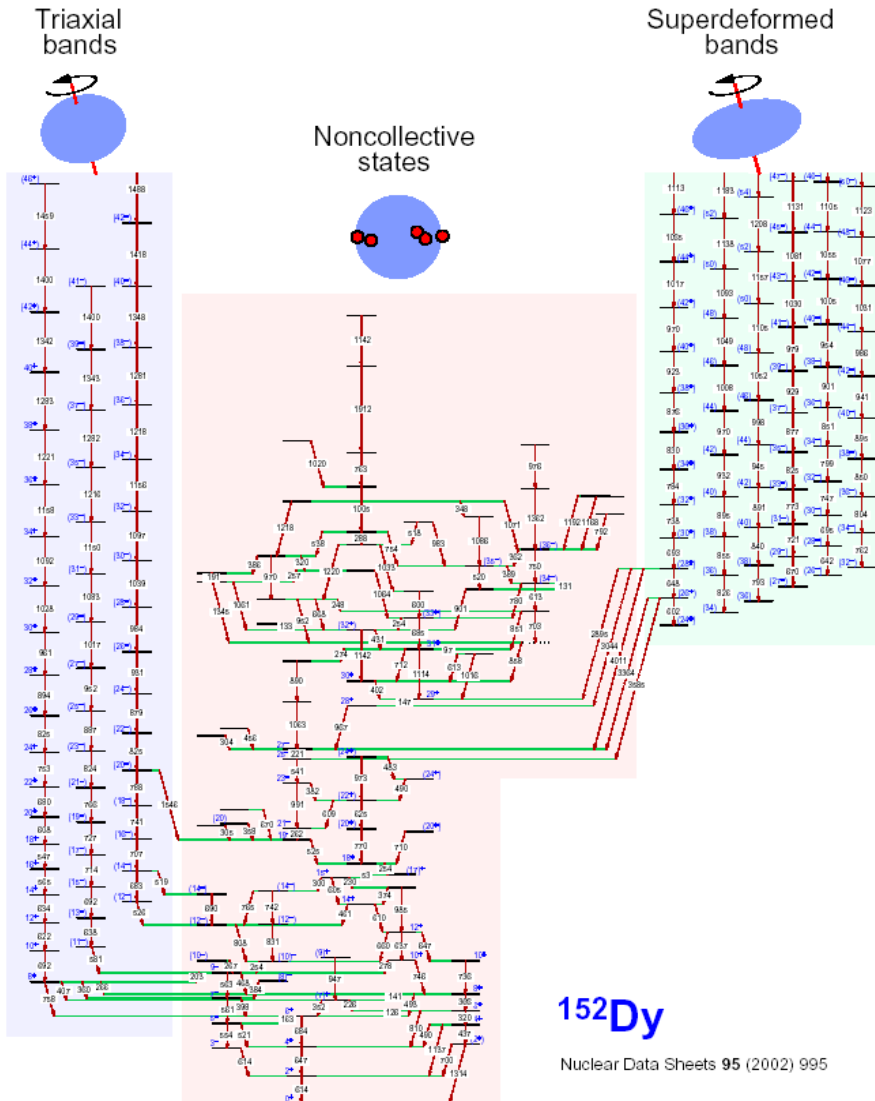


- SD bands exist in cerium ( $Z = 58$ ) nuclei with a major/minor axis ratio of 3:2. This band in  $^{132}\text{Ce}$  (THE original SD band - discovered by the Liverpool Nuclear Physics Group) is now seen up to spin approaching  $70\hbar$  - one of the highest spins ever seen in the atomic nucleus!

# Superdeformed Systematics

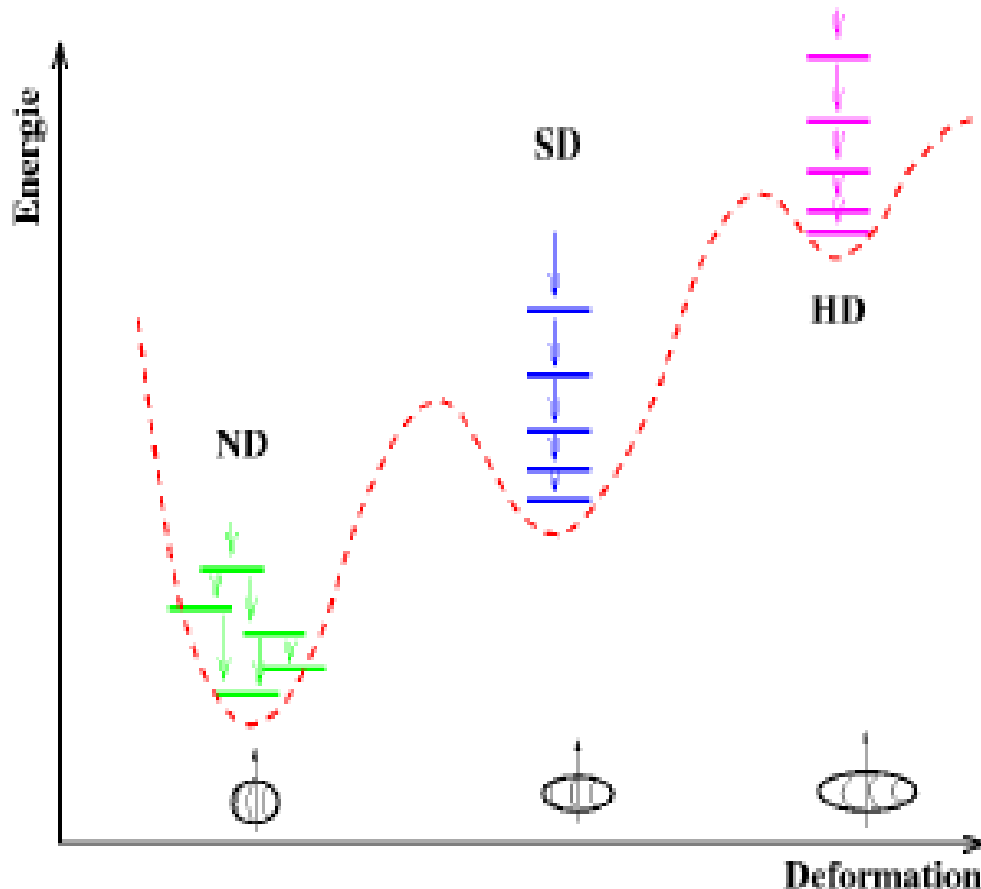


# Shape Coexistence



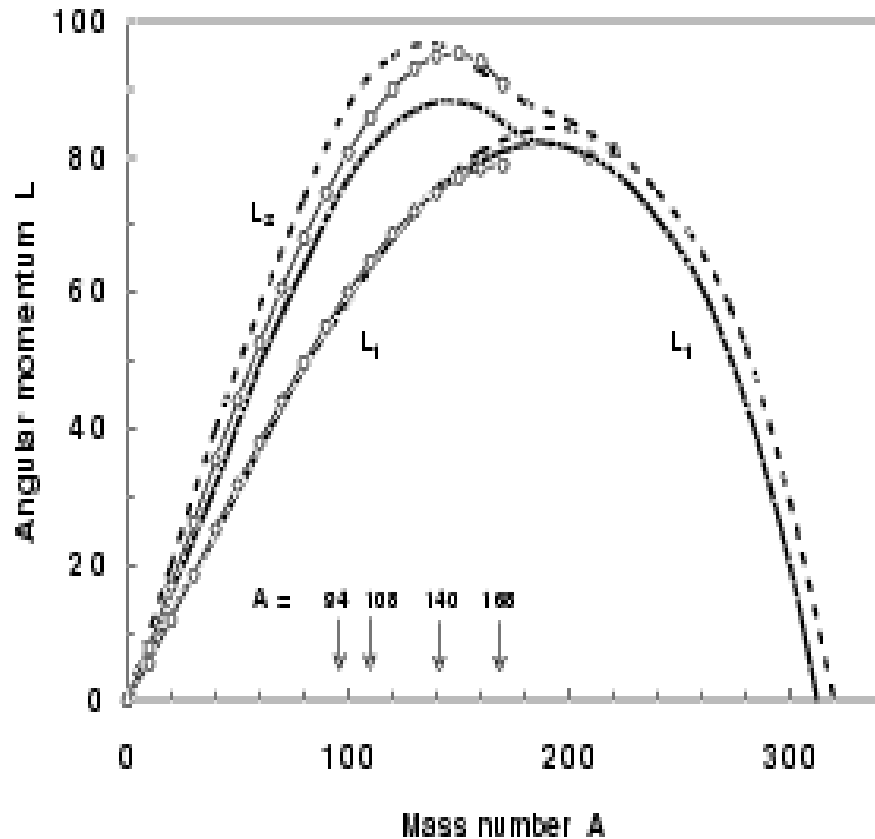
- For a given nuclear system at a given value of spin, a number of configurations can exist
- These configurations may have different shapes
- Weakly deformed **triaxial** and **oblate** shapes **coexist** in  $^{152}\text{Dy}$  along with the **superdeformed** shape
- Each shape has a (local) 'minimum' in the nuclear 'total energy surface'

# Hyperdeformation



- Superdeformation represents a **secondary** minimum in the nuclear potential energy, with typically a **2:1** axis ratio
- Hyperdeformation represents a **third** minimum, with an axis ratio **3:1**

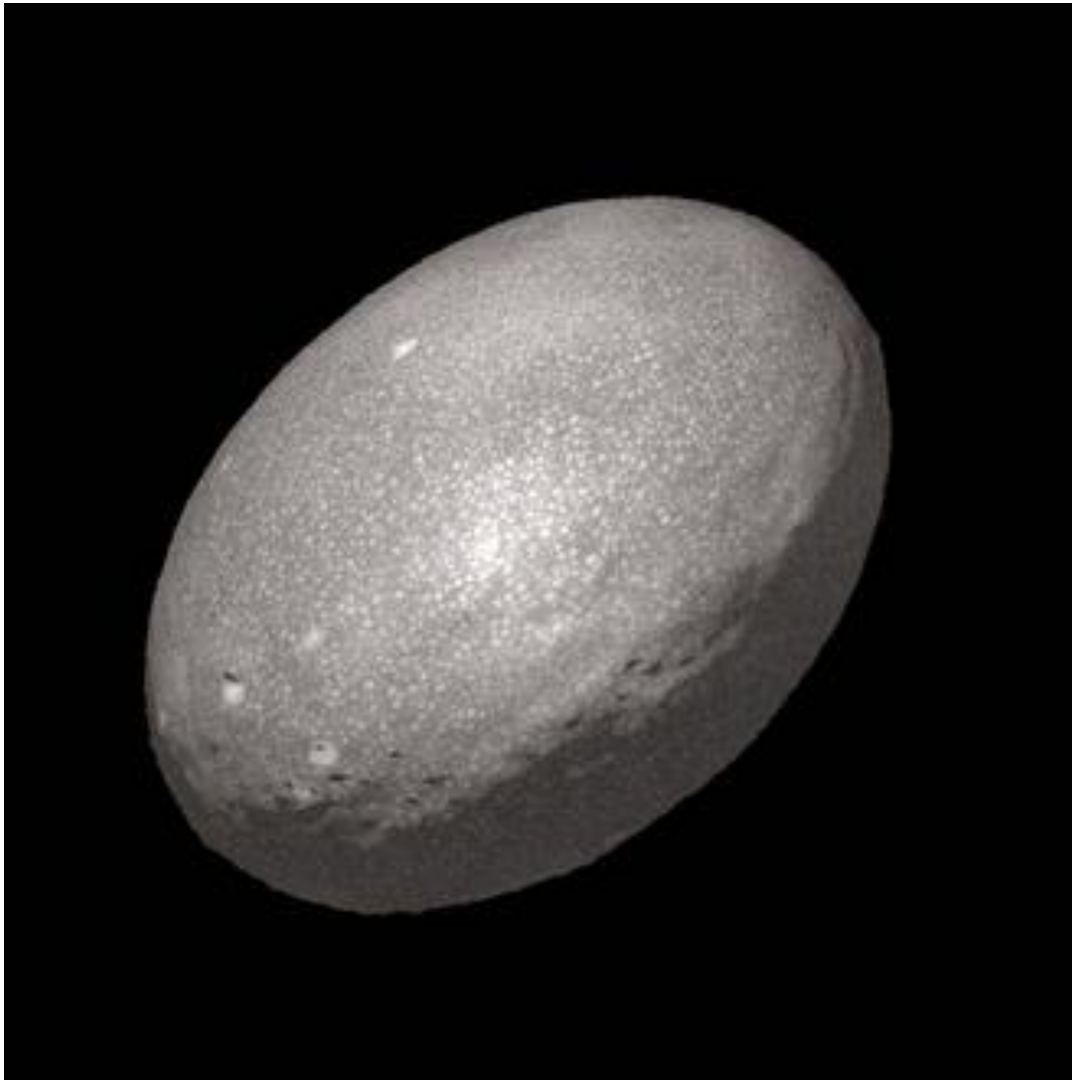
# Critical Angular Momentum



Nuclei with mass 130-150 can accommodate the most spin

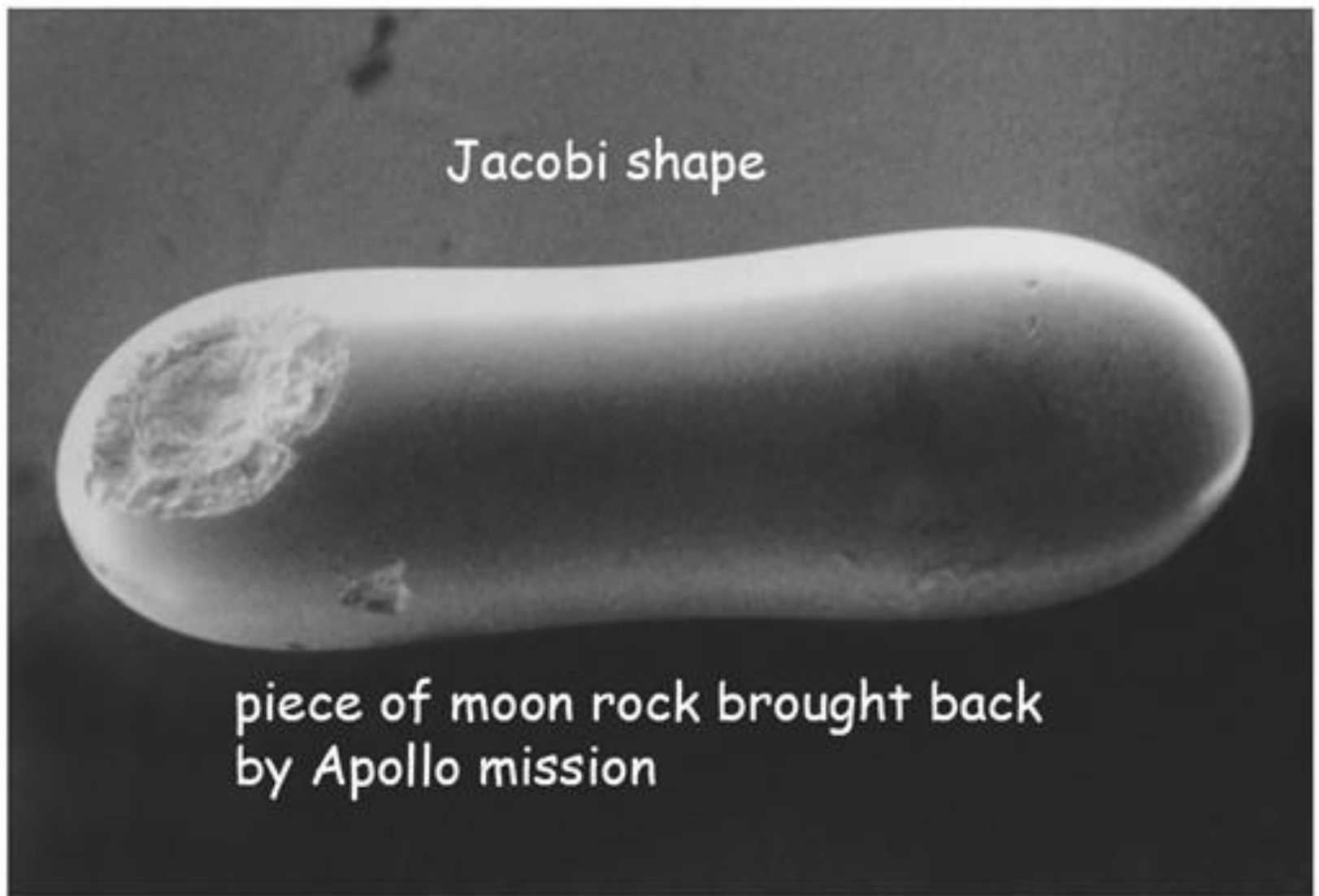
- Nuclei can only attain a finite amount of spin before they fly apart (fission)
- Just before this fission is a predicted region of extended triaxial ( $x \neq y \neq z$ ) shapes
- This is known as the **Jacobi** regime
- Such behaviour also occurs for macroscopic objects

# Jacobi Shape (classical)



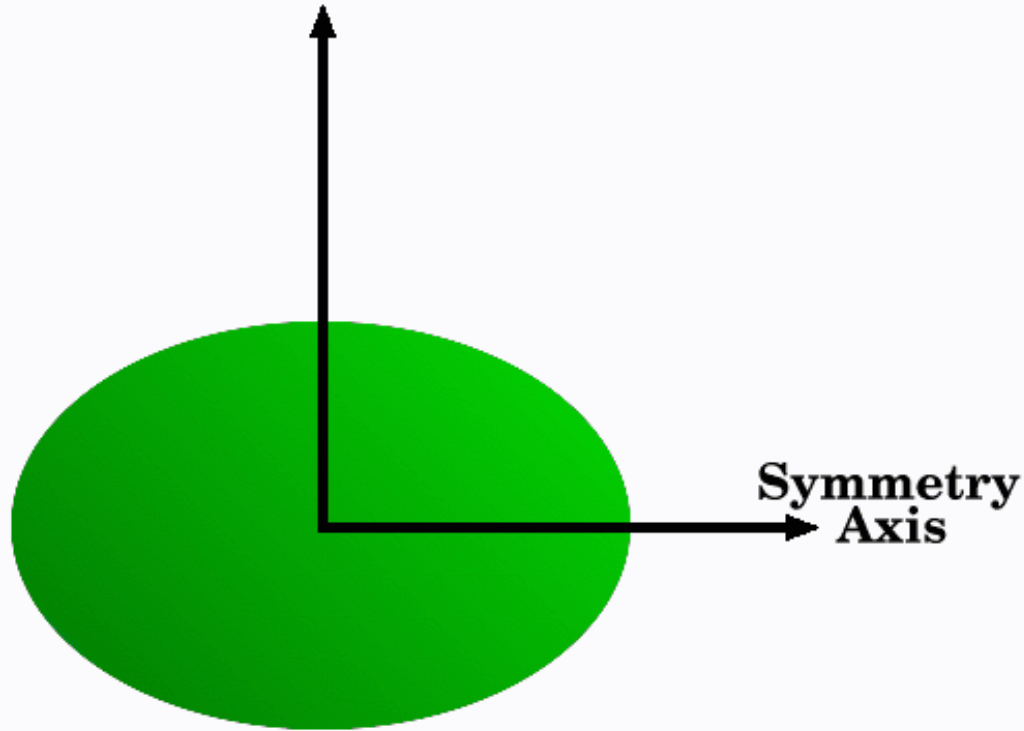
- Stable equilibrium shape with three different lengths of principal axes (i.e. **triaxial**)
- Can arise when a self-gravitating fluid body of uniform density rotates with a constant angular velocity

# On The Way To Fission



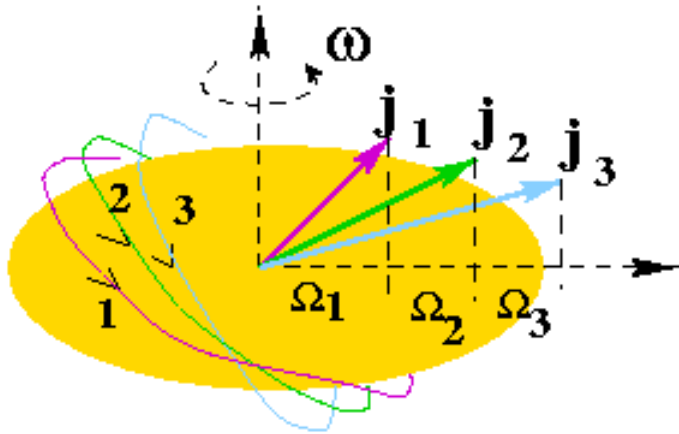
# Splitting of $\Omega$ States

## $i_{13}^{\frac{1}{2}}$ Orbitals With a Prolate Core

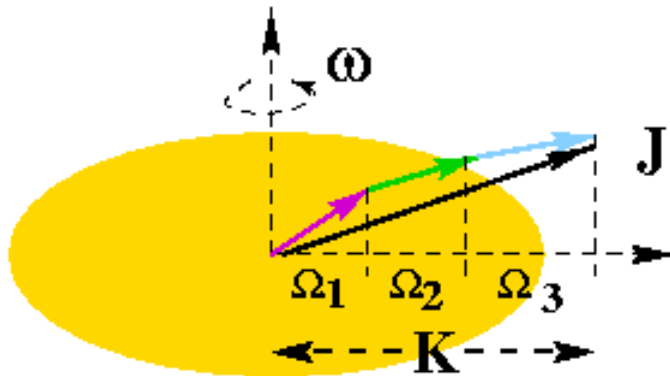


David Campbell  
Florida State  
University

# High K ( $I_z$ ) Bands



- If there are **many** unpaired nucleons outside the closed shell then alignment with the **x-axis** becomes difficult because the valence nucleons lie closer to the **z-axis**, i.e. they have high  $\Omega$  values



- The sum **K** of these projections onto the deformation (**z**) axis is now a good quantum number
- Isomeric band heads when **I = K**

$$K = I_z = \sum j_z = \sum \Omega$$

# Nuclear Isomerism

- If a nuclear state lives (much) longer than expected it is called an isomer
  - **Seniority Isomers** ("seniority" = no. unpaired particles)
  - **Spin Trap Isomers** (yrast trap, e.g.  $10^+$  below  $8^+$ )
  - **K Isomers** (reorientation of spin vector)
  - **Shape Isomers** (fission isomers, shape coexistence)
- The decay of the isomeric state is hindered

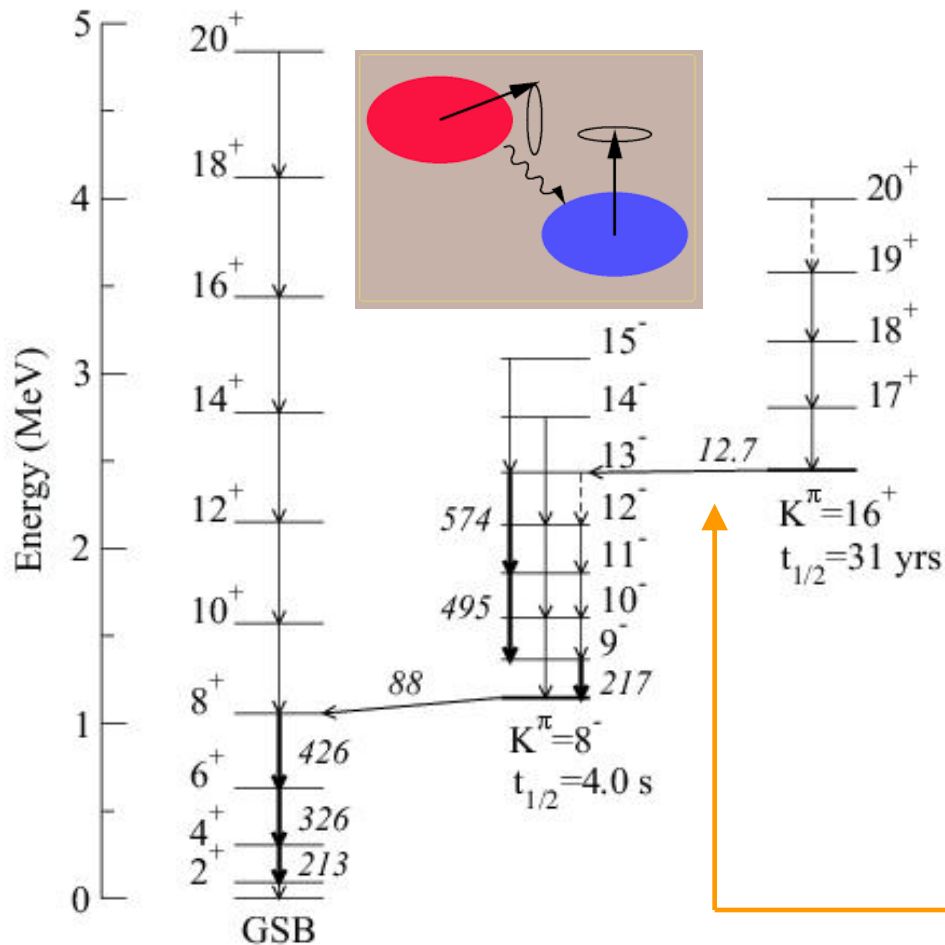
# K Forbidden Transitions

- It is difficult for rotational bands with high  $K$  values to decay to bands with smaller  $K$  since the nucleus has to change the orientation of its angular momentum.
- For example, the  $K^\pi = 8^-$  band head in  $^{178}\text{Hf}$  is isomeric with a lifetime of 4 s. This is much longer than the lifetimes of the rotational states built on it.
- The  $K^\pi = 8^-$  band head is formed by breaking a pair of protons and placing them in the 'Nilsson configurations':

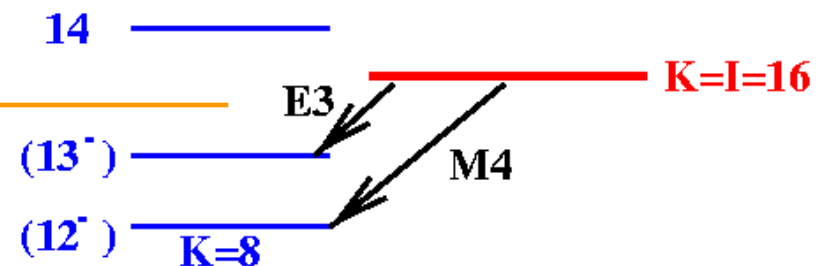
$$\Omega [N n_3 \Lambda] = 7/2 [4 0 4] \text{ and } 9/2 [5 1 4]$$

- In this case:  $K = 7/2 + 9/2 = 8$  and  $\pi = (-1)^{N(1)} \cdot (-1)^{N(2)} = -1$

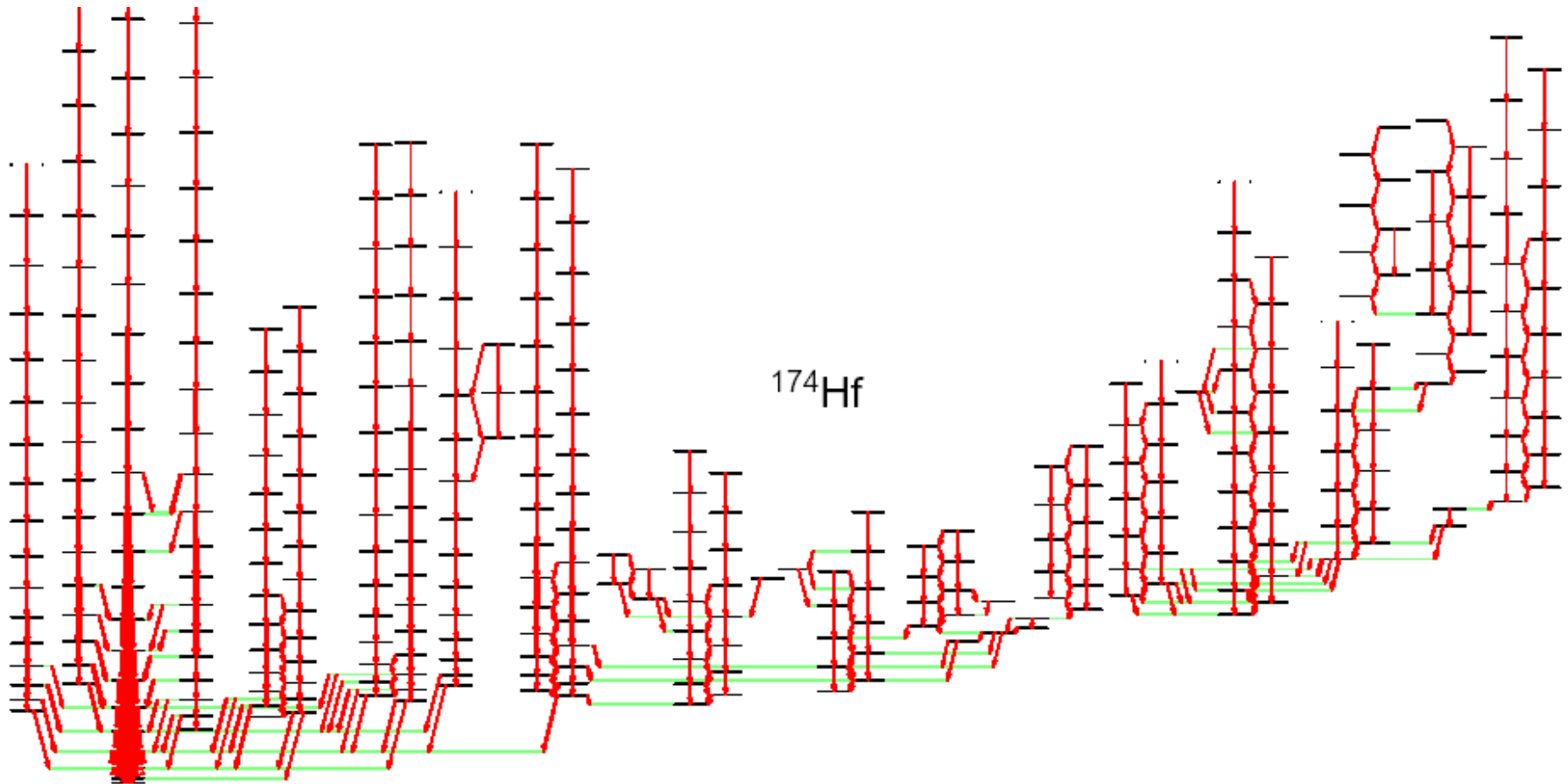
# $K^\pi = 8^-$ and $16^+$ Isomers in $^{178}\text{Hf}$



- A low lying state with spin  $I = 16$  and  $K = 16$  in  $^{178}\text{Hf}$  is isomeric with a half life of **31 years** !
- It is an example of both an yrast trap (lowest state for a given spin) and a K isomer



# High K bands in $^{174}\text{Hf}$



- This nucleus has 347 known levels and 516 gamma rays !

# Nuclear Wobbling

- This type of rotation is predicted to only occur in **triaxially** deformed nuclei
- The nucleus rotates around the principal axis having the **largest** moment of inertia and this axis executes **harmonic** oscillations about the space-fixed angular momentum vector
- Its analogue in classic mechanics is an asymmetric spinning top
- Precession of the Earth

# Wobbling Motion and Triaxiality

- Wobbling is a fundamental mode due to **triaxiality** which occurs when the axis of collective rotation does not coincide with one of the principal axes
- For a deformed rotor the Hamiltonian is:

$$H_{\text{rot}} = (\hbar^2/2\mathfrak{I}_x) I_x^2 + (\hbar^2/2\mathfrak{I}_y) I_y^2 + (\hbar^2/2\mathfrak{I}_z) I_z^2$$

- For a well-deformed but triaxial nucleus with  $\mathfrak{I}_x \gg \mathfrak{I}_y \neq \mathfrak{I}_z$  the energy of the wobbling rotor is:

$$E(I, n_W) = (\hbar^2/2\mathfrak{I}_x) I(I+1) + \hbar\omega_W(I) (n_W + \frac{1}{2})$$

where  $n_W$  is the number of wobbling phonons and  $\omega_W$  is the wobbling frequency

# Wobbling Frequency

- The wobbling frequency is related to the rotational frequency as:

$$\omega_W = \omega_{\text{rot}} \sqrt{ [ (\mathcal{I}_x - \mathcal{I}_y)(\mathcal{I}_x - \mathcal{I}_z) / (\mathcal{I}_y \mathcal{I}_z) ] }$$

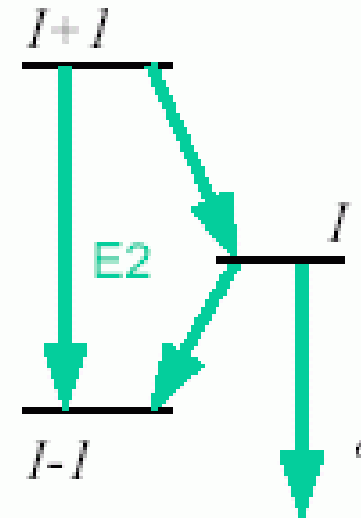
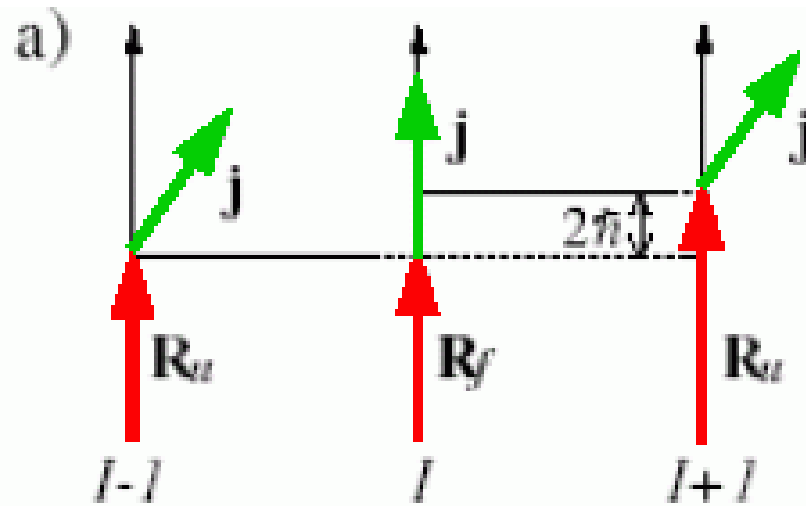
with

$$\omega_{\text{rot}} = \hbar I / \mathcal{I}_x$$

- Note for an axially symmetric prolate nucleus,  $\mathcal{I}_z$  goes to zero and  $\omega_W \rightarrow \infty$ , i.e. there is no wobbling motion
- A family of wobbling bands is expected for  $n_W = 0, 1, 2, \dots$

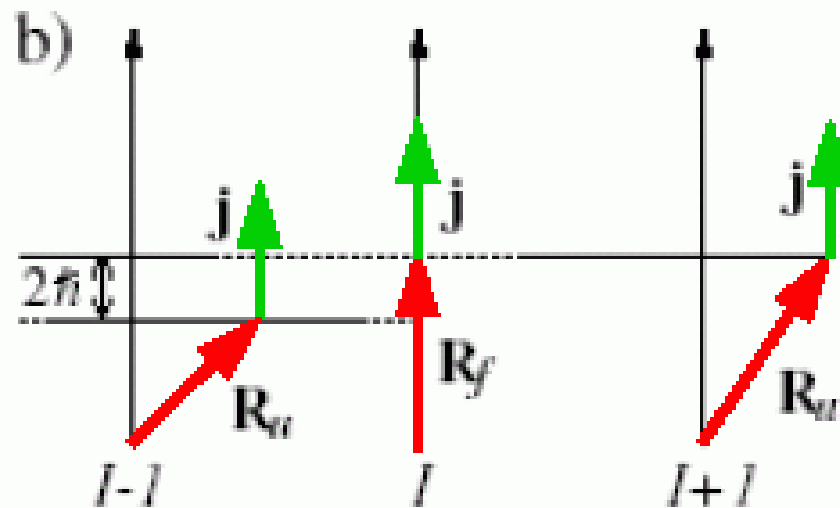
# Wobbling Motion

Cranking:

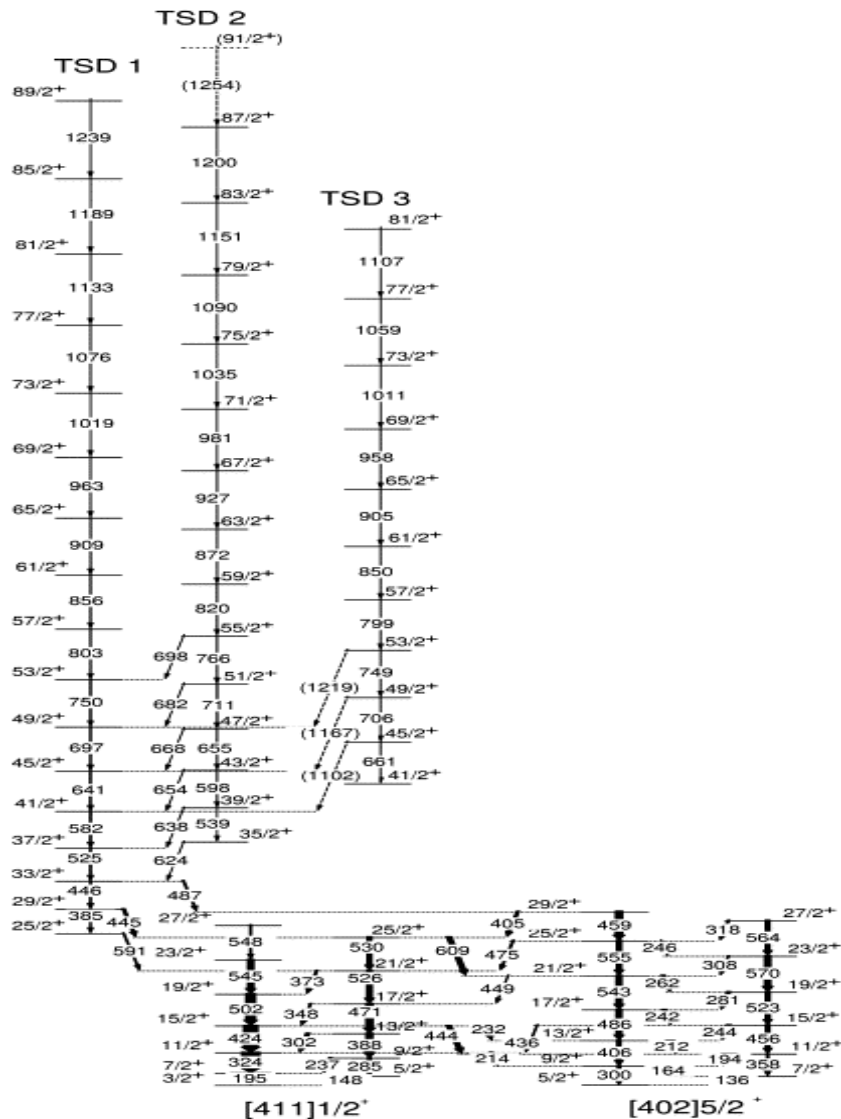


Wobbling:

-Requires  
Triaxialty!

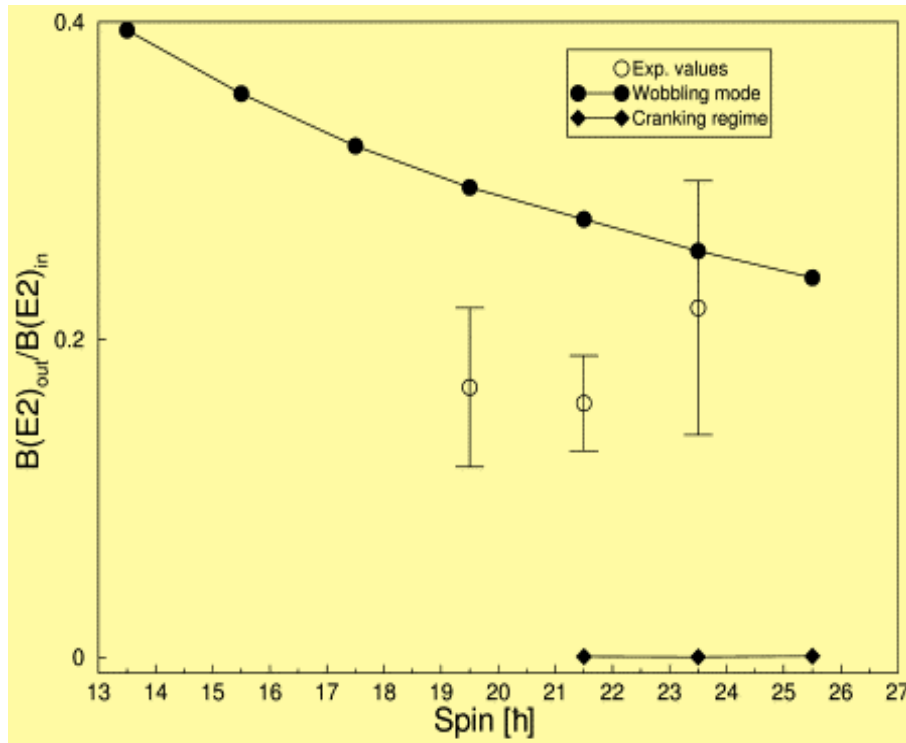


# Wobbling Bands in $^{165}\text{Lu}$



- A family of wobbling bands is expected to show very similar internal structure
- TSD (Triaxial SuperDeformed) bands 1, 2 and 3 in  $^{165}\text{Lu}$  represent bands with 0, 1 and 2 wobbling phonons, respectively

# Electromagnetic Properties



- A characteristic signature of wobbling motion is the occurrence of  $\Delta I = \pm 1$  interband transitions with unusually large  $B(E2)_{out}$  values that compete with the strong  $\Delta I = 2$  inband transitions,  $B(E2)_{in}$
- $\Delta n_w = 2$  transitions are forbidden
- Measured multipole mixing ratios for the interband  $\Delta I = 1$  transitions in  $^{165}\text{Lu}$  show them to be  $\sim 90\%$  E2 and only  $\sim 10\%$  M1 !

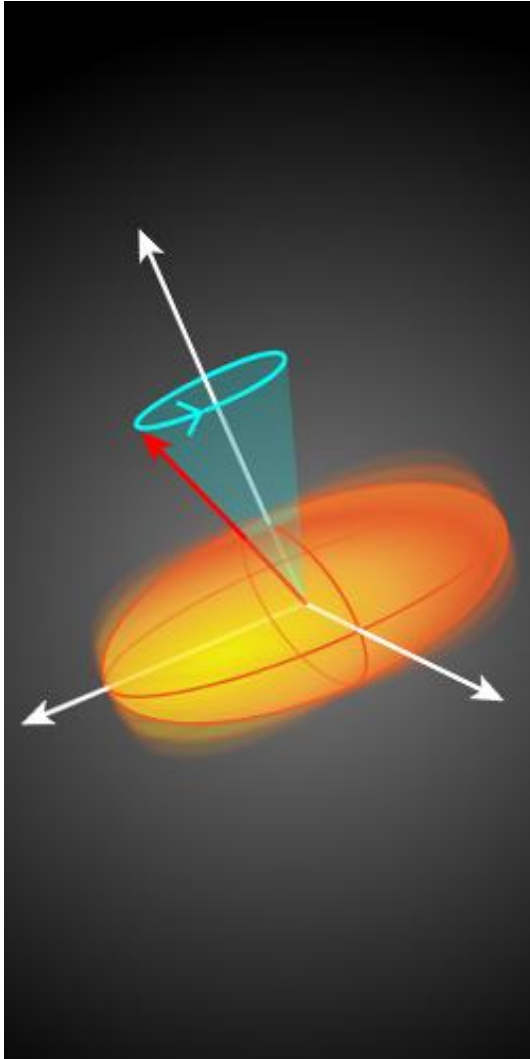
# $^{105}\text{Pd}$ : Transverse Wobbler

- Precise measurements of angular distributions and linear polarisations in  $^{105}\text{Pd}$  have also pointed to  $\Delta I = 1$  interlinking transitions with predominantly  $E2$  (non-stretched) character
- The angular momentum of the odd neutron ( $h_{11/2}$ ) is perpendicular to the wobbling axis, but aligned with the **short** axis
- Hence the name **transverse wobbler!**
- Phys. Rev. Letter (2019)

# $^{187}\text{Au}$ : Longitudinal Wobbler...

- Like other nuclei with an odd number of nucleons,  $^{187}\text{Au}$  can be modelled as a spinning **ellipsoid** with an **independent** nucleon orbiting within it
- In its ground state, the system rotates smoothly about one of the ellipsoid's axes. But at higher energies,  $^{187}\text{Au}$ , like other **triaxial** nuclei with three **unequal** principal axes, can exhibit a more complex motion
- This motion comes from the nucleon tugging on the nucleus, making its rotation axis **wobble** like an unbalanced spinning top

# ...Longitudinal Wobbling



- Alignment between the odd nucleon's orbital axis and the nuclear **intermediate** axis leads to **longitudinal** wobbling
- This unusual rotational mode of nuclear excitation was first predicted in 2014
- Phys. Rev Letter (2020)

# Summary

- Generation of angular momentum
- Backbending and band termination
- Ultrahigh spin
- High-K Isomers
- Wobbling motion