

Nuclear Structure from Gamma-Ray Spectroscopy

2013 Postgraduate Summer School

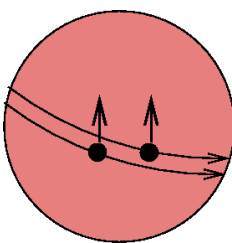
Lecture 2: Low-spin nuclear structure

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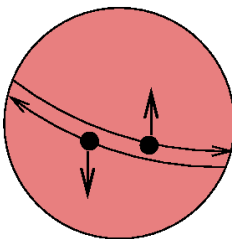
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Time Reversed Orbits



Pauli forbidden



**Time reversed:
two interactions
per revolution**

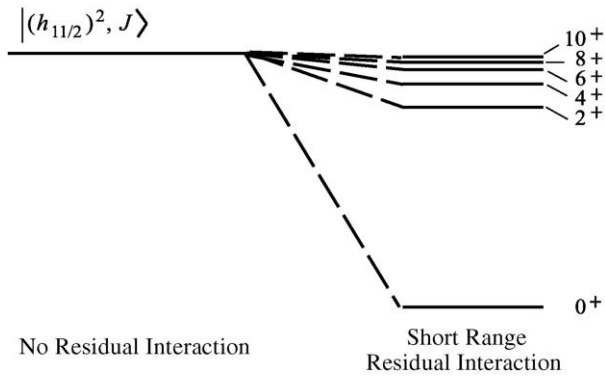
- The greatest overlap would occur if two particles could orbit in the same level
- Not allowed (PEP) !
- The next greatest overlap occurs for particles in 'time reversed' orbits
- The spins cancel to give $I^\pi = 0^+$

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Coupling Two Particles



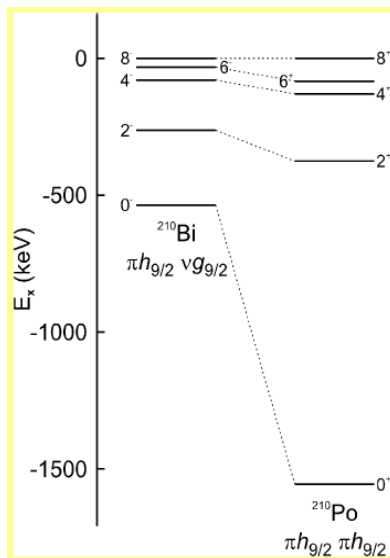
- The short-range (pairing) residual interaction yields an energetically favoured 0^+ state

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Favoured 0^+ Ground State



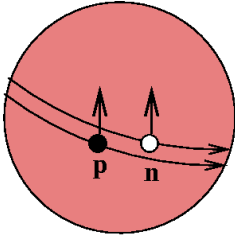
- In ^{210}Po the configuration outside the doubly closed shell core of ^{208}Pb is $(\pi h_{9/2})^2$.
- If there were no interaction between these two protons, i.e. if they behaved like independent particles, the various $(h_{9/2})^2$ spin couplings, which reflect the orbital alignments, would lead to states degenerate in energy.
- Correlated pair of two protons
- Energy gain $\approx 2\Delta$

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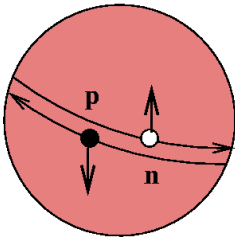
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Neutron-Proton Pairing



Spin $I = 1$
Isospin $T = 0$



Spin $I = 0$
Isospin $T = 0$

- The concept of superconductivity, related to like nucleon pairs coupled to spin $I = 0$ and isospin $T = 1$, can be extended to neutron-proton pairs with $T = 0$
- The greatest overlap occurs if the particles are in the same orbitals
- Strong neutron-proton pairing can occur for nuclei with $N = Z$

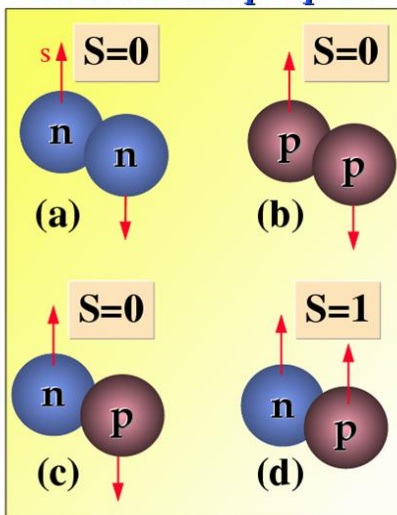
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Nucleon Pairing

nucleonic Cooper pairs



- The **isovector** ($T=1$) n-p pairing (c) is similar to the n-n (a) and p-p (b) pairing
- The **isoscalar** ($T=0$) n-p pairing (d) is clearly different

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Moment of Inertia

- Deformation provides an element of **anisotropy** allowing the definition of a nuclear **orientation** and the possibility of observing rotation

- Classically the energy associated with rotation is:

$$E_{\text{rot}} = \frac{1}{2} \mathfrak{I} \omega^2 = I^2 / 2 \mathfrak{I} ; \omega = I / \mathfrak{I}$$

- Collective rotation involves the **coherent** contributions from many nucleons and gives rise to a **smooth** relation between energy and spin:

$$E = (\hbar^2/2\mathfrak{I}) I[I + 1]$$

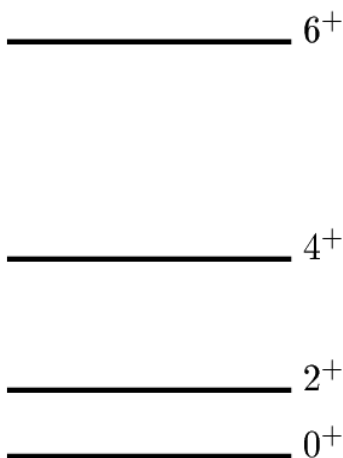
which defines the '**static**' moment of inertia, sometimes denoted $\mathfrak{I}^{(0)}$

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Energy Levels of a Rotor



- The energy levels of a rotor are proportional to $I(I+1)$

- The ratios of energy levels for a rotor are:

$$E(4+)/E(2+) = 3.333$$

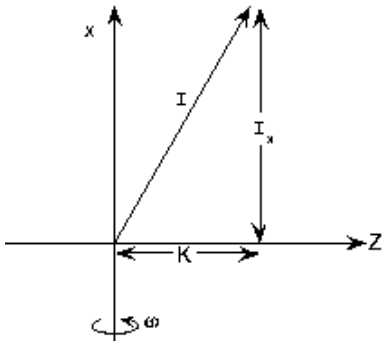
$$E(6+)/E(2+) = 7.0$$

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Rotational Frequency



- The **intensive** variable ω (rotation about the x axis) is related to the **extensive** variable I by the relation:

$$\hbar\omega = dE/dI_x$$

$$\approx \frac{1}{2}[E(I+1) - E(I-1)]$$
- Here I_x is the projection of I onto the rotation axis (x):

$$I_x = \sqrt{[I(I+1) - K^2]} \hbar$$

The rotational frequency ω is distinct from the oscillator quantum ω_0 . In practice $\omega \ll \omega_0$ and the collective rotation can be considered as an **adiabatic** motion

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Rigid Body Moment of Inertia

- The rigid-body moment of inertia for a spherical nucleus is:

$$\mathfrak{I}_{\text{rig}} = (2/5) MR^2 = (2/5) A^{5/3} m_N r_0^2$$

where m_N is the mass of a nucleon ($M = A m_N$) and

$$R = r_0 A^{1/3} \text{ with } r_0 = 1.2 \text{ fm}$$

- For a deformed nucleus:

$$\mathfrak{I}_{\text{rig}} = (2/5) A^{5/3} m_N r_0^2 [1 + 1/3 \delta]$$

where $\delta = \Delta R / R_0$

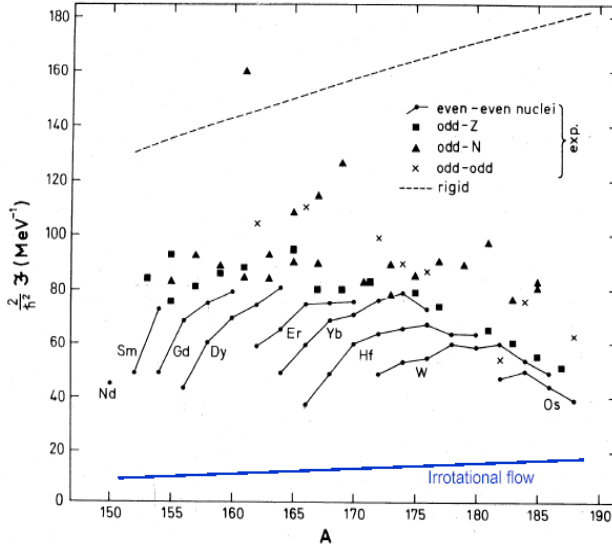
- Typically nuclear moments of inertia are less than **50%** of the rigid-body value at low spin

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Nuclear Moments of Inertia



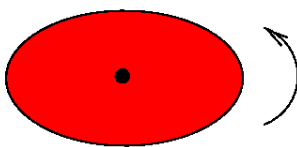
- Nuclear moments of inertia are lower than the rigid-body value - a consequence of nuclear pairing

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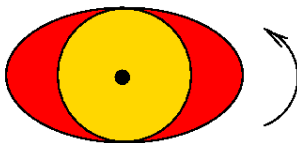
Nuclear Rotation



Rigid body

- The assumption of the ideal flow of an incompressible nonviscous fluid (Liquid Drop Model) leads to a hydrodynamic moment of inertia (surface waves):

$$I_{\text{hydro}} = I_{\text{rig}} \delta^2$$



Nucleus

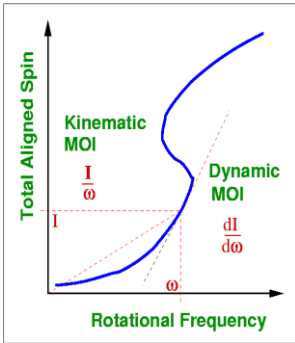
- This estimate is much too low !
- We require short-range pairing correlations to account for the experimental values

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Kinematic and Dynamic MoI's



- Assuming maximum alignment on the x-axis ($I_x \sim I$), the kinematic moment of inertia is defined:

$$\mathfrak{I}^{(1)} = (\hbar^2 I) [dE(I)/dI]^{-1} = \hbar I/\omega$$

- The dynamic moment of inertia (response of system to a force) is:

$$\mathfrak{I}^{(2)} = (\hbar^2) [d^2E(I)/dI^2]^{-1} = \hbar dI/d\omega$$

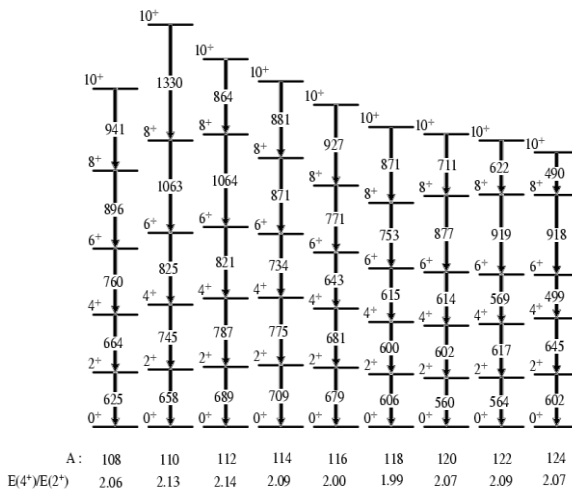
- Note that $\mathfrak{I}^{(2)} = \mathfrak{I}^{(1)} + \omega d\mathfrak{I}^{(1)}/d\omega$
- Rigid body: $\mathfrak{I}^{(1)} = \mathfrak{I}^{(2)}$ Nucleus at high spin: $\mathfrak{I}^{(1)} \approx \mathfrak{I}^{(2)}$

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Vibration or Rotation?



- The simple ratio of the 4^+ and 2^+ energy levels of an even-even nucleus gives an indication of the types of excitation

- For a vibrator:

$$E \propto n$$

$$E(4^+)/E(2^+) = 2.0$$

- For a rotor:

$$E \propto I(I+1)$$

$$E(4^+)/E(2^+) = 3.33$$

Te ($Z = 52$) systematics show that they are vibrational

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Development of Collectivity

- Another limiting value of the $E(4^+)/E(2^+)$ ratio is 2.5, corresponding to a γ -soft rotor, or γ -unstable oscillator ($O(6)$ limit of the interacting boson model: IBM)

- Adding protons to tin:

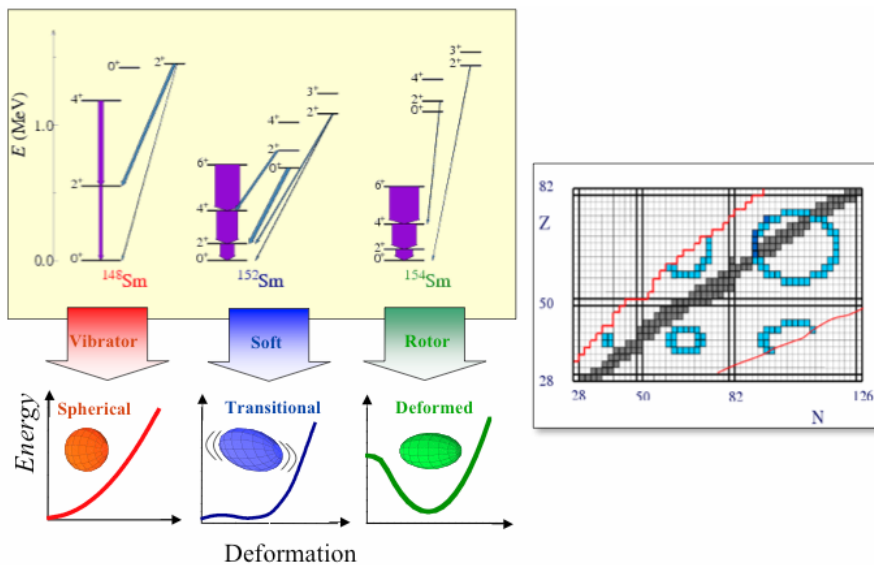
<u>Nucleus</u>	<u>$E(4^+)/E(2^+)$</u>	<u>Behaviour</u>
^{116}Sn (Z=50)	1.65	spherical
^{118}Te (Z=52)	1.99	vibrational
^{120}Xe (Z=54)	2.47	γ -soft
^{122}Ba (Z=56)	2.89	transitional
^{124}Ce (Z=58)	3.15	rotational

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Transitional Nuclei

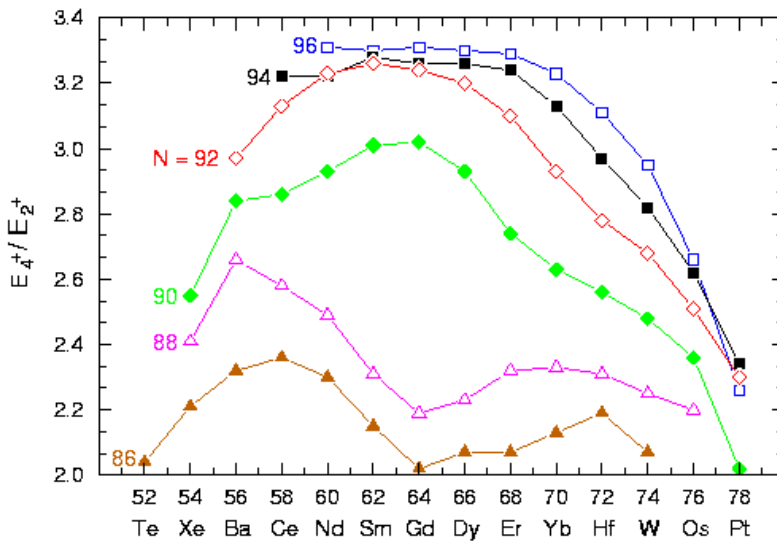


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E(4+)/E(2+) Values

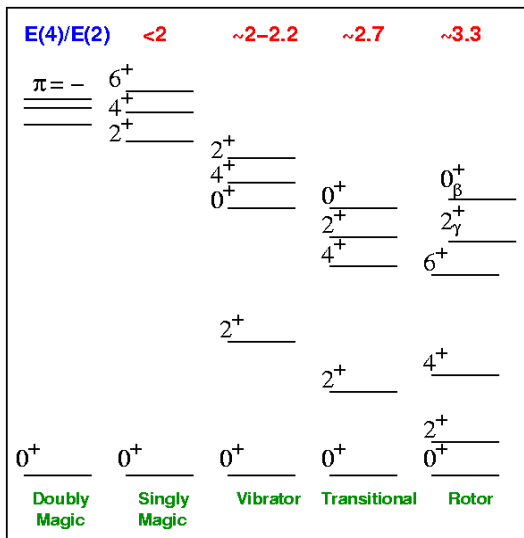


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Evolution of Structure



- This diagram shows the evolution of level structure from closed shell (doubly magic, spherical) to midshell (rotational, deformed) nuclei
- The corresponding E(4+)/E(2+) ratios are also shown

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Particle-Vibration Coupling

3^- ——— 2.612

^{208}Pb

0^+ ——— 0 MeV



$3/2^+ \rightarrow 15/2^+$

^{209}Bi

————— $9/2^-$

▪ For an **odd-A** nucleus near a closed shell with small deformation, the odd particle may couple to the surface vibrations of the core

- The Hamiltonian is: $H = [H_{\text{int}} + H_{\text{vib}}] + H_{\text{coup}} = H_0 + H_{\text{coup}}$
- If we assume the interaction $H_{\text{coup}} \rightarrow 0$, the motions are **decoupled** from one another and the eigenfunctions will take a product form: $H_0\Psi = E\Psi$ with $\Psi = \Psi_{\text{int}} \Psi_{\text{vib}}$
- Consider coupling an $h_{9/2}$ proton to the 3^- state in ^{208}Pb , forming states in ^{209}Bi . Seven 'degenerate' states are formed by coupling spin vectors **3** and **9/2**

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Rotation-Vibration Model

- The **RVM** model considers a well deformed (**static**), **axially symmetric** even-even nucleus and allows small fluctuations (**dynamic**) about the equilibrium shape β_0
- After a 'few' approximations the energy spectrum may be written as:

$$E = \frac{1}{2}\epsilon_R [I(I+1) - K^2] + \epsilon_\beta n_\beta + \epsilon_\gamma n_\gamma$$

where the ϵ parameters are energies associated with rotations and vibrations

- ϵ_R is related to β_0 and the nuclear moment of inertia \mathfrak{I}

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RVM Quantum Numbers

- The quantum numbers are constrained such that:

$$K = 0, 2, 4, \dots$$

$$I = 0, 2, 4, \dots \quad \text{for } K = 0$$

$$= K, K+1, K+2, \dots \quad \text{for } K \neq 0$$

$$n_\beta = 0, 1, 2, \dots$$

$$n_\gamma = K/2, K/2 + 2, K/2 + 4, \dots$$

- So what are the possible low-lying energy levels ?

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RVM Band Structure

- For $K = n_\beta = n_\gamma = 0$, we expect a set of levels:

$$E = \frac{1}{2}\epsilon_R I(I+1)$$

with $I = 0, 2, 4, \dots$ 'ground-state band' represents pure rotation

- A rotational band can be built on a β vibration by setting $n_\beta = 1$. The energy levels are:

$$E = \epsilon_\beta + \frac{1}{2}\epsilon_R I(I+1)$$

again with $I = 0, 2, 4, \dots$ ' β band' (a_{20} varies)

- To include a γ vibration requires a nonzero K . So beginning with $K = 2$ and $n_\gamma = 1$, the levels are:

$$E = \epsilon_\gamma + \frac{1}{2}\epsilon_R [I(I+1) - 4]$$

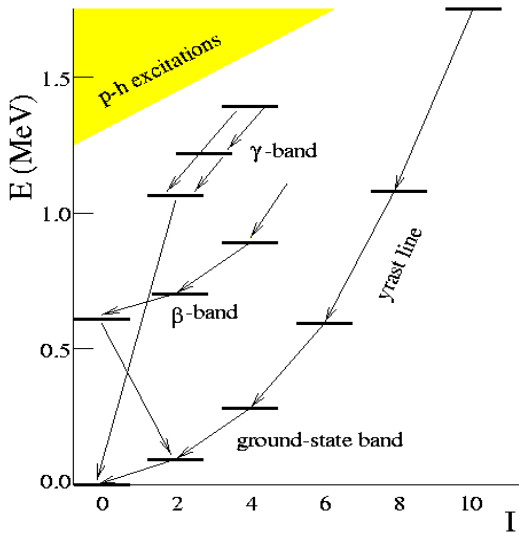
this time with $I = 2, 3, 4, \dots$ ' γ band' (a_{22} varies)

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β and γ Bands



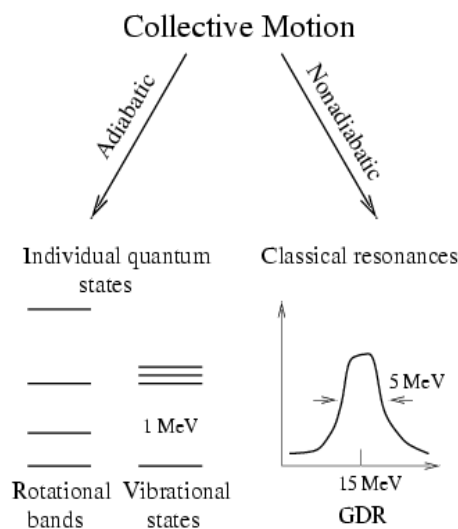
- β -vibrational and γ -vibrational bands coexist with the rotational ground-state band in deformed nuclei
- Such bands are found predominantly in the regions:
 - $150 \leq A \leq 190$ and
 - $A \geq 230$
 which are far from shell closures

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Nonadiabatic Vibration



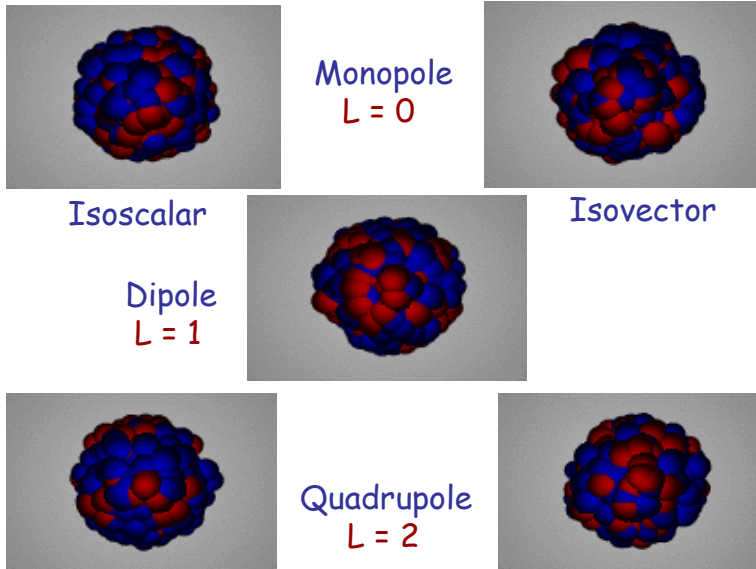
- For the surface modes of vibration, the frequency (velocity) of the oscillations is much smaller than that of the individual nucleonic motion
- The motion is 'adiabatic' (as is nuclear rotation) and individual quantum levels are evident
- However, 'nonadiabatic' collective motion can occur: 'giant resonances'

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Giant Resonances



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Reflection Asymmetry

- If a nucleus is 'reflection asymmetric' (i.e. the odd multipole deformation parameters are non-zero, e.g. $\beta_3 \neq 0$ is the most important) then the nuclear wavefunction in its intrinsic frame is not an eigenvalue of the parity operator:

$$\Psi^2(x, y, z) \neq \Psi^2(-x, -y, -z)$$

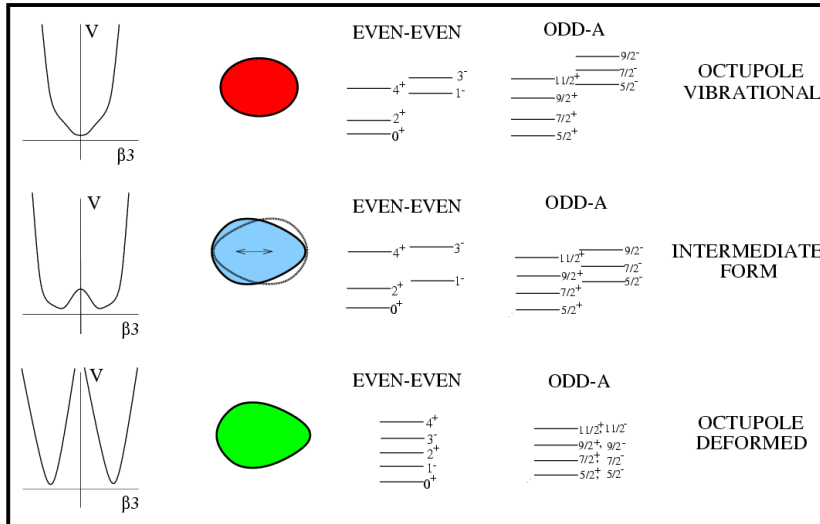
- If $\beta_3 \neq 0$ for a nucleus it is said to possess octupole deformation
- The deformation can however be static, $\langle \beta_3 \rangle \neq 0$, or dynamic, $\langle \beta_3 \rangle = 0$ (oscillating octupole shape)

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Octupole Band Structures

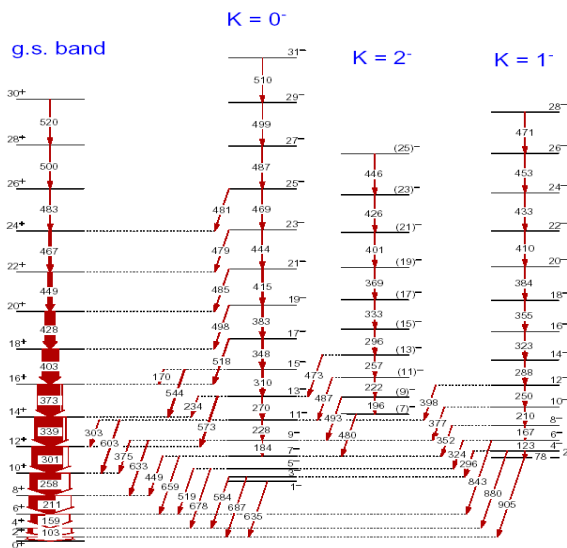


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Octupole Vibrations in ^{238}U



- This nucleus shows three octupole vibrational bands with different K values

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Parity Splitting

- For a static octupole shape, the negative parity states are **interleaved** (midway between) with the positive parity states

- A measure of such a feature is the '**parity splitting**', defined as:

$$\delta E = E(I)^- - \frac{1}{2} [E(I+1)^+ + E(I-1)^+]$$

- This quantity generally **decreases** towards zero with increasing spin and suggests that **rotation** may **stabilise** the **octupole** shape

- A similar quantity is the difference in alignment:

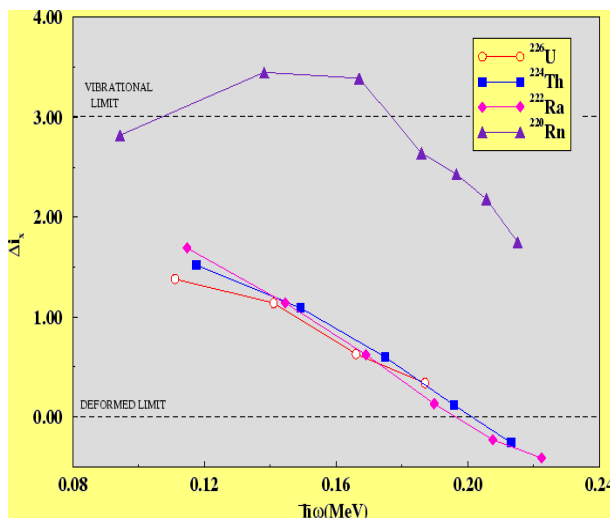
$$\Delta i_x = i_x^- - i_x^+$$

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Octupole Vibration or Deformed?



- For an octupole vibrational phonon coupled to the positive-parity states:

$$\Delta i_x = 3 \hbar$$

- For a static octupole deformation:

$$\Delta i_x = 0$$

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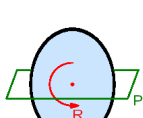
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Reflection (A)symmetry

K = angular momentum projection on symmetry axis

K = angular momentum projection on symmetry axis



K = 0

P: parity (reflection)
R: rotation by 180°
T: time reversal

6⁺ _____

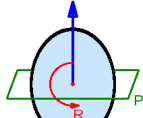
4⁺ _____

2⁺ _____

0⁺ _____

K = 0

1 band



K > 0

P: parity (reflection)
RT: rotation by 180°
AND time reversal (which reverses K)

(K+6)^π _____

(K+5)^π _____

(K+4)^π _____

(K+3)^π _____

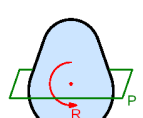
(K+2)^π _____

K^π _____

(K+1)^π _____

K > 0

2 bands



K = 0

RP: rotation & reflection
T: time reversal

6⁺ _____

5⁻ _____

4⁺ _____

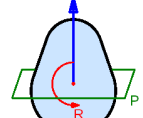
3⁻ _____

2⁺ _____

0⁺ _____

K = 0

2 bands



K > 0

RPT: need all three operations

(K+5)⁻ _____ (K+5)⁺ _____

(K+4)⁺ _____ (K+4)⁻ _____

(K+3)⁻ _____ (K+3)⁺ _____

(K+2)⁺ _____ (K+2)⁻ _____

(K+1)⁻ _____ (K+1)⁺ _____

K⁺ _____ K⁻ _____

K > 0

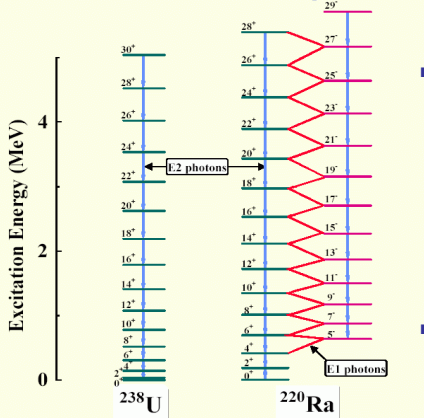
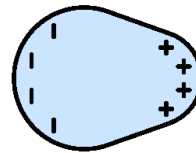
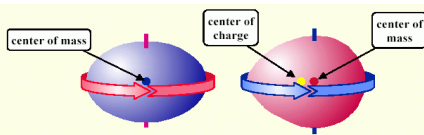
4 bands

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Electric Dipole Moment



- In a nucleus with octupole deformation, the centre of mass and centre of charge tend to separate, creating a non-zero electric dipole moment
- Bands of opposite parity connected by strong E1 transitions occur

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Enhanced E1 Transitions

- In heavy nuclei, E1 strengths typically lie between 10^{-4} and $10^{-7} W_u$
- In nuclei with octupole deformation, the E1 strengths can be much higher: $10^{-3} - 10^{-2} W_u$
- The intrinsic dipole moment of an octupole deformed nucleus is:

$$D_0 = C_{LD} A Z e \beta_2 \beta_3$$

with the liquid drop constant $C_{LD} = 0.0007 \text{ fm}$

- In a Strutinsky type approach, macroscopic and microscopic effects can be considered and:

$$D = D_{\text{macro}} + D_{\text{shell}}$$

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Experimental Dipole Moments

- Experimental values of D_0 can be obtained by measuring $B(E1)/B(E2)$ ratios, related simply to γ -ray energies and intensities

- The $B(E1)$ reduced transition rate is:

$$B(E1; I \rightarrow I-1) = (3/4\pi) e^2 D_0^2 |\langle I_i K_i 1 0 | I_f K_f \rangle|^2$$

- The $B(E2)$ reduced transition rate is:

$$B(E2; I \rightarrow I-2) = (5/16\pi) e^2 Q_0^2 |\langle I_i K_i 2 0 | I_f K_f \rangle|^2$$

- Hence if Q_0 is known (e.g. from the quadrupole deformation β_2) then a value for D_0 can be extracted, i.e:

$$D_0 = \sqrt{[5B(E1)/16B(E2)]} Q_0$$

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