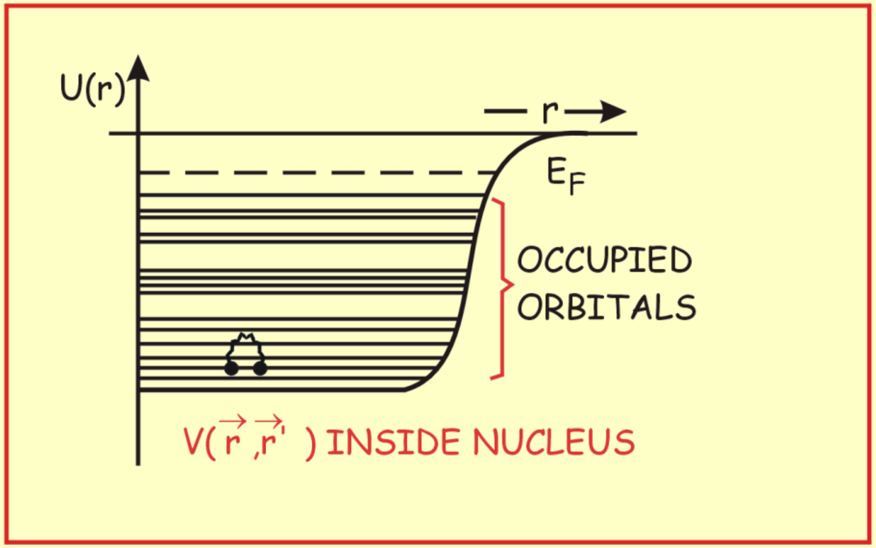
The atomic nucleus: a bound system of interacting nucleons

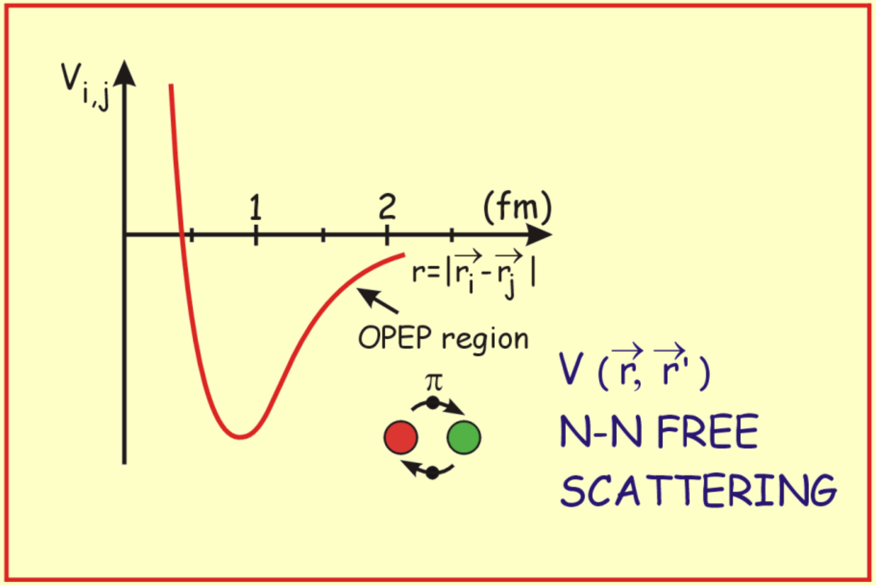
1. Nuclear forces and very light nuclei

2. The nuclear shell model: from independent particle motion towards modern applications

3. Nuclear deformation and collective motion: phenomenological models and self-consistent mean-field theory

Problems using realistic NN forces in nuclei





Many-body Hamiltonian-Nuclear mean-field

- Start from many-body Hamiltonian

$$H=\sum_{i=1}^{A} \frac{\vec{p}_i^{\,2}}{2m_i} + \frac{1}{2}\sum_{i,j=1}^{A} V_{ij}\,(|\vec{r}_i-\vec{r}_j|)$$

- Introduction of mean-field $U(r_i)$

$$H=\underbrace{\sum_{i=1}^{A}\left(\frac{\vec{p}_i^{\,2}}{2m_i} + U(r_i)\right)}_{h_o(i)} + \underbrace{\sum_{i<j=1}^{A} V_{ij}\,(|\vec{r}_i-\vec{r}_j|) - \sum_{i=1}^{A} U(ri)}_{\text{Residual interaction}}$$

$$h_o\varphi_\alpha(\vec{r}_i)=\varepsilon_a\varphi_\alpha(\vec{r}_i) \qquad \Rightarrow \qquad \{\varphi_\alpha(\vec{r}_i),\ \varepsilon_a\}$$

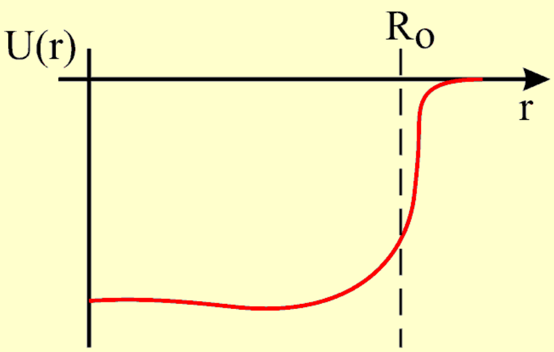
Spherical field

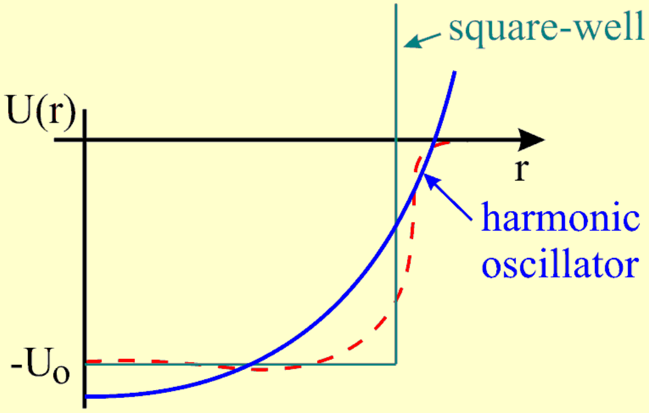
$\alpha \equiv \{n_a, l_a, j_a, m_a\}$

Single-particle wave function,

Single-particle energy

Central problem: choice of U(r)





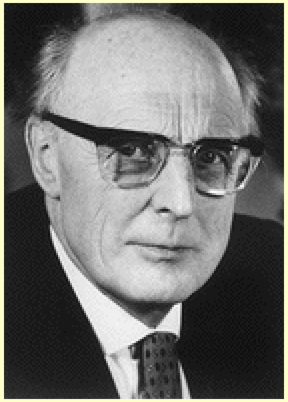
$U(r) = -U_0 \; (r \lessgtr R_0)$
$U(r) = \frac{1}{2} m\omega^2 r^2$
} analytical solutions

Solve central problem Schrödinger eq. $[\frac{\vec{p}^2}{2m} + U(r)]\,\varphi(\vec{r},\vec{s}) = \varepsilon\varphi(\vec{r},\vec{s})$

$\varphi(\vec{r},\vec{s}) = R_{n\ell}(r) Y_\ell^m(\theta,\varphi)\chi_{\frac{1}{2}}^{m_s}(\sigma)$

Harmonic oscillator: $(N+{}^3/_2)\,\hbar w$ with $N=2(n-1)+\ell$

n: radial q.n.; ℓ : orbital angular momentum q.n.



Idea of shell-model:

- M. Goeppert-Mayer (1949)
- H. Jensen, O. Haxel and H. E. Suess (1949)

$$U(r)=\frac{1}{2}\, m\omega^2 r^2+\alpha\, \vec{\ell}.\vec{\ell}+\beta\, \vec{\ell}.\vec{s}$$

On Closed Shells in Nuclei. II

MARIA GOEPPERT MAYER
Argonne National Laboratory and Department of Physics, University of Chicago, Chicago, Illinois
February 4, 1949

THE spins and magnetic moments of the even-odd nuclei have been used by Feenberg[1,2] and Nordheim[3] to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum. If strong spin-orbit coupling, increasing with angular momentum, is assumed, a level assignment different from either Feenberg or Nordheim is obtained. This assignment encounters a very few contradictions with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers 50, 82, and 126 occur at the place of the spin-orbit splitting of levels of high angular momentum.

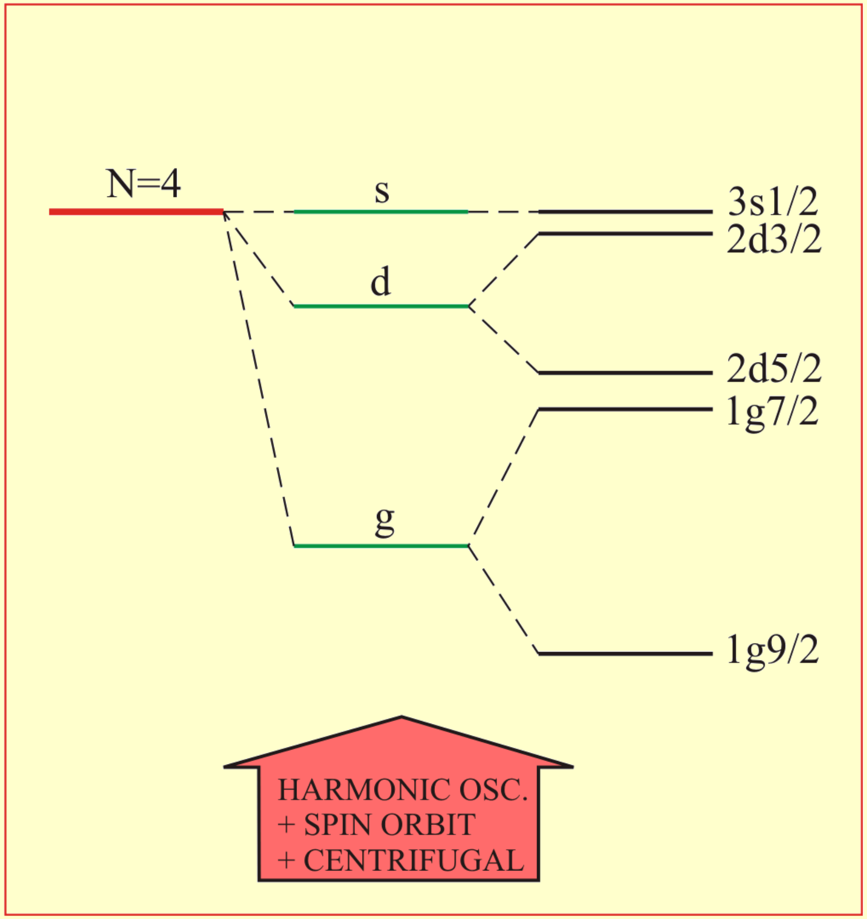
Phys.Rev.75,1969 (1949)

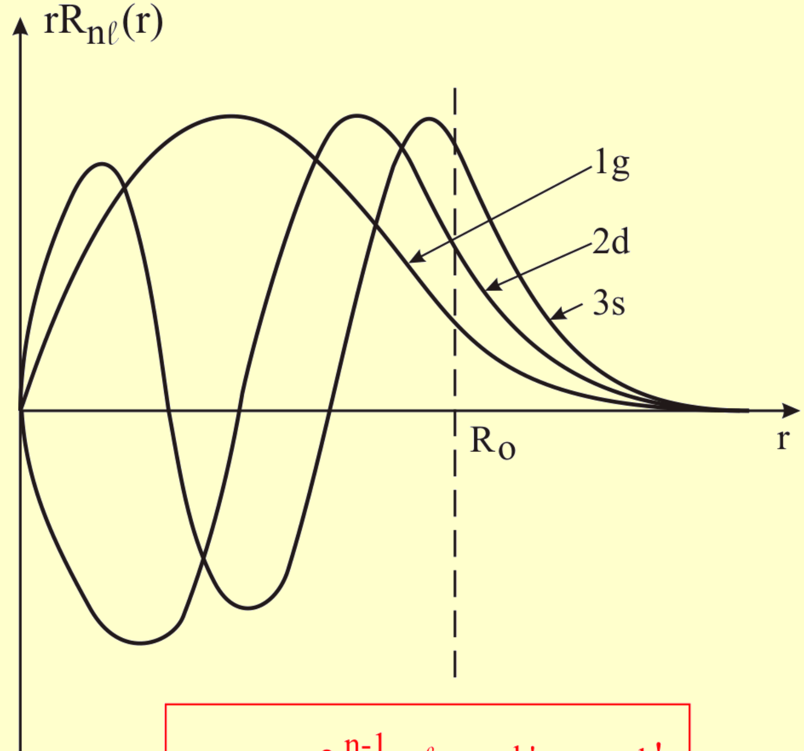
On the "Magic Numbers" in Nuclear Structure

OTTO HAXEL
Max Planck Institut, Göttingen
J. HANS D. JENSEN
Institut f. theor. Physik, Heidelberg
AND
HANS E. SUESS
Inst. f. phys. Chemie, Hamburg
April 18, 1949

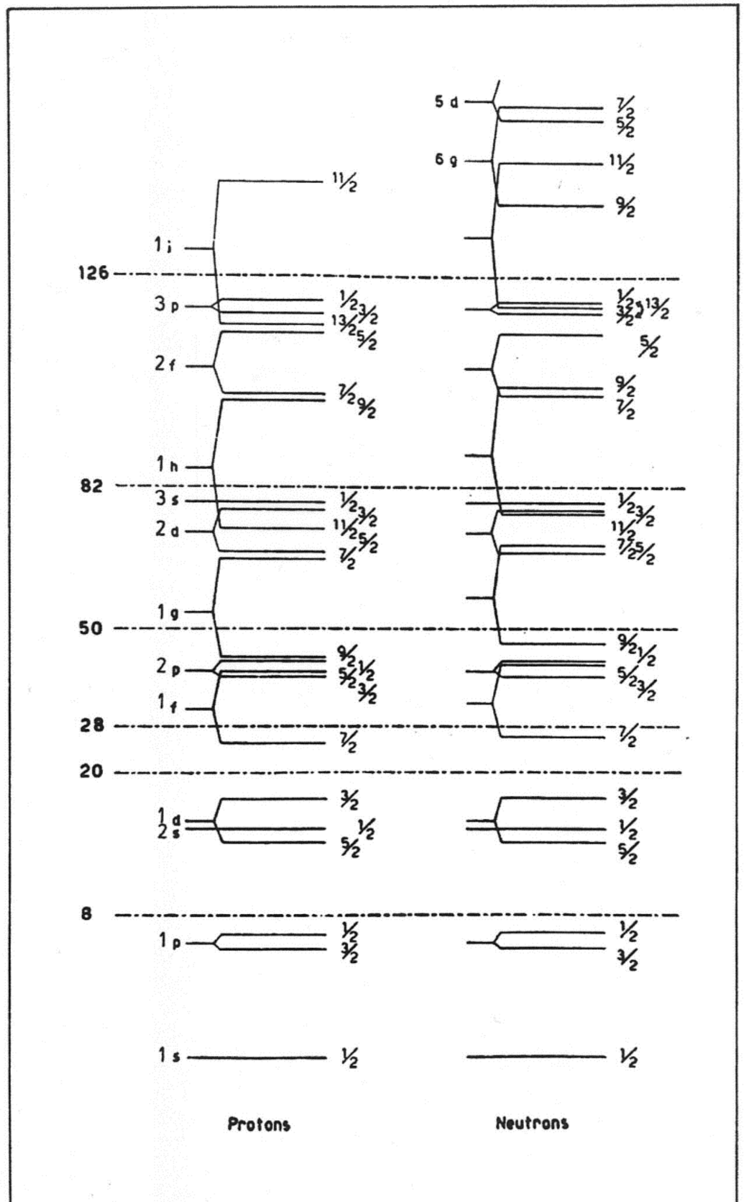
A SIMPLE explanation of the "magic numbers" 14, 28, 50, 82, 126 follows at once from the oscillator model of the nucleus,[1] if one assumes that the spin-orbit coupling in the Yukawa field theory of nuclear forces leads to a strong splitting of a term with angular momentum l into two distinct terms $j=l\pm\frac{1}{2}$.

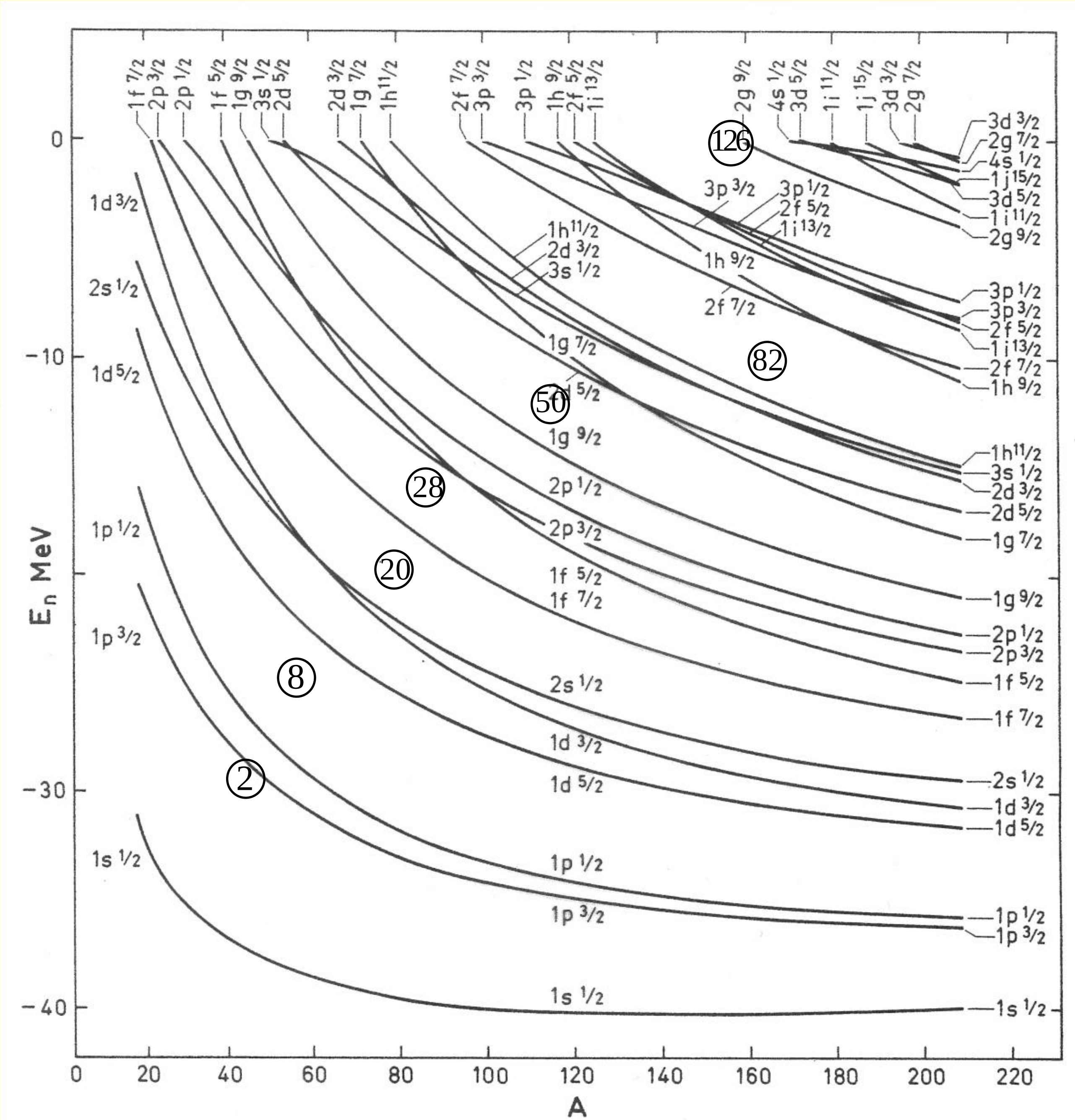
Phys.Rev.75,1766(1949)



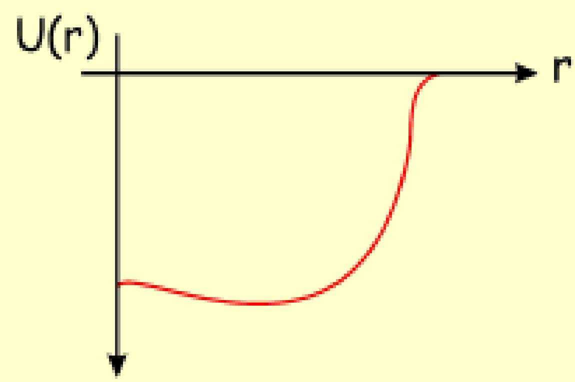


$$r^{\ell+1}.e^{-\nu r^2}\sum_{k'=0}^{n-1} a_{k'}^{\ell}\,(-1)^{k'}(2\nu r^2)^{k'}$$


5 d
6 g
1 j
126
3 p
2 f
1 h
82
3 s
2 d
1 g
50
2 p
1 f
28
20
1 d
2 s
8
1 p
1 s
Protons
Neutrons



Woods-Saxon potential.



Construction of basis wave functions

- $Y_l^{m_l}(\theta,\varphi)$ and $\chi^{1/2}_{m_s}$

- $\varphi_{n,l,j,m}(\vec{r},\sigma) = R_{nl}(r)[Y_l \otimes \chi^{1/2}]^j_m$

ang. momentum coupling

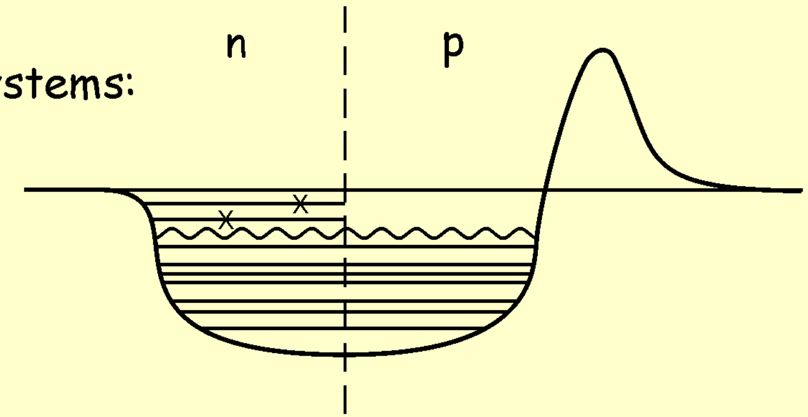
$: \sum_{m_l,m_s} \langle l\, m_l, 1/2\, m_s | jm \rangle Y_l^{m_l} \chi^{1/2}_{m_s}$

- Two-particle wave functions $(j_a,j_b)J$

$\psi(j_a j_b, JM) \Rightarrow$ J all values $|j_a - j_b| \leq J \leq j_a + j_b$

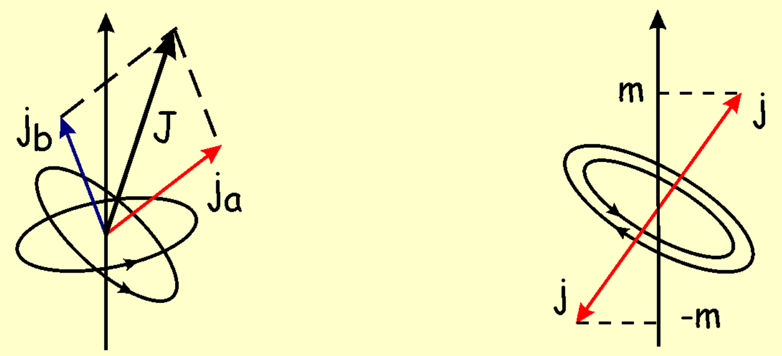
$\psi(j^2, JM) \Rightarrow$ J: 0,2,4,6,.... (even only)

Two nucleon systems:



$H=h_o(1) + h_o(2) + V(1,2)$

$E(j_a j_b,JM) = \varepsilon_{ja} + \varepsilon_{jb} + \langle j_a j_b,JM | V(1,2) | j_a j_b,JM \rangle$



$|j_a - j_b| \leq J \leq j_a + j_b$

$J=0, 2, 4,\ldots 2j-1$

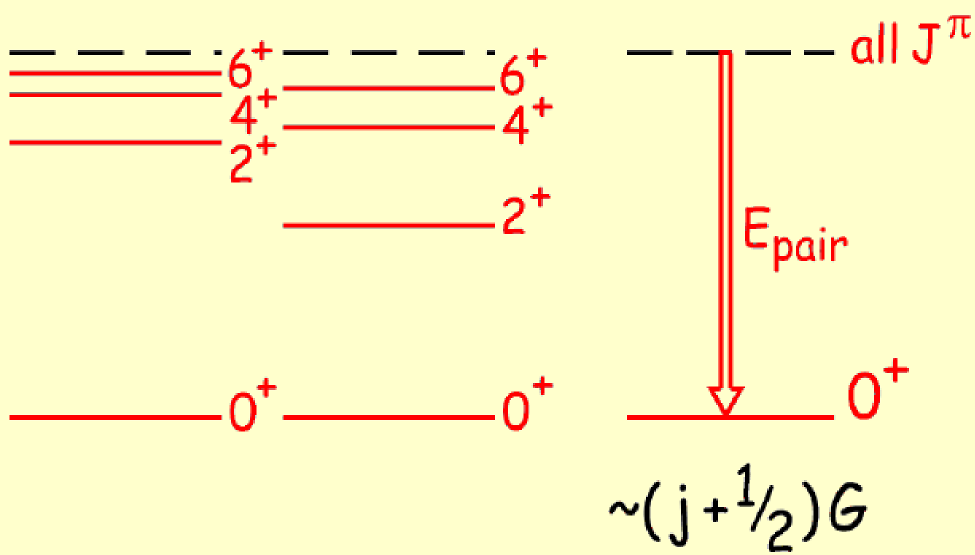
$\Delta E_{(j_a j_b)J}$

$\Delta E_{(j)^2 J}$

Radial overlap of s.p. orbitals

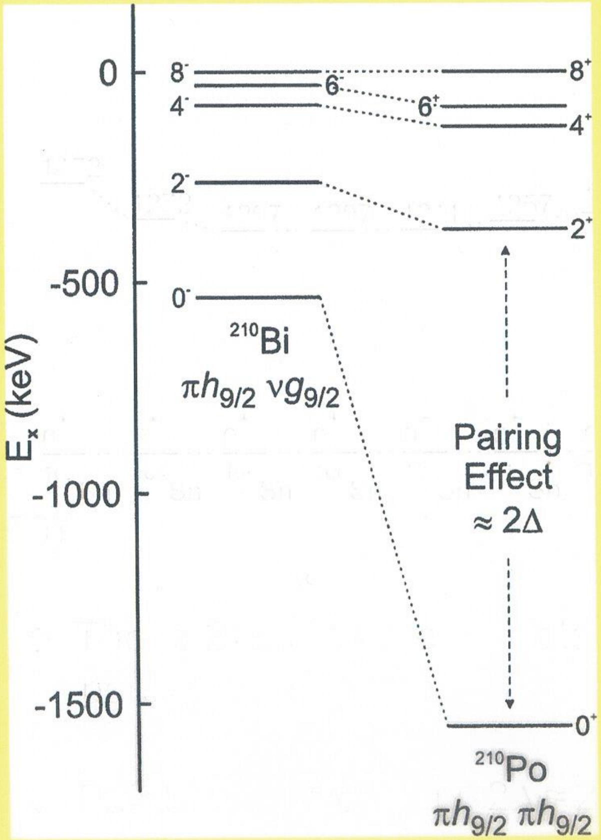
The two-body interaction (finite range, $\delta(\vec{r}_1 - \vec{r}_2)$,....) all show a "pairing" property

$$\langle j^2, JM | V_\delta | j^2, JM \rangle = A(j) \begin{pmatrix} j & j & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2$$



(ii) Energy spectra for nuclei near closed shells (A ± 2, A ± 4) show a clear gap for the 0^+ g.s.

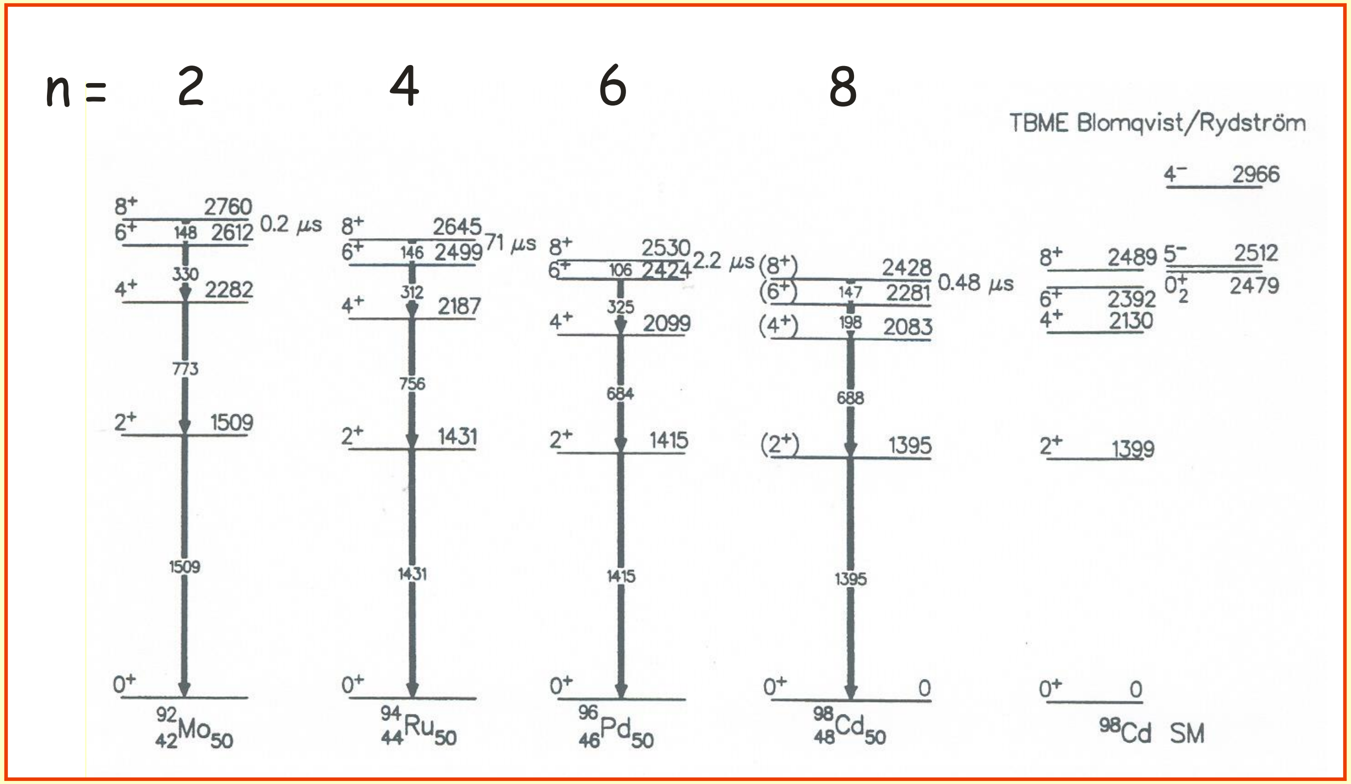
Example: $^{210}_{84}Po_{126}$ and $^{210}_{83}Bi_{127}$ adjacent to $^{208}_{82}Pb_{126}$



- In ^{210}Po the configuration outside the doubly-closed shell core of ^{208}Pb is $(1h_{9/2})^2$. If there were no interaction between the two π's constituting the pair, i.e. if they behaved like independent particles, the various $(1h_{9/2})^2$ spin couplings, which reflect the orbital alignments, would lead to states degenerate in energy.

  → correlated pair of two π's

- Pairing effect ≈ 2Δ



N=50 nuclei

Filling the $1g_{9/2}$ orbital

$(1g_{9/2})^{-2}$

CONFIGURATION

$(2p_{1/2})^{-2}$

$(2p_{1/2}^{-1}\, 1g_{9/2})$

Adding more particles 3,4,...,n, angular momentum becomes more involved (use of angular momentum coupling methods)

An alternative method: constructing a Slater determinant wave function: state with fixed M (no fixed J).

$$\psi(1,2,..A)=\frac{1}{\sqrt{A!}}\begin{bmatrix} \varphi_{\alpha_1}(\vec{r}_1) & \varphi_{\alpha_1}(\vec{r}_2) & \dots & \varphi_{\alpha_1}(\vec{r}_A) \\ \varphi_{\alpha_2}(\vec{r}_1) & \varphi_{\alpha_2}(\vec{r}_2) & \dots & \varphi_{\alpha_2}(\vec{r}_A) \\ \vdots & & & \\ \varphi_{\alpha_A}(\vec{r}_1) & \varphi_{\alpha_A}(\vec{r}_2) & \dots & \varphi_{\alpha_A}(\vec{r}_A) \end{bmatrix}$$

↓

A very convient way for computing

ex, 4 particles in sd-shell

| 1d5/2,-1/2; 1d5/2,-3/2; 1d3/2, +3/2; 2s1/2,+1/2>(M=0)

A very usefull approach is a bit-representation, known as the M-scheme.

0	1	1	0	0	0	0	0	0	1	0	1
-5	-3	-1	1	3	5	-3	-1	1	3	-1	1
$1d_{5/2}$						$1d_{3/2}$				$2s_{1/2}$	

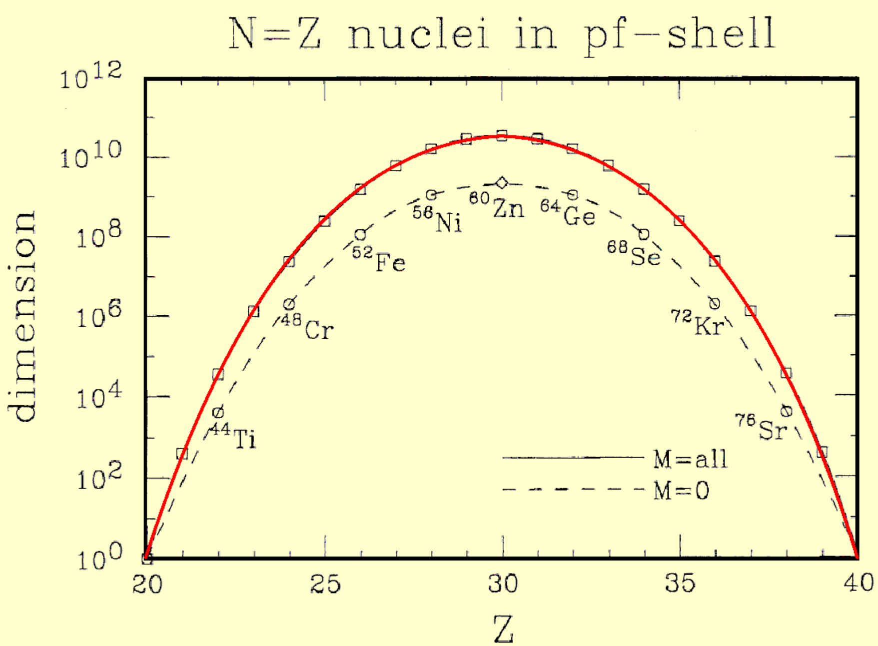
$=2^1 + 2^3 + 2^{10} + 2^{11} = 3082$

- Counting # basis states (approx.)

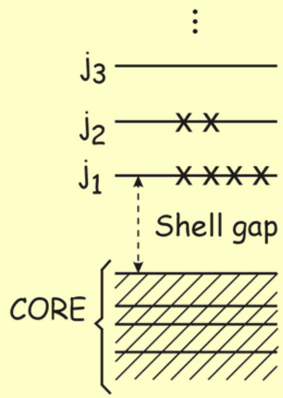
n particles, N orbitals $\simeq \binom{N_p}{n_p}\binom{N_n}{n_n}$

ex. $^{60}Zn(fp)$:10p, 10n;20 orbitals

$\binom{20}{10}\binom{20}{10} \simeq 3.4.10^{10}$



NUCLEAR RESIDUAL INTERACTIONS...



SEPARATION OF A NUCLEONS

→ 'INERT' CORE AND A NUMBER OF VALENCE NUCLEONS

→ CONSTRUCT A 'BASIS' IN THE VALENCE SPACE $\varphi_{J,i}$

$$\varphi_{J,i} = \{(j_1)^{n1}_{J_1}\ (j_2)^{n2}_{J_2} \dots\ \}_J$$

AND SOLVE EIGENVALUE EQUATION

$$\hat{H}\psi_{J,k} = E_{J,k}\,\psi_{J,k}$$

$$\psi_{J,k} = \sum_i a_J^{k,i}\,\varphi_{J,i}$$

PARAMETERS:

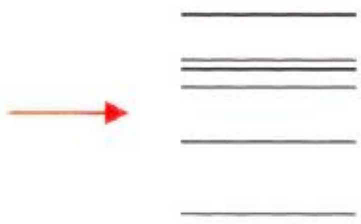
- SINGLE-PARTICLE ENERGIES ε_j
- TWO-BODY n-n INTERACTION $\langle j_1 j_2, J | V_{1,2} | j_3 j_4, J \rangle$

→ MANY STRUCTURES ARISE

Calculate Hamiltonian matrix $H_{ij}=\langle\phi_j|H|\phi_i\rangle$

— Diagonalize to obtain eigenvalues

$$\begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1N} \\ H_{21} & H_{22} & & \\ \vdots & & \ddots & \\ H_{N1} & & \cdots & H_{NN} \end{pmatrix}$$



Use of various algorithms: Jacobi method (small), Householder method (few 100-1000), Lanczos method for “very big” matrices (see numerical schemes).

$\star$ VERY - LIGHT NUCLEI ($A \lesssim 12$): AB - INITIO CALCULATIONS

$\star$ SHELL - MODEL

1p shell : Cohen, Kurath (1965): 15 2b m.e.

2s, 1d shell : Brown, Wildenthal (OXBASH) (1988, 2007): 63 2b m.e.

2p, 1f shell : Hjorth-Jensen, Kuo, Osnes (1995): Bonn (C) potential (1996)
Madrid - Strasbourg, (ANTOINE) (Kuo, Brown-KB force (1996))
KB (fp), KB3, KB3G,...
Honma, Otsuka, Brown, Mizusaki: GXPF1,...(2004)
} 195 2b m. e.

ex. ^{56}Ni: full fp shell: 10^9 (all M states) (2007)

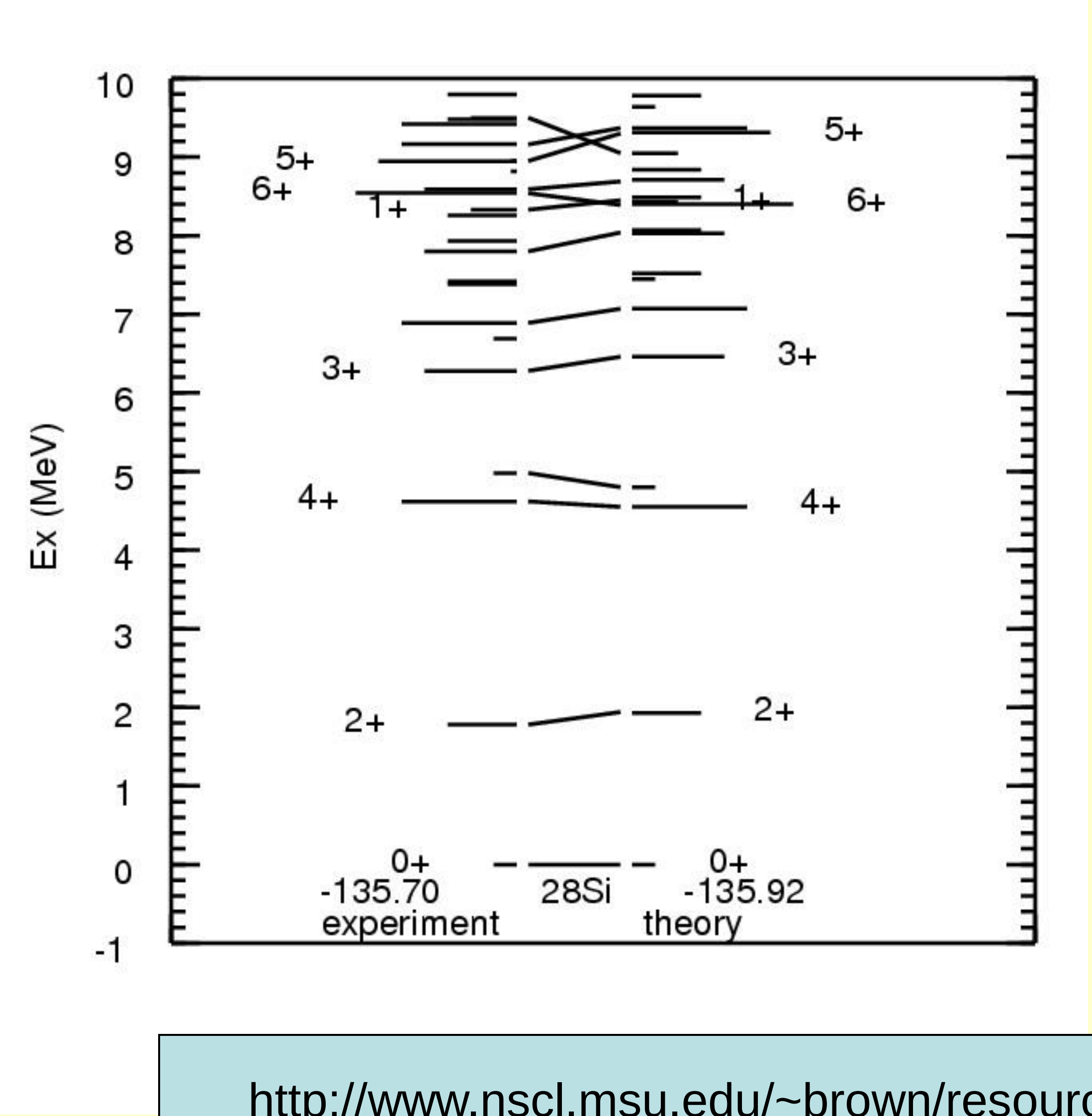
2p, 1f, $1g_{9/2}$: ? without reach 2.4×10^{20}(all M states)

$\star$ RESTRICTIONS

$C^{n_\pi}_{N_\pi} \cdot C^{n_\nu}_{N_\nu}$

n_π (n_ν) : number of active protons (neutrons)

N_π (N_ν) : number of single-particle (j,m) states for protons (neutrons)

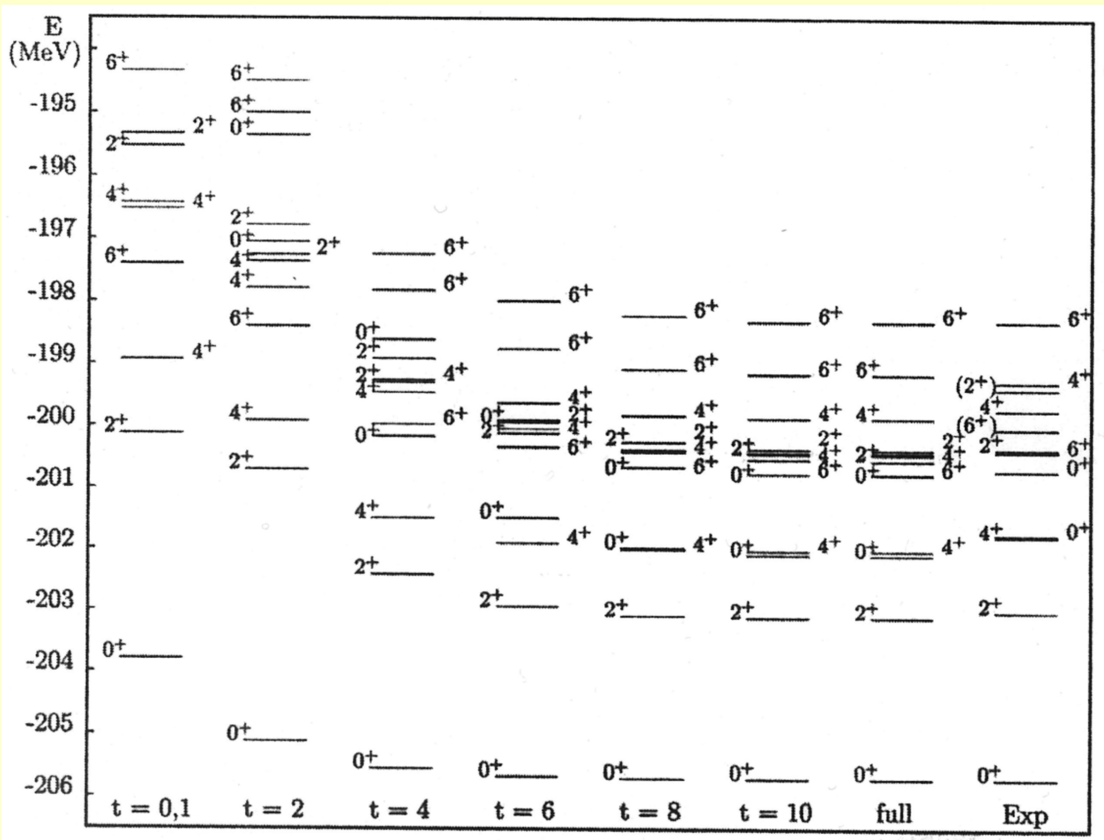


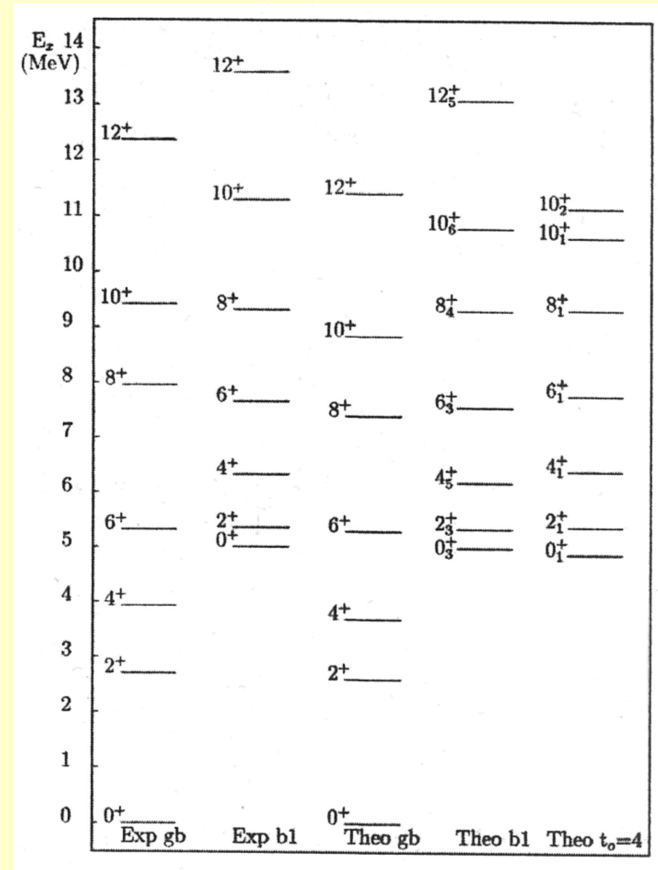
^{28}Si using the USD-A Hamiltonian for the sd shell

B.A.Brown and W.A.Richter, Phys.Rev.C74(2006),034315

http://www.nscl.msu.edu/~brown/resources/usd-05ajpg/si28.jpg

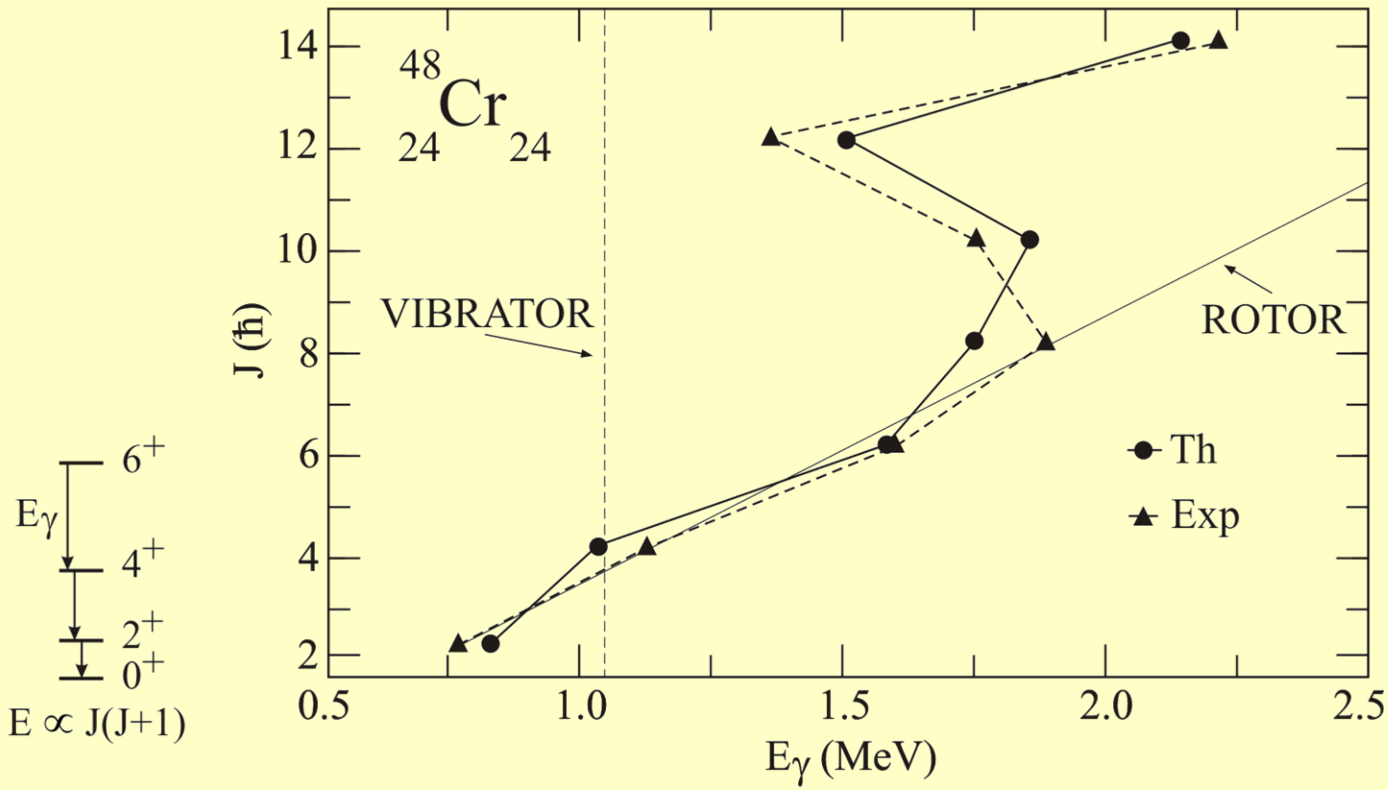
Full fp shell-model study of ^{56}Ni (Horoi et al., PRC 73(2006), 061305)





$(1f_{7/2})^{16-t}(2p_{3/2}\,1f_{5/2}\,2p_{1/2})^t$

STATE-OF-THE ART SHELL-MODEL CALCULATION IN fp SHELL



Strasbourg-Madrid
(CAURIER, NOWACKI, ZUKER, POVES et al.)

Round up of the shell model approach

- Shell model represent a powerful theoretical model to describe low-energy nuclear spectroscopy
- Having got $E_{J,k}$, $\Psi_{J,k}$ one can calculate matrix elements of operators to compare with experiment (spectroscopic factors, static and transition electromagnetic moments - Q, μ, B(E2), …, weak decays - β, $\beta\beta$, lifetimes, etc)
- There is a link to the NN interaction, although more developments in the effective interaction theory is required

The shell model as unified view of nuclear structure
E.Caurier et al, Rev. Mod. Phys. 77 (2005) 427