The atomic nucleus: a bound system of interacting nucleons

1. Nuclear forces and very light nuclei

2. The nuclear shell model: from few nucleon correlations towards modern applications

3. Nuclear deformation and collective motion: phenomenological models and self-consistent mean-field theory

Dynamics of the liquid drop

Theory developed by A. Bohr and B.Mottelson - 1949

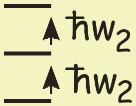
$$R = R_0 \left(1+\sum_{\lambda,\mu} \alpha_{\lambda\mu}(t)\ Y_\lambda^\mu(\theta,\varphi)\right)$$

$\alpha_{\lambda\mu}(t)$: small amplitude coördinates

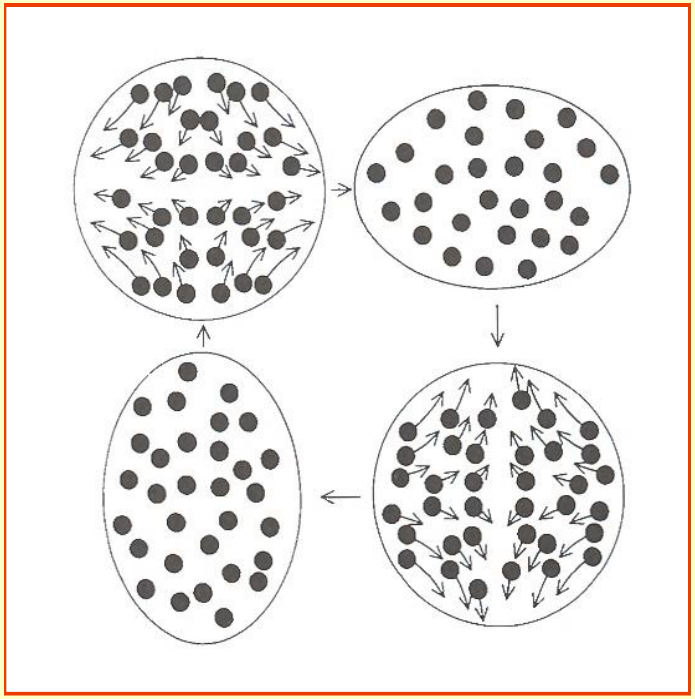
$\lambda = 0$: Compression mode
$\lambda = 1$: Shift of c.o.m.-c.o. charge
$\lambda = 2$: Quadrupole mode

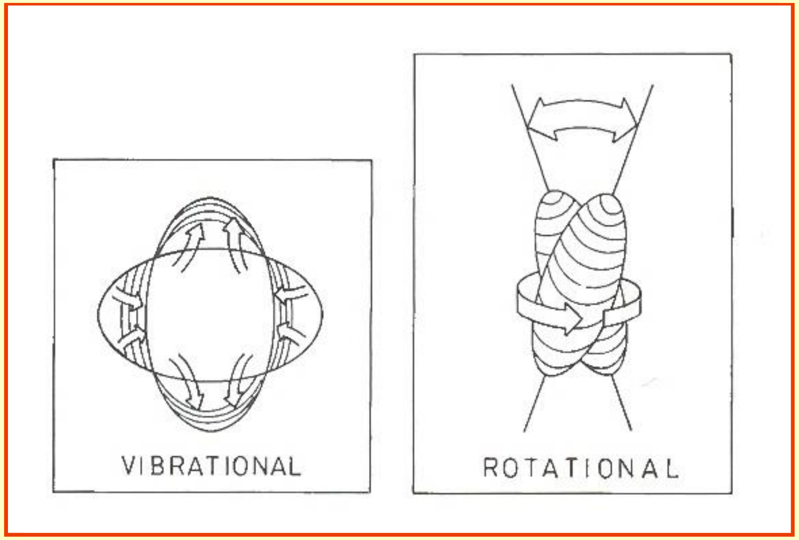
$$H = \frac{1}{2} \sum_{\lambda\mu} B_\lambda\ |\dot{\alpha}_{\lambda,\mu}|^2 + \frac{1}{2} \sum_{\lambda\mu} C_\lambda |\alpha_{\lambda,\mu}|^2$$

→ Quantize motion

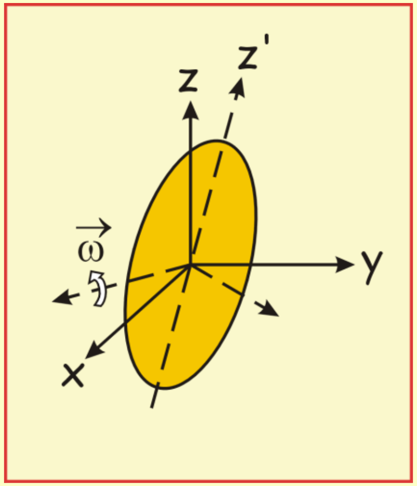


HARMONIC VIBRATIONS




VIBRATIONAL
ROTATIONAL

Deformed shape



$$\underset{\substack{\uparrow \\ \text{intrinsic}}}{a_{\lambda\mu}} = \sum_{\mu'} D_{\mu'\mu}(\Omega)\underset{\substack{\uparrow \\ \text{lab.}}}{\alpha_{\lambda\mu'}}$$

$$a_{20}=\beta\cos\gamma;\ a_{22}(a_{2,-2})=\frac{\beta}{\sqrt{2}}\sin\gamma,\ a_{21}(a_{2,-1}) = 0$$

$$R(\theta,\varphi) = Ro[1 + \beta\sqrt{\frac{5}{16\pi}}\ \{\cos\gamma(3\cos^2\theta-1)$$

$$+ \sqrt{3}\sin\gamma.\sin^2\theta.\cos2\varphi\}]$$

γ = 0°, 120°, 240° → prolate shape

γ = 60°, 180°, 300° → oblate shape

$$\delta R_k=Ro\sqrt{\frac{5}{4\pi}}\ \beta\cos\left(\gamma-\frac{2\pi}{3}k\right)$$

k=1, 2, 3

$$H = T(\beta,\gamma) + U(\beta,\gamma) \longrightarrow \text{Potential energy surface}$$

$$\swarrow$$

$$T_{rot} + \frac{1}{2} B_2(\dot{\beta}^2 + \beta^2\dot{\gamma}^2)$$

$$\Downarrow$$

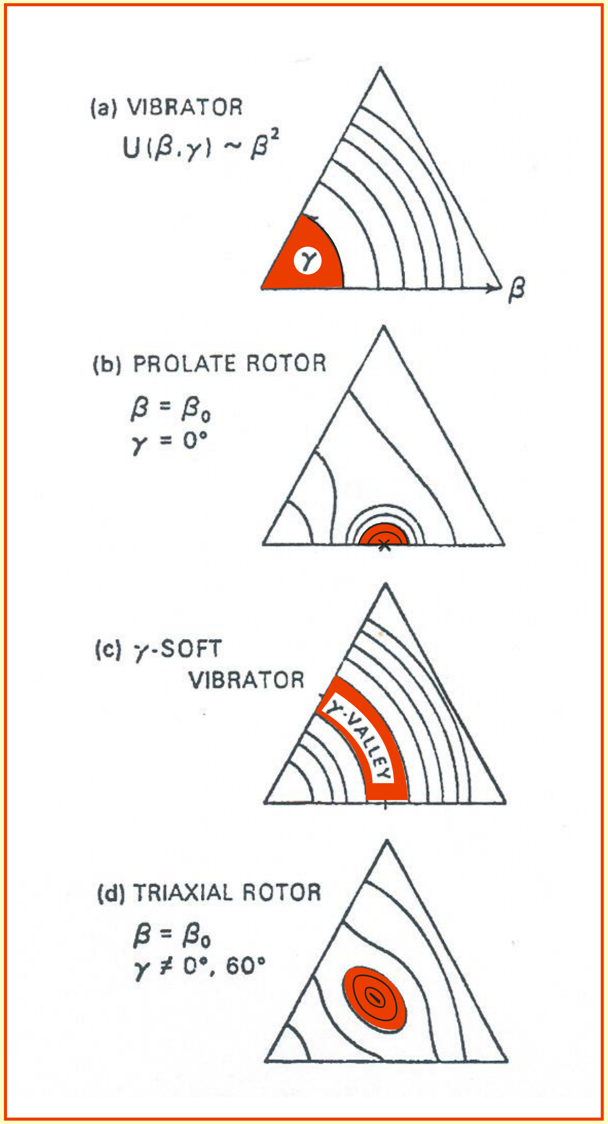
$$\frac{1}{2}\sum_{k=1}^{3} J_k \omega_k^2$$

Bohr Hamiltonian (Bohr-Mottelson)

$$\hat{H} = \frac{-\hbar^2}{2B_2}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin3\gamma}\frac{\partial}{\partial\gamma}\sin3\gamma\frac{\partial}{\partial\gamma} \right]$$

$$+\hat{T}_{rot.} + U(\beta,\gamma)$$

$$\Rightarrow \Psi(\beta,\gamma,\Omega)$$

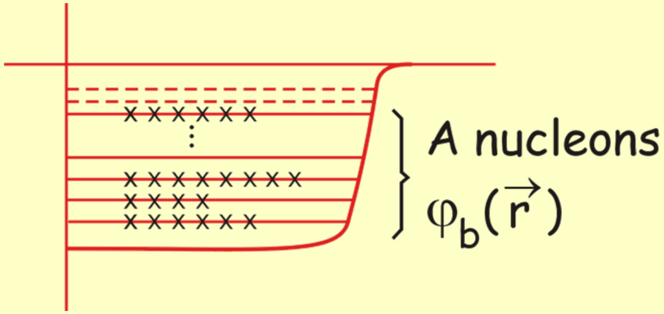


How to determine the nuclear mean field – a self-consistent approach

Hartree-Fock method

$$U(\vec{r}) = \int \rho(\vec{r}') V(\vec{r}, \vec{r}') \, d\vec{r}'$$

$$\rho(\vec{r}') = \sum_{b\{occ.\}} |\varphi_b(\vec{r}')|^2$$



Solve H. F. equations in self-consistent way, starting from $V(\vec{r}, \vec{r}')$ and initial guess for $\varphi_i(\vec{r})$

$$-\frac{\hbar^2}{2m} \Delta_i \varphi_i(\vec{r}) + U(\vec{r}_i) \varphi_i(\vec{r}) = \varepsilon_i \varphi_i(\vec{r})$$

Direct (Hartree) term

$$-\frac{\hbar^2}{2m} \Delta_i \varphi_i(\vec{r}) + \sum_{b\{occ\}} \int \varphi_b^*(\vec{r}') \, V(\vec{r}, \vec{r}') \, \varphi_b(\vec{r}') \, \varphi_i(\vec{r}) \, d\vec{r}'$$

Exchange(Fock)term

$$-\sum_{b\{occ\}} \int \varphi_b^*(\vec{r}') \, V(\vec{r}, \vec{r}') \, \varphi_b(\vec{r}) \, \varphi_i(\vec{r}') \, d\vec{r}' = \varepsilon_i \varphi_i(\vec{r})$$

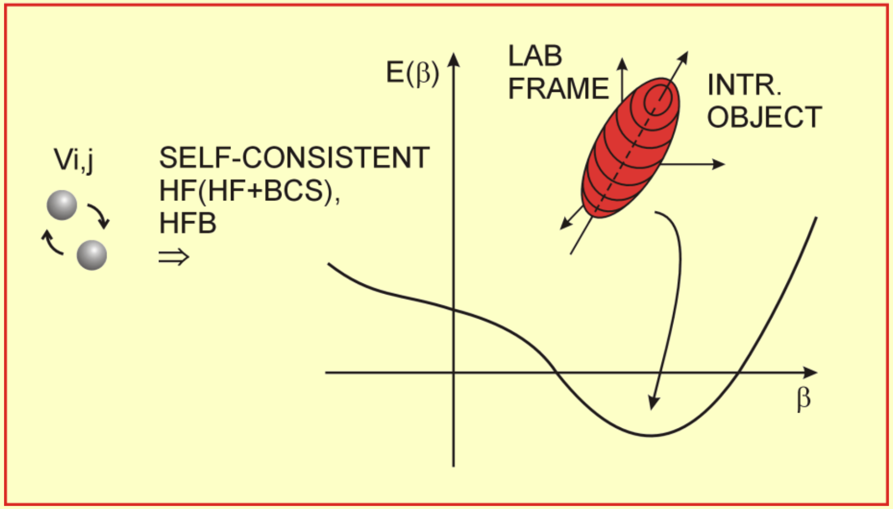
Iterate equations ⇒ convergence

Nucleons interacting with 2-body interaction $V_{i,j}$ generate a mean field U_i in the nucleus
→ shell structure.

$$\Phi^{HF}(1, 2,..A) = \text{Slater det.}\{\varphi_{\alpha_1}(1)\,\varphi_{\alpha_2}(2)\ldots \ldots\varphi_{\alpha_A}(A)\}$$

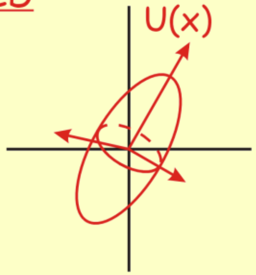
Energy minimum $E^{HF} = \langle \Phi^{HF}|\hat{H}|\Phi^{HF}\rangle$
→ deformed energy minimum

Conclusion : most nuclei have a "deformed" equilibrium shape



QUADRUPOLE DEFORMED FIELD

$$U(\vec{r}) = U_{h.o.}(r) + \varepsilon_2 r^2 \underset{\substack{\downarrow \\ r^2(3\cos^2\theta-1)}}{Y_2^0}(\theta)$$

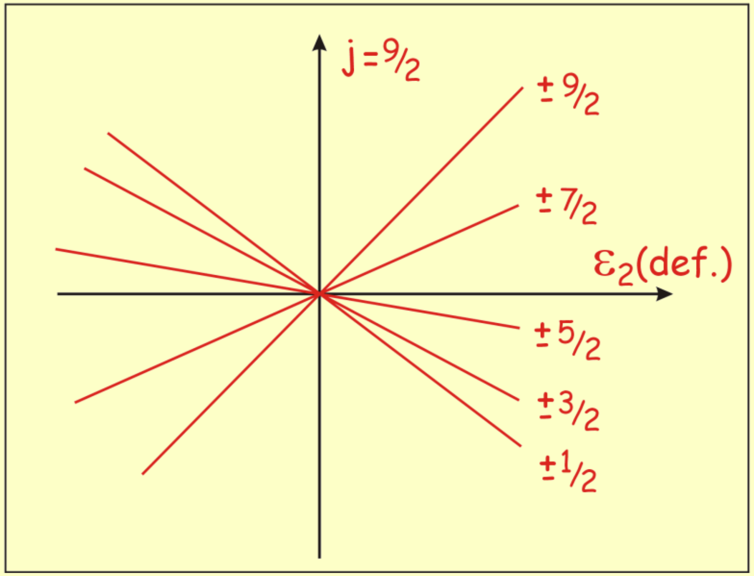


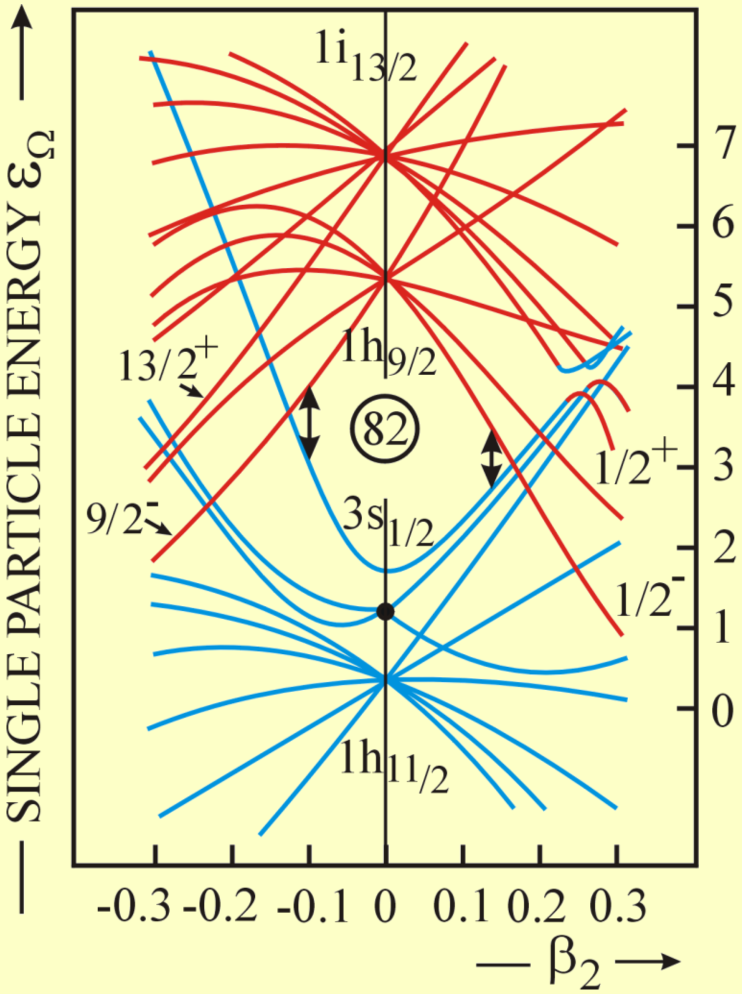
$$\left(\omega_x=\omega_y=\omega_o\,(1+2/3\,\varepsilon_2)^{1/2};\ \omega_z=\omega_o(1-4/3\,\varepsilon_2)^{1/2}\right)$$

ENERGY CORRECTION TO SPHERICAL S.P. ENERGY

$$\langle n\,(\ell\tfrac{1}{2})\,jm|H_{def}|n(\ell\tfrac{1}{2})jm\rangle$$

$$= ct.\ \varepsilon_2\frac{3m^2-j(j+1)}{j(j+1)}$$





APPROACH : NATURAL
DESCRIPTION VIA
DEFORMED MEAN - FIELD

(Nilsson, Deformed WS,HF(B)..)

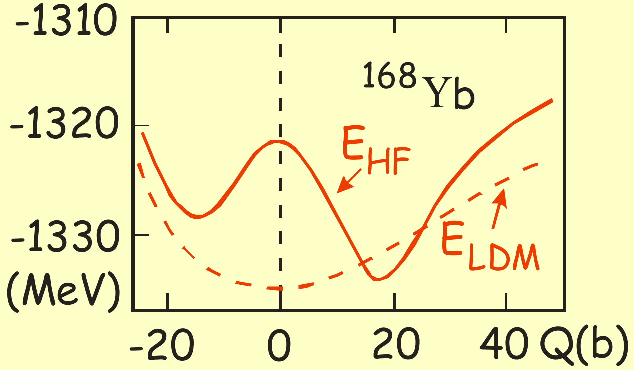
ONE CAN NOW EVALUATE THE TOTAL ENERGY (HARTREE-FOCK) AS A FUNCTION OF THE DEFORMATION VARIABLE ε_2

$$\hookrightarrow E(\varepsilon_2) = \langle \hat{H} \rangle = \sum_{h\{occ.\}} \varepsilon_h(\varepsilon_2) - \frac{1}{2} \sum_{h,h'\{occ.\}} \langle hh'|V|hh' \rangle$$

FOR HARMONIC OSCILLATOR POTENTIAL

$\langle t_i \rangle = \langle U_i \rangle = 1/2\varepsilon_i$

$$\hookrightarrow E(\varepsilon_2) = 3/4 \sum_{h\{occ.\}} \varepsilon_h(\varepsilon_2)$$



Hartree-Fock +BCS method (or HFB)

Step 1: A nucleon-nucleon effective force needs to be chosen, describing the total BE for the whole mass region (spherical and deformed nuclei). Original work of Skyrme (late '50) and Gogny (early '70)

Step 2: Single-particle wave functions and occupation numbers are derived in a self-consistent way through a variational method applied to the energy with constraints on the nuclear shape (multipole moments) and pairing properties

Step 3: The many-body wave function $\Phi(q)$ is built from the independent (quasi) –particle states: Slater determinant

Step 4: Restoration of broken symmetries: states with fixed number of protons (Z) and neutrons (N), isospin (T), spin (J)

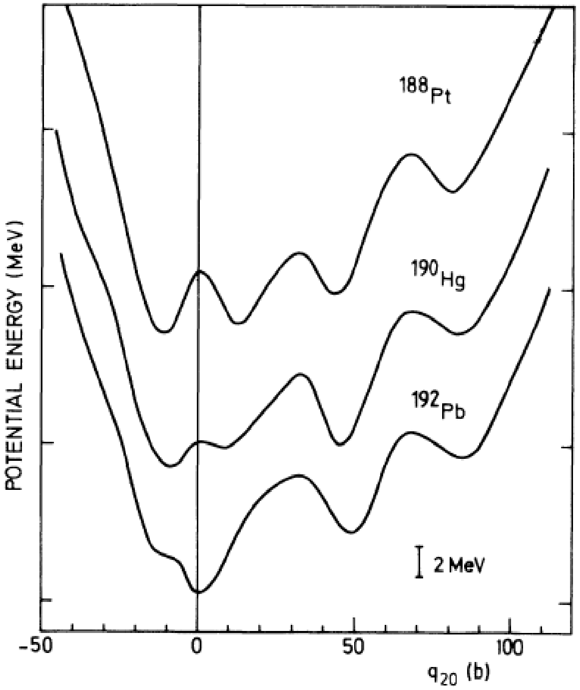
M. Bender, P.-H. Heenen and P.-G. Reinhard, Rev. Mod. Phys. 75, 121 (2003)

Comparing advances in constrained HF(B) calculations: 2 decades

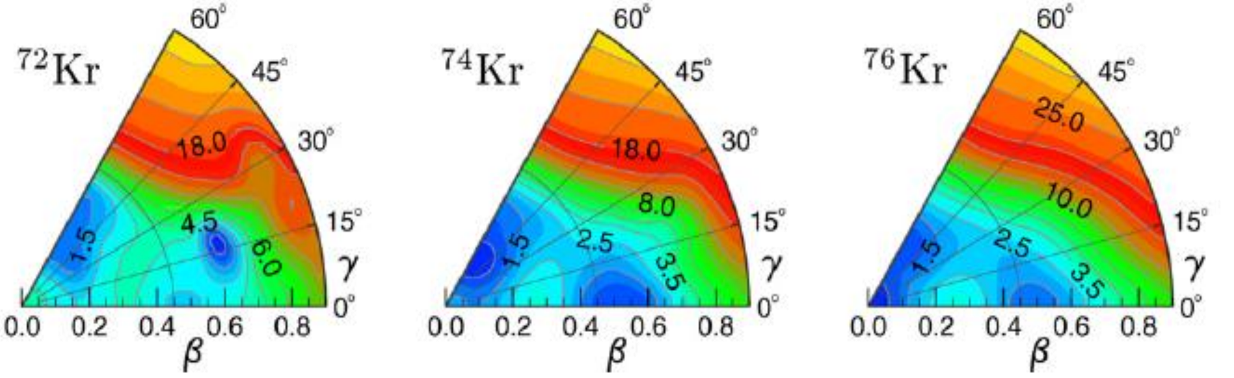
$$\langle \Phi^{HF} | \mathbf{H} - \lambda_Z \mathbf{Z} - \lambda_N \mathbf{N} - \mu_0 \mathbf{Q}_{20} - \mu_2 \mathbf{Q}_{22} | \Phi^{HF} \rangle$$

Variational method for the energy adding constraints on the multipole moments determines the Hartree-Fock single-particle wave functions.

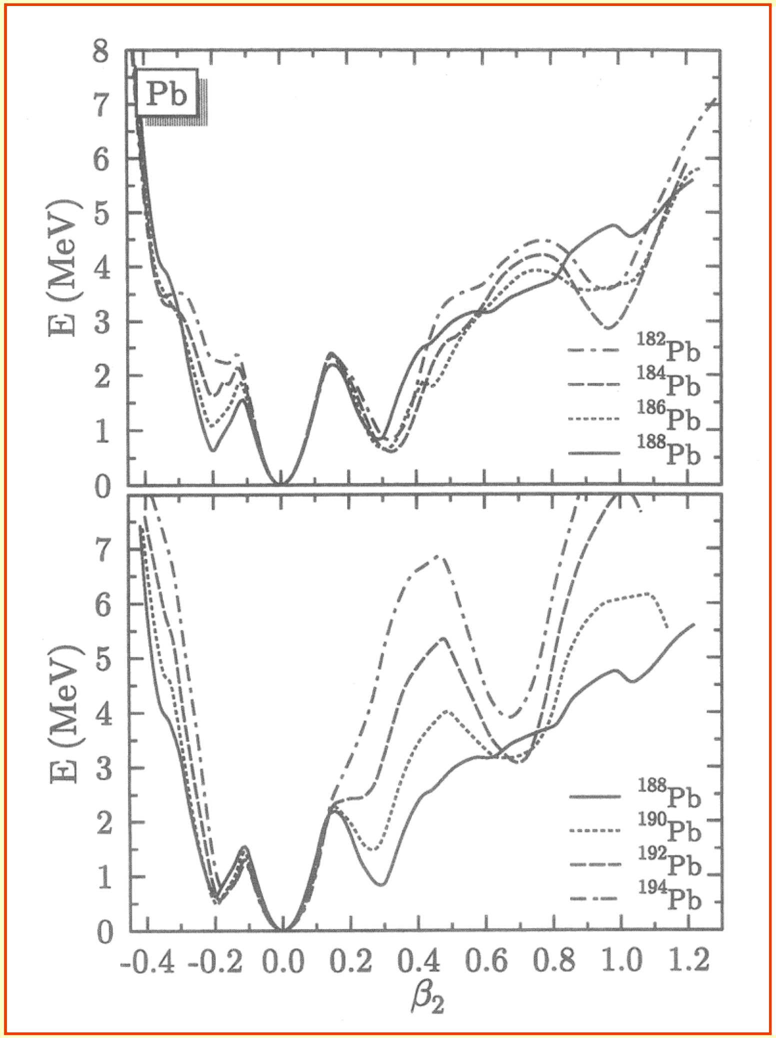
$$V(q) = \langle \Phi | \mathbf{H} | \Phi \rangle - E_{ZP}(q)$$



Girod et al., PRL(1989)

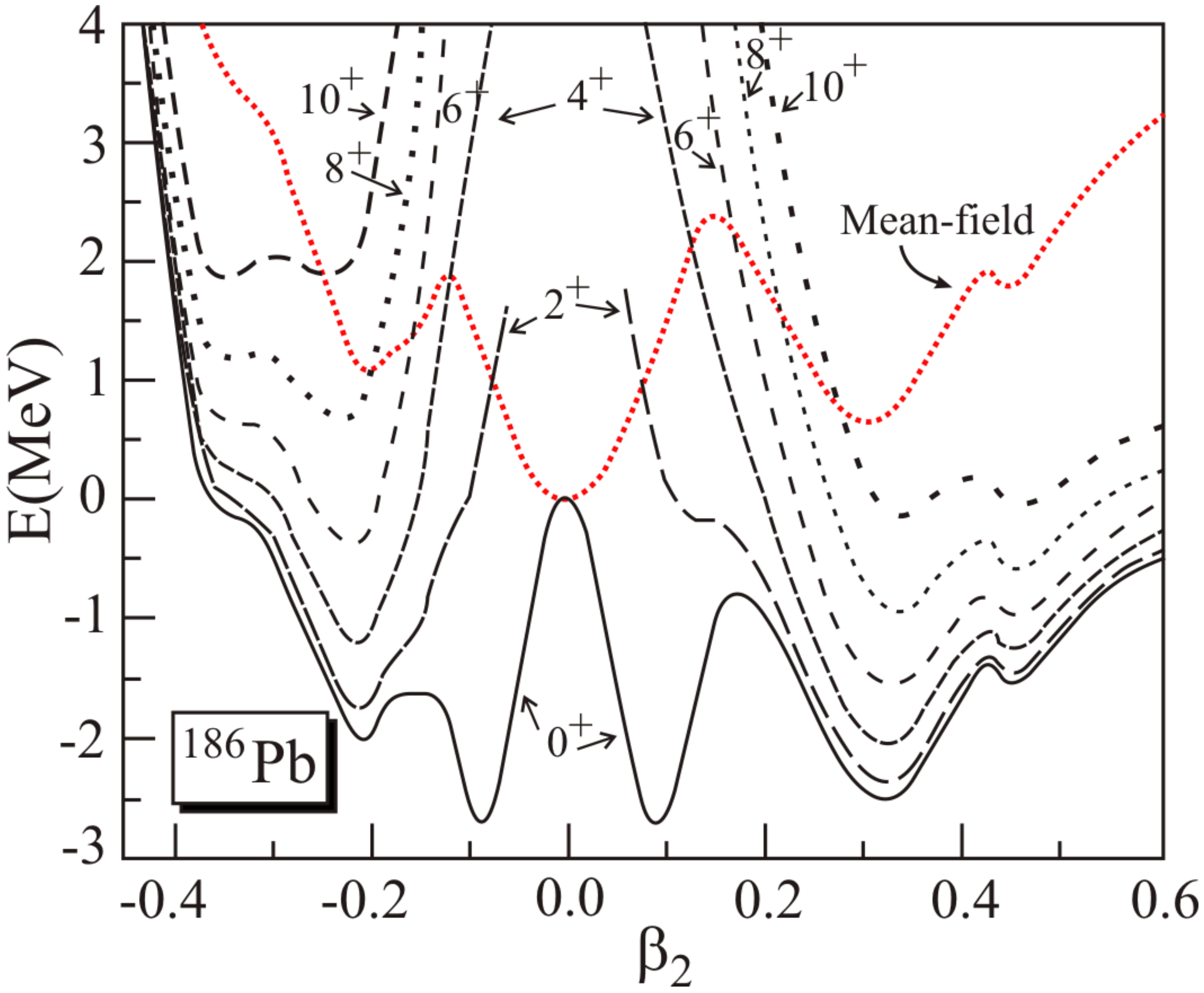


Girod et al., PLB(2009)

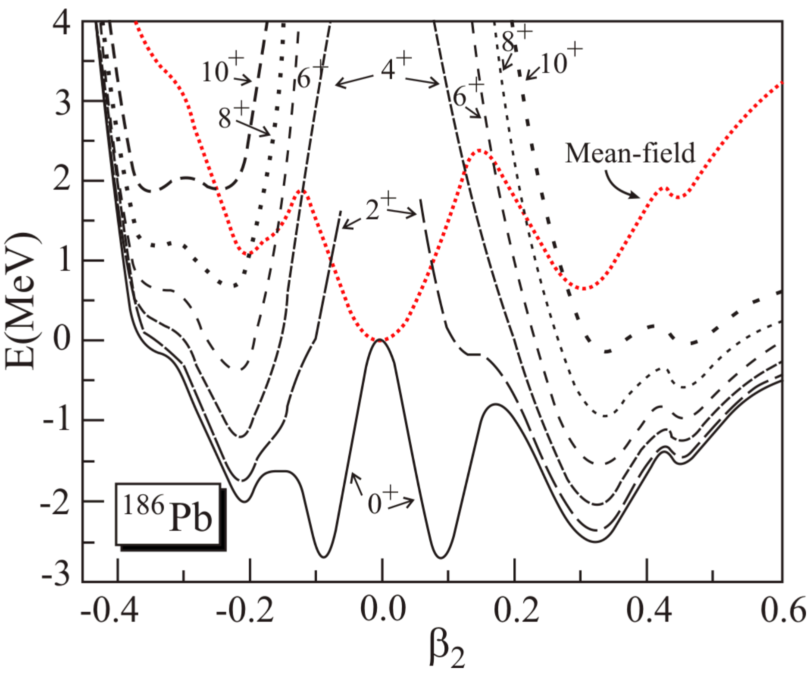


Particle number projected deformation energy curves for $^{182\text{-}194}$Pb

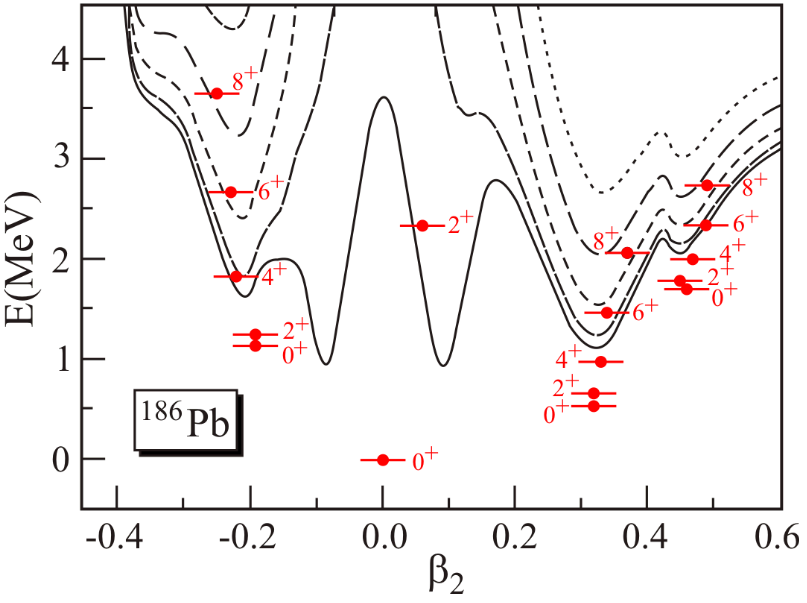
M. Bender et al., PRC 69 (2004) 064303



Particle- and angular momentum (J) projected energy curves (J=0,2,4,6,8,10) as a function of quadrupole deformation

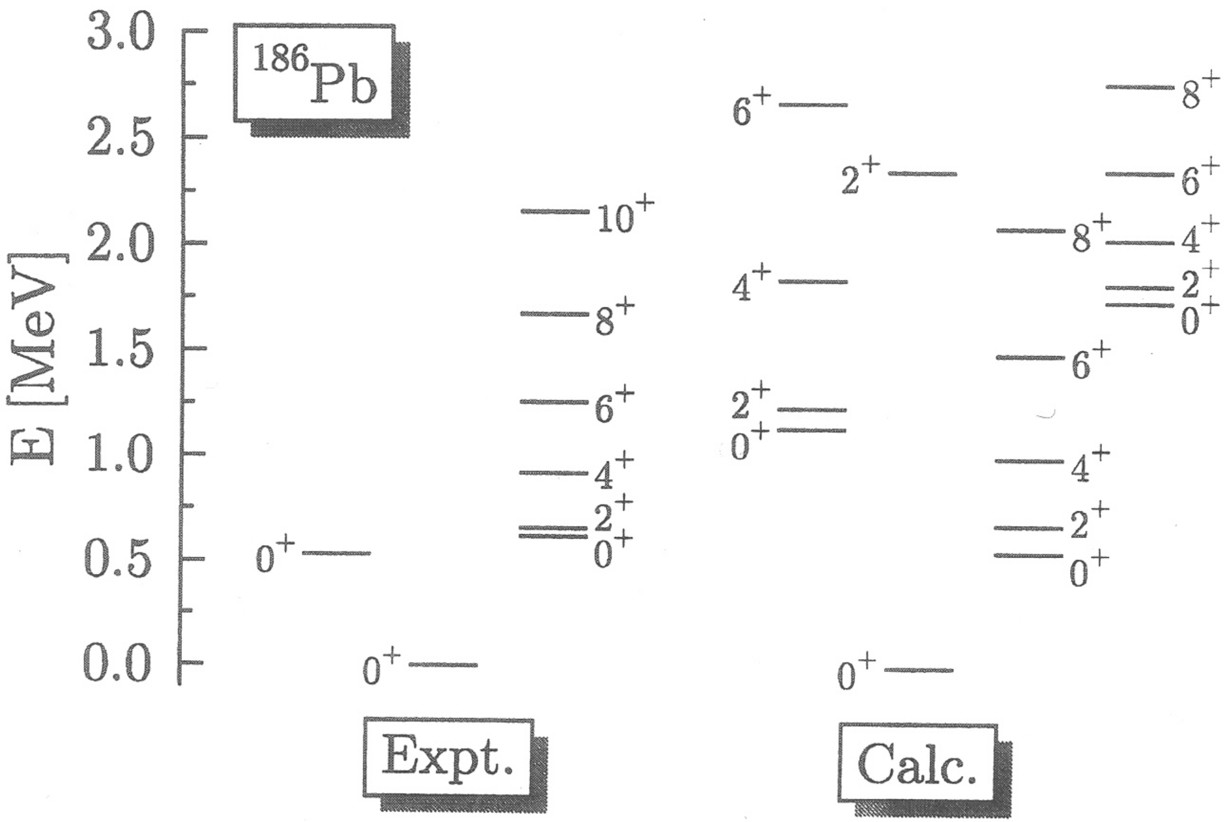


Energy curves for angular momentum states (JM) bands as a function of deformation $|JM;q\rangle,$

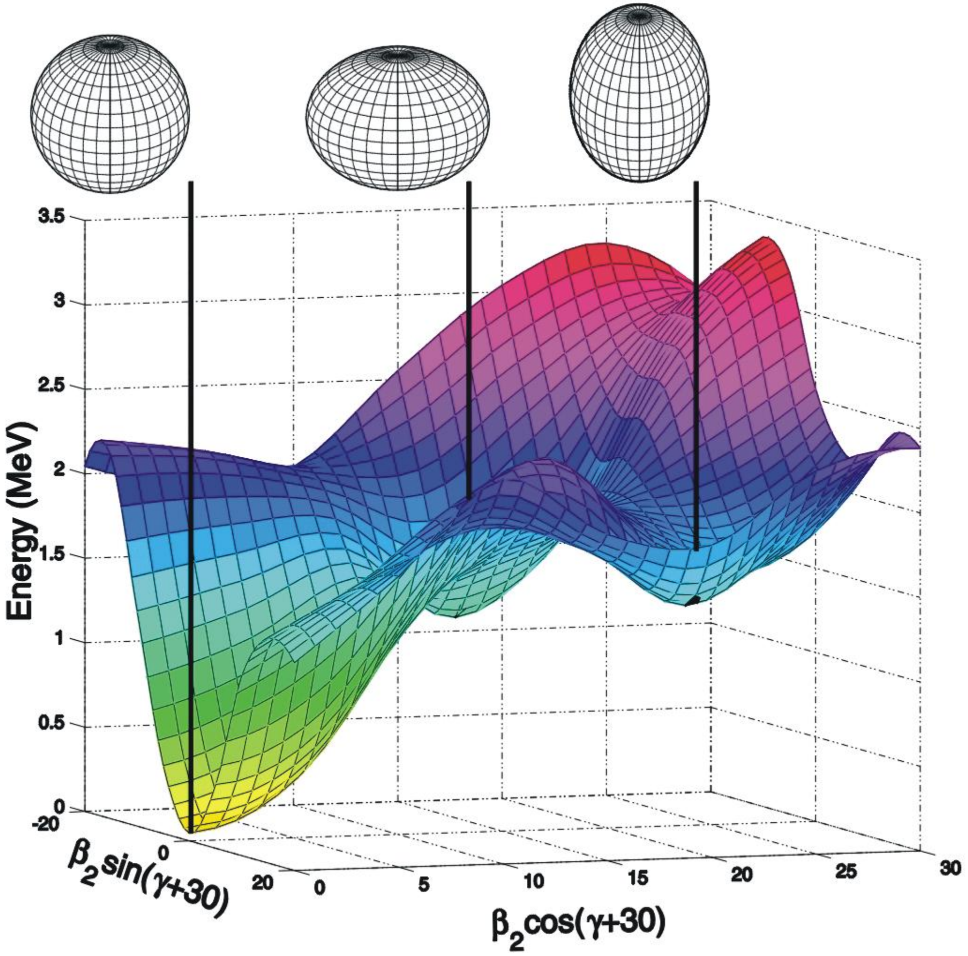


Energy spectrum for the lowest bands (given as the red lines). The energy spectrum is given relative to the energy of the ground state.

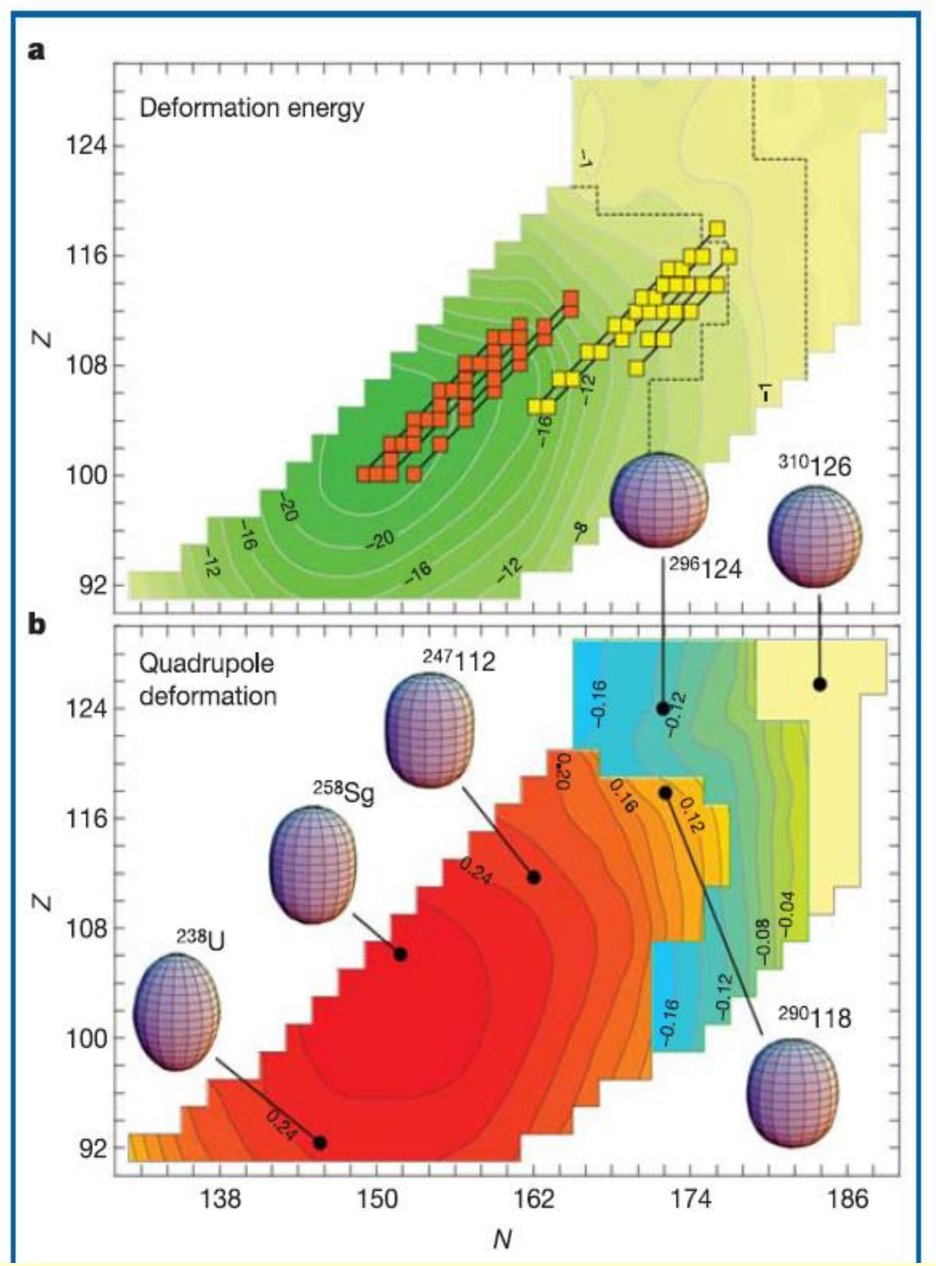
$$|JM;k\rangle = \sum_q f_{J,k}(q)\,|JM;q\rangle,$$



M. Bender et al., PRC 69 (2004) 064303



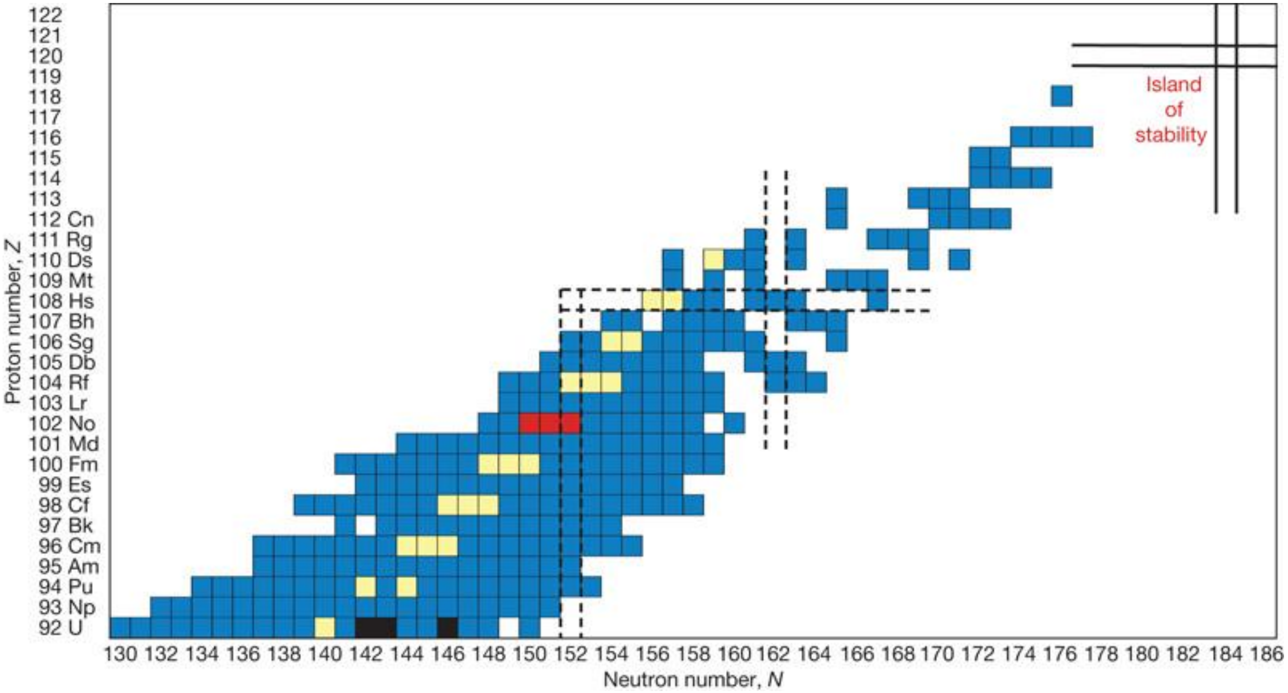
Potential Energy Surface for ^{186}Pb



Deformation properties of the even-even superheavy Nuclei calculated using Self-consistent mean-field Methods, using the Skyrme Force SLy4

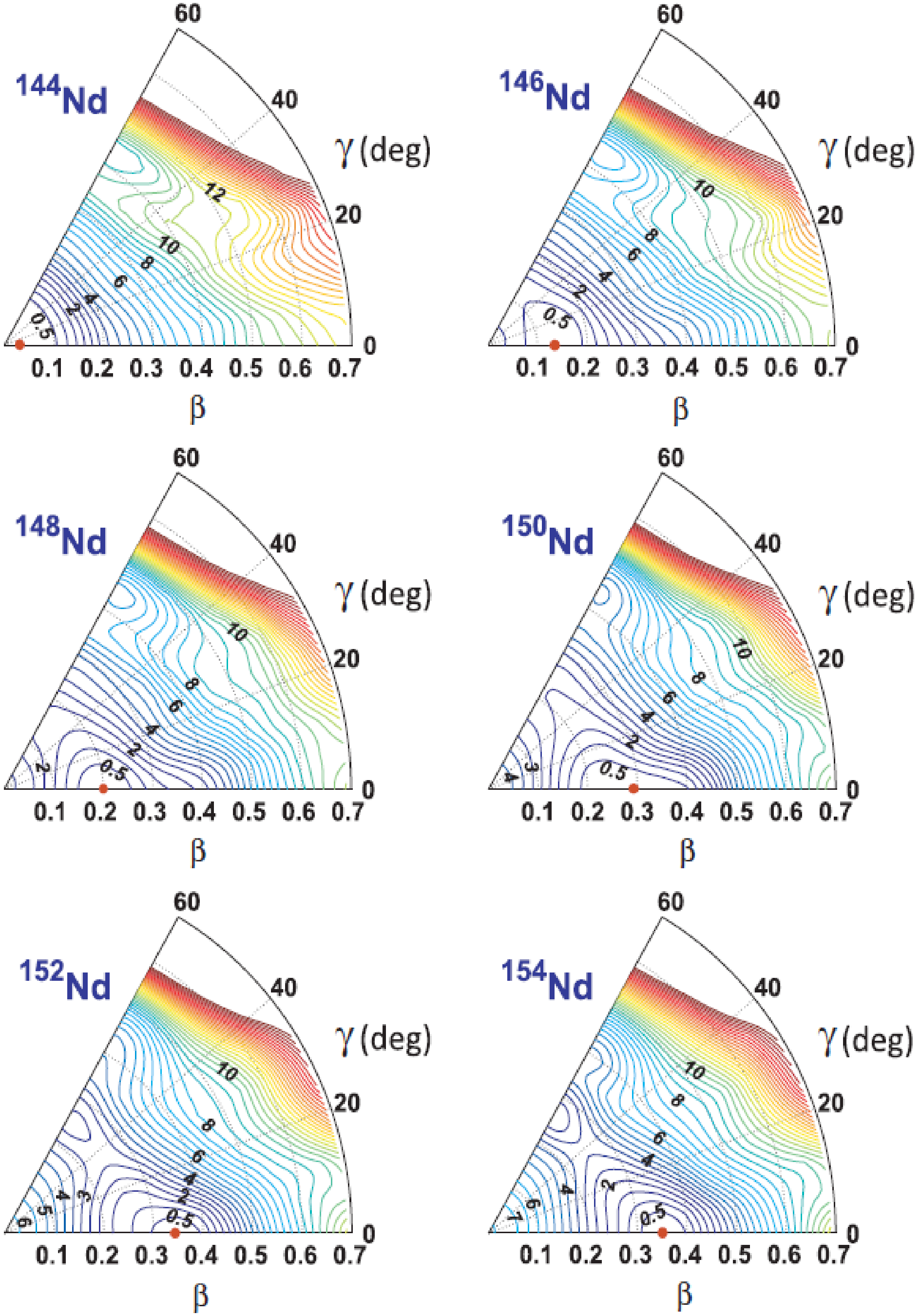
S. Cwiok, P.-H. Heenen
And W. Nazarewicz,
Nature, vol.433 (2005),705

Part of the chart of nuclides between uranium and element 118.



M Block *et al.* *Nature* **463**, 785-788 (2010) doi:10.1038/nature08774

CLOSING THE CIRCLE…



Constructing a Bohr Hamiltonian, this time with a potential energy surface and inertial functions constructed from a deformed mean field (microscopic) approach.

Results from solving a collective Bohr Hamiltonian over the full (ß,γ) plane starting from a constrained self-consistent RMF calculation

Li, Nikšic et al., PRC79(2009)

Shape changes in even-even Nd isotones (Z=60) in passing N=90.

Study books

1. Shell-Model Applications in Nuclear Spectroscopy, North-Holland Publ. Co, 1977- P.J. Brussaard and P. W. N. Glaudemans
2. Simple Model of Complex Nuclei; Harwood Acad. Publ., 1993- I. Talmi
3. The nuclear shell-model, study Ed., Springer Verlag 1994 - K. Heyde

. . . . .

4. Shapes and Shells in Nuclear Structure; Cambridge Univ. Press, 1995 - S.G. Nilsson and I. Ragnarsson
5. Nuclear Structure, vol I and II, 1998, World Scientific , A. Borh and B. Mottelson

. . . . .

6. The nuclear Many-Body Problem, Springer-Verlag, 1980-P. Ring and P. Schuck

. . . . .

7. Fundamentals of Nuclear Models, World Scientific, 2007-D. J. Rowe and J. L. Wood (forthcoming)