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Unveiling the many facets of the atomic nucleus: from Rutherford to exotic nuclei

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Abstract

After a discussion of the early steps in the period 1930–1950 that disclosed the presence of both independent particle and collective effects in the atomic nucleus, we highlight the major advances describing the atomic nucleus within a shell-model context as well as starting from a mean-field approach. We also discuss the importance of symmetries in recognizing the most important degrees of freedom in the nuclear many-body system as well as connecting shell-model and collective model approaches.

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1. Introduction

In the field of nuclear theory, Nobel prizes were rewarded to E P J Wigner, M Goeppert–Mayer and J H D Jensen for their discoveries concerning a shell structure in the atomic nucleus (Nobel Prize in 1963) and to A Bohr, B Mottelson and J Rainwater for discovering the deep connection between collective motion and single-particle motion in the atomic nucleus (Nobel Prize in 1975). It turns out that those two major discoveries, with the original papers dating from 1949 and 1950–1953, have put their mark on the later work and the evolution of our understanding of atomic nuclei over the last 60 years with as a major theme, trying to reconcile collective motion with individual nucleon motion within a mean field, incorporating residual interactions. In order to do so, nuclear physicists needed also to understand the nucleon–nucleon (NN) interaction as originating from free NN scattering, as well as their effective interaction inside the atomic nucleus.

2. The early period: from Rutherford to the Nobel Prize work on nuclear structure

Our knowledge of the atomic nucleus has mainly been driven by ingenious experimental work. The start came with the α-scattering experiments by Rutherford (1911), experiments that set the length scale characteristic for the extension of the atomic nucleus. Since then, a number of key experiments have disclosed the essential degrees of freedom, at the same time giving rise to an increasing theoretical activity that evolved hand in hand with the experimental developments.

Let us consider some of the very early steps. The discovery of the neutron by Chadwick (1932) gave rise, within months to the concept of charge symmetry using the Pauli spin matrices and the SU(2) structure (Heisenberg 1932). Experiments providing indications of particular stability for light α-like nuclei were at the origin of an extension to combine isospin and intrinsic spin into the SU(4) Wigner supermultiplet scheme (Wigner 1937), proposing full charge independence of the NN interaction. The latter scheme was at the origin of a first primitive kind of nuclear shell model, worked out by Feenberg and Phillips (1937), using simple forces with exchange character (Heisenberg 1932, Majorana 1933, Bartlett 1936), describing the binding energies from ^{6}He to ^{16}O. Discoveries following from radioactive decay studies and inspecting the abundances of the stable elements pointed out the extra stability of nuclei that contained a particular number of protons and/or neutrons at 50, 82 and 126, configurations that posed serious problems to the early shell model. In 1934, Fermi succeeded to perform nuclear reactions using neutrons, in particular slow ones, thereby covering almost all of the then known stable nuclei (Fermi *et al* 1934) and extended the experimental knowledge on atomic nuclei in a major way. These results resulted in the concept of a compound nucleus, resulting from the strong interactions between protons and neutrons (Bohr 1936) which allowed a description of neutron cross-sections as well as the statistical characteristics at high excitation energy in the nucleus. These concepts resulted in a new picture of the atomic nucleus to be considered as a charged liquid drop (von Weizsäcker 1935, Bethe and Bacher 1936) and was at the origin of a theoretical description of the phenomenon of nuclear fission (Bohr and Wheeler 1939). On the other hand, the high precision obtained in atomic physics studies of the hyperfine structure in atoms gave results on magnetic dipole moments and electric quadrupole moments (Schüler and Schmidt 1935, Schmidt 1937) that pointed toward independent particle motion characterizing the odd nucleon in odd-A nuclei and the presence of an electric quadrupole charge distribution for the nucleus (Casimir 1936). A systematic study of quadrupole moments by Townes *et al* (1949) was inconsistent with a single nucleon moving in a central potential indicating the need of an induced deformation.

By the end of 1940, it was clear that the experimental data were pointing toward two almost contradictory facets of the atomic nucleus: independent particle motion versus collective structure. In the field of nuclear theory, Nobel Prizes were awarded to M Goepert–Mayer and J Hans D Jensen 'for their discoveries concerning nuclear shell studies' (Haxel *et al* 1949, Mayer 1949) and to A Bohr, B Mottelson and J Rainwater 'for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection' (Rainwater 1950, Bohr 1952, Bohr and Mottelson 1953).

It turned out that those two major steps forward, however, resulted in seemingly conflicting views with on one side the individual nucleon motion and on the other side collective degrees of freedom. There have been major developments since on a theoretical understanding of both the shell model and mean-field methods, intimately connected through underlying symmetries over a period of ∼60 years. This would have been impossible without the enormous developments of experimental methods for both the acceleration and detection of particles and nuclei, first starting from stable nuclei, later moving away from stability with the extensive use of radioactive beams.

3. The shell model: from independent particle motion to strongly correlated systems

3.1. Phenomenological effective interactions

It is needless to say that the understanding of how correlations are developing within an interacting system of nucleons is tightly connected to a firm understanding of the nucleon interactions that are active inside the atomic nucleus.

Our understanding of NN interactions around 1950 was still rather restrained and started from the presence of an attractive part (Yukawa 1935) at long range and a repulsive short-range component that was needed in order to describe n–p and p–p scattering (Jastrow 1951). Therefore, first shell-model calculations made use of the philosophy not to calculate the two-body interaction matrix elements but rather use the two-body matrix elements (TBME) $\langle j_a j_b, JT|V|j_c j_d, JT\rangle$ and the single-particle energies ε_{j_a} as parameters to be fitted to the experimental data through the use of the eigenvalue equations (Talmi and Unna 1960). The early work concentrated on the light p-shell nuclei (Cohen and Kurath 1965), followed by the sd shell mass region (Wildenthal and Chung 1979, Brown and Wildenthal 1988) with an update by Brown and Richter (2006), and, more recently, reaching the full fp shell (Honma *et al* 2002) and spanning the ^{56}Ni–^{100}Sn region (using the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell-model space) (Honma *et al* 2009)), the latter implying huge computing needs. Powerful codes have been developed over the years allowing to reach model spaces up to $\sim 10^{10}$ basis states. These studies all have shown the robust and consistent structure of the shell-model methods using one major harmonic oscillator shell. The methods used, however, remain phenomenological in nature with respect to the NN interaction since matrix elements of the force, taken as numbers, are treated as parameters when fitting the calculated nuclear properties to the large body of experimental data. This approach is fine in as much as it has also predictive power, which means that with increasing number of experimental data, the calculated results predicted later experimental results.

This approach considering only one major shell, confronted with new data, has been shown to fail in a number of regions, starting with the $N = 20$ shell closure. At the ISOLDE separator, installed at the CERN synchrocyclotron (SC) and using its external proton beam line at 10.5 GeV (24 GeV in later experiments), it became possible to explore the properties of unstable nuclei (Klapisch *et al* 1968). Thibault *et al* (1975) succeeded to carry out direct mass measurements on $^{26-32}$Na nuclei ($Z = 11$) going beyond $N = 20$ and found an increase in the S_{2n} value hinting for increased binding energy at $N = 20$ instead of a swift drop in the vicinity of the $Z = 20$, $N = 20$ ^{40}Ca nucleus. These results were supported through measurements of charge radii (Huber *et al* 1978) and the energy of the first excited 2^+ state in the Mg nuclei, following β^--decay of the Na nuclei (Détraz *et al* 1979). These results showed that the original idea of only treating active protons and neutrons limited to the 8–20 sd valence shell had its limitations (Wildenthal and Chung 1980). Similar cases have been shown to arise near neutron number $N = 8,\ 20,\ 28$ nuclei, for heavier Sn ($Z = 50$) and Pb ($Z = 82$) nuclei and, even for doubly-closed shell nuclei, such as ^{16}O, ^{40}Ca, ^{56}Ni, etc (Heyde and Wood 2011). Powerful though the above method to set up a phenomenological effective set of interaction matrix elements is, deep insight in its microscopic basis was lacking.

3.2. Microscopic effective interactions

In order to have a well-understood shell-model approach, one should start from realistic NN forces. The early realistic forces were mainly phenomenological in nature constrained by fitting to the experimental information obtained from free NN scattering experiments (parameterized through phase shifts in the various $(l, S)J$ collision channels, total and differential cross-sections, polarization data) and set up in the early 1960s by Hamada and Johnston (1962), Tabakin (1964), Reid (1968). In these interactions, the repulsive short-range character and the long-range one pion exchange potential (OPEP) are essential ingredients. With the experimental discovery of the more heavy mesons (ρ, ω, etc) in the 1960s, during the 1970s, and mid-1980s, NN forces were derived that were for a large part based on the exchange of mesons (π, 2π, ρ, etc) resulting in the Paris (Lacombe *et al* 1980) and Bonn (Machleidt *et al* 1987, Machleidt 1989) potentials. The modern realistic two-body interactions are obtained by fitting to the 4301 data points at scattering energies below 350 MeV in the Nijmegen NN-scattering database (covering all of the experimental work carried out between 1955 and 1992) which is complete in the sense that all relevant phase shifts and mixing parameters are determined (Stoks *et al* 1993). The presently used modern and also equivalent potentials are the Argonne V18 potential AV18 (Wiringa *et al* 1995), the Nijmegen I+II and Reid-93 potentials (Stoks *et al* 1994) and the improved CD-Bonn potential (Machleidt *et al* 1996, Machleidt 2001) fitting the database with a χ^2/datum of almost 1. These potentials describe equally well the properties in three- and four-body nuclei.

Because of the repulsive short-range properties of these realistic n–n forces, perturbation theory cannot be used and one (i) needs to evaluate the reaction G-matrix (methods developed by (Brueckner 1955, Goldstone 1957, Bethe *et al* 1963) during the late-1950s, early 1960s), and (ii) implement this G-matrix within the shell-model in order to describe a given mass region so that specific nuclear correlations can be treated using perturbation theory. These two steps have been carried out in great detail (Kuo and Brown 1966, 1968) (applied to the sd-shell near ^{16}O, in particular, and for the 1f–2p shells) showing that the effective in-medium interaction constructing the G-matrix and the subsequent very important core-polarization effects is manifested in light and medium-heavy nuclei. Similar studies have been carried out for the sd and the 1f–2p shell starting from meson-theoretical potentials (Hjorth-Jensen *et al* 1995). These so-called microscopic effective NN interactions have been very much explored during the last 25–30 years in light to medium-heavy nuclei.

A major drawback with this approach, however, is the fact that the so constructed in-medium effective forces do not lead to the correct saturation (binding energy) properties when proton and neutron number are changing through long series of isotopes or isotones. This can be noticed in the case of the Ca nuclei comparing the energy gap between the $1f_{7/2}$ and $2p_{3/2}$ orbitals moving from ^{41}Ca toward ^{49}Ca (Martinez-Pinedo *et al* 1997) filling up the $1f_{7/2}$ orbital with eight neutrons. The starting value changes from 1.8 to 2.06 MeV (using the Kuo–Brown interaction) with the experimental value observed at 4.81 MeV: a most striking discrepancy. This so-called monopole problem has been recognized early on and cured by Pasquini and Zuker (1978) and Poves and Zuker (1981) in an empirical way such that the theoretical mean field fits with the experimental data (in the above case using the KB3 force with modified energy centroids). This is even more dramatically illustrated comparing the full energy spectrum of ^{49}Ca which results in a far too compressed (and overbound) energy spectrum using KB force, as compared to the monopole corrected KB3 forces (Martinez-Pinedo *et al* 1997). It has been recognized over the last 5–10 years that three-body forces are almost surely behind this problem and exploratory studies point toward a coherent description using realistic two- and three-body forces (Zuker 2003).

More detailed examples on the accomplishments and present status of modern shell-model methods, as described in sections 3.2 and 3.3, are given by Caurier *et al* (2005).

3.3. Microscopic effective interactions: a new approach

In view of the above problems when starting from realistic NN forces, and noticing the lack of three-body forces, the question has been brought up to connect the forces used in nuclei with the deeper level of quantum chromodynamics (QCD) and derive the basic form of the NN interaction. This has been initiated by early work of Weinberg (1990, 1991) which gave rise to a formulation in the framework of effective field theory (EFT). Here a Lagrangian is proposed which contains the nucleon and pion fields and all possible interactions consistent with chiral symmetry. At low energies, the heavy mesons and nucleon resonances are integrated out of the theory. Thus an effective Lagrangian is obtained and allows a power counting scheme that is based on the number of couplings in this Lagrangian. This form is then fitted to $\pi\pi$, πN scattering and NN low-energy data (Stoks *et al* 1993) and enables the EFT to predict other processes. Major work has been carried out since the 1990s by Van Kolck (1999) for a review of work in the 1990s, and more recently by Meissner and the Bochum/Jülich(Bonn) group (Epelbaum *et al* 2009) and Machleidt and co-workers (2003). A major result is that three-body NN interactions can now be derived consistently with the two-body NN interaction. This is an important step in order to link the realistic NN forces to the underlying theory of QCD, albeit in a low-energy limit. A recent review paper appeared on this particular subject (Bernard and Meissner 2007). This domain of research is trying to build a bridge between the NN force that can be used to describe a large collection of interacting nucleons (atomic nuclei) and the NN force as resulting from the underlying chiral symmetry of QCD as proposed by Weinberg (1990).

A second major point to consider is the fact that all high-precision NN realistic interactions (including the chiral N^{3}LO interactions) are constrained only by means of the two-nucleon scattering phase shifts up to a relative momentum of $k < 2.1\,\mathrm{fm}^{-1}$ as well as by the deuteron properties. It turns out that the matrix elements $V_{\mathrm{NN}}(k,k)$ have a largely different high-momentum behavior with is not relevant to describe the low-energy nuclear structure properties (Bogner *et al* 2003). Using renormalization methods, a low-momentum interaction $V_{\mathrm{low}-k}$ results from integrating out the high-momentum components above a certain threshold $\Lambda \sim 2.1\,\mathrm{fm}^{-1}$. The outcome is that the $V_{\mathrm{low}-k}(k, k)$ matrix elements, independent of the starting realistic force, show an identical variation with momentum k (see also Bogner *et al* 2003). The big advantage is that this low-momentum interaction is smooth enough (no longer the high-k components) so that it serves as a kind of G-matrix which can be used to calculate the local (model space) corrections using perturbation theory (Schwenk and Zuker 2006, Bogner *et al* 2010).

Quite some work has been carried out using this approach, in particular when starting from NN, NNN, etc forces that are consistent with low-energy chiral symmetry breaking. Consequently, at best one would not need any more data for further fitting and there is hope for minimal input from data to calculate nuclear structure properties far from stability (Otsuka *et al* 2010, Holt *et al* 2011). Otsuka *et al* (2010) have compared nuclear binding energies for the even–even O isotopes using both phenomenological interactions (USD-B (Brown and Richter 2006) and SDPF-M (Utsuno *et al* 1999)) with calculations using a G-matrix NN calculation, starting from the Bonn-C realistic interaction (Hjorth-Jensen *et al* 1995). These latter calculations result in overbinding and only after adding the effect of three-body forces (mainly the Δ resonance) are experimental data reproduced. On the other hand, starting from the $V_{\mathrm{low}-k}$ part and including three-body effects up to N^2LO order, the experimental can be well described pointing out the need to use a consistent treatment of two- and three-body forces.

3.4. Other shell-model methods

In view of the quickly increasing computational problems that obey combinatorial scaling in the number of singe-particle orbitals and active nucleons, once reaching nuclei in the medium-heavy region and beyond, regular diagonalizaton of the energy matrix ceases. A shell model Monte Carlo (SMMC) method was developed by Koonin co-workers (Johnson *et al* 1992, Koonin *et al* 1997) circumventing these restrictions, however retaining the rigor, flexibility and predictive power of the more standard shell model. The method essentially transforms the nuclear many-body problem into a set of one-body problems in fluctuating auxiliary fields. This approach proved very important and allowed to study both the ground-state and thermal properties of atomic nuclei, albeit only for summed strengths, important for applications of nuclear physics behavior in stellar conditions. Monte Carlo methods have, on the other hand, also been used to generate coherent states as an optimal basis to describe low-lying collective states (Monte Carlo shell model—MCSM). This method, using standard diagonalization techniques to obtain nuclear wavefunctions and corresponding observables, was developed by Otsuka co-workers (Honma *et al* 1995, Otsuka *et al* 2001) and has been applied with success to light and medium-heavy (p shell, s–d shell, f–p shells and light rare-earth nuclei).

4. The nuclear mean field: from static nuclear properties to beyond mean-field collective correlations

The challenge in formulating a general description of collective modes of motion (vibrations, rotations, etc) and their underlying intrinsic structure, this time starting from the NN interaction, and in order to describe both the nuclear binding energies, characterized by the energy scale of $\sim$ 1 GeV, as well as the local effects that are situated at the energy scale of $\sim$ 1–5 MeV, is a big one. Starting from realistic NN interactions that contain a strong repulsive component at the short distance scale, the application of Hartree–Fock (HF) and Hartree–Fock–Bogoliubov (HFB) methods did not lead to a correct description of binding energies, radii, densities. The complexity in using these realistic nucleon forces and complex calculation methods seemed to run into a dead end.

The insight brought by Skyrme (1956, 1959) that the effective NN interaction acting inside the nucleus had to be taken as a zero-range two-body interaction, including a momentum dependence plus a density-dependent term, proved to offer promising results when describing global nuclear properties extending throughout the nuclear mass chart. Vautherin and Brink were able to propose a powerful and elegant method to solve the HF equations for both spherical (Vautherin and Brink 1970, 1972) and deformed nuclei (Vautherin 1973). These calculations formed the start of systematic studies throughout the whole nuclear mass region. The connection between the use of such effective interactions based on Skyrme-type interactions and microscopic many-body theory, starting from realistic interactions and invoking a density matrix expansion, was first studied by Negele (1970) and elucidated further by Negele and Vautherin (1972, 1975). A few years later, a version that could cope as well with the mean-field properties as with the NN pairing correlations in nuclei using density-dependent HFB theory was brought upfront by Gogny (1973, 1975). The force he suggested (also called D1, see Dechargé and Gogny (1980)) contained a central finite-range interaction, a zero-range density-dependent part as well as a spin–orbit term as outlined by Skyrme in his original papers. Around the same time, Walecka (1974) developed a relativistic quantum field theory with nucleons and scalar and vector (σ, ω) mesons and derived its mean-field solution. This proved in later years to be a highly successful approach and was extended in a relativistic HF approach, this time including the mean-field contributions in a self-consistent way (Serot and Walecka 1986), with later use of energy-density functionals including also the pairing part of the nuclear interactions (Vretenar *et al* 2005).

These seminal papers gave rise to a very extensive field of HF and HFB studies using Skyrme energy functionals to which a zero-range density-dependent pairing energy had to be added in order to treat nuclei with protons and neutrons outside of closed shell configurations (since 1972, tens of variants on the original Skyrme have been constructed with various applications (Erler *et al* 2011)), using the Gogny-force (since 1973) and using relativistic mean-field methods (Niksic *et al* 2011). Over the years—since these major developments that originated in the early 1970s—the early studies have been transformed in highly performing numerical codes that allow to cover a large part of the ground-state properties of atomic nuclei, from light nuclei ($A > 16$) up to superheavy nuclei (Bender *et al* 2003). More recently, efforts are undertaken toward an *ab initio* derivation of density functionals starting from the underlying NN interactions and using many-body perturbation theory (Gebremariam *et al* 2010, Kortelainen *et al* 2010, Stoitsov *et al* 2010) and (Drut *et al* 2010, Dobaczewski 2011, Raimondi *et al* 2011) for recent reviews.

The early mean-field studies concentrated on the static energy contribution. It is interesting to appreciate the steps taken forward starting from the rather simple energy surface calculations (Girod *et al* 1989) only using the axial quadrupole deformation as compared with the detailed energy surfaces, this time covering the full (β, γ) plane (Girod *et al* 2009) for the Kr nuclei, over a period spanning $\sim$ 20 years to take just one example.

More recently, the need to go beyond the static mean-field level, mainly based on the original concepts of Hill and Wheeler (1953), Griffin and Wheeler (1957), called generator-coordinate-method (GCM), in order to study nuclear collective dynamics has been recognized and developed. The numerical implementation to restore symmetries, broken in mean-field HF and HFB studies, has taken more than 20 years of improvements and developing new and better computing algorithms (Valor *et al* 2000, Rodriguez-Guzman *et al* 2002). The largely increased computing power has at present led to codes that are able, starting from intrinsic HF(B) states over the complete (β, γ) plane, to carry out GCM calculations with states projected on the full triaxial angular momentum (Bender and Heenen 2008, Yao *et al* 2009, Rodriguez and Egido 2010). Those calculations showed rather consistently the variation in nuclear shape as a function of proton and neutron number moving all through the nuclear mass table from light nuclei (^{16}O) toward the heaviest and superheavy nuclei (Bender *et al* 2003).

An interesting approach to study the dynamics of the full five-dimensional collective model (5DCH) results stems from the approximation to use Gaussians (GOA) in order to describe the overlap of the mean fields at different deformations $J(q, q') = \langle\Phi(q)|\Phi(q')\rangle$ (Ring and Schuck 1980) and expanding the Hamiltonian matrix elements up to second order in $(q - q')$. This shows that the Hill–Wheeler equations are formally equivalent to a collective Schrödinger equation. The collective mass parameters and the inertial factors can subsequently be described on a microscopic basis. This approach can be regarded as a modern version of the Kumar and Baranger model (Baranger and Kumar 1968, Kumar 1974) and is now extensively used, using both Gogny forces and using relativistic energy density functionals (EDF), to explore the systematic variation in the nuclear low-lying collective properties and explore possibilities of shape-phase transitions (Delaroche *et al* 2010, Niksic *et al* 2011).

5. The importance of symmetries: recognizing the major degrees of freedom in nuclei

Symmetry has proven before, through the early work of Heisenberg (1932), Wigner (1937), Racah (1943, 1949), that properties of nuclei, built from protons and neutrons could be unified in elegant ways. It is, however, important to study the role played by symmetries to connect the shell-model method and rotational properties in nuclei, the latter generated naturally from a mean-field approach. Elliott and Flowers (1955), carrying out shell-model calculations for light nuclei (p-shell and s–d shell), showed the appearance of rotational structures in nuclei as light as ^{20}Ne and ^{24}Mg. Elliott was able to shown in two seminal papers (Elliott 1958a, b) that the states, considering a degenerate and complete sd harmonic oscillator shell, could be classified according to the representations of SU(3) and moreover that besides this very elegant group-theoretical representation, the states could be shown to originate from a given intrinsic state. The realization that collective rotational motion was associated with ‘intrinsic’ states was an early reconciliation of shell-model and collective motion. Elliott moreover found out that the quadrupole–quadrupole force was at the basis of these results and was using ideas of dynamical symmetries to describe the energy splitting of the multiplet members through this quadrupole force (keeping the underlying SU(3) symmetry intact). Thus, a deep connection was uncovered linking the nuclear shell model with collective nuclear rotational properties as observed in light nuclei and proved, in the context of large-scale shell-model calculations, more generally applicable.

A most interesting result of the large-scale shell-model calculations is the fact that for nuclei in the fp shell, bands reminiscent of collective rotational motion could be realized. A deeper analyses of the wave functions indeed pointed out that, e.g., in ^{48}Cr the results are consistent with a remarkable constant intrinsic quadrupole moment starting up to spin $J = 10$ where backbending appears (Poves 2004). It could be shown that a generalization of Elliott’s SU(3) idea also holds in other regions of the nuclear mass table applying a ‘quasi-SU(3)’ scheme (Zuker *et al* 1995) thereby pinning down in more detail the relation between the nuclear shell model including the interactions active within the shell model, on one side, and nuclear collective rotational motion, on the other side.

Even though Elliott’s SU(3) model is restricted to a single harmonic oscillator shell, it had become clear over the years that even doubly-closed shell nuclei: ^{16}O, ^{40}Ca, ^{56}Ni were exhibiting well-developed rotational bands. It was long known (Brown 1964) that a microscopic origin could be given as mp-mh proton and neutron excitations across the closed shells, giving rise to rotational bands. With the increasing computing possibilities using shell-model codes, ^{40}Ca was studied in great detail (Caurier *et al* 2007) considering up to 10p–10h excitations from the upper sd shell into the full fp shell giving rise to both deformed and superdeformed bands. A group theoretical extension of the SU(3) model to include pair excitations was formulated by Rosensteel and Rowe (1977, 1980) resulting in the non-compact Sp(3,R) group. It turned out that this group emerges as an appropriate group for a many-body theory of collective motion but is at the same time a dynamical group for the harmonic oscillator, thus guiding us toward the shell model (Rowe 1985). Consequently, a strong link between more realistic large-scale shell model calculations and an underlying group theoretical structure becomes established.

There have been attempts to describe the nuclear collective properties, resulting from the Bohr Hamiltonian, starting from the group theoretical structure of the five-dimensional oscillator (Gneuss and Greiner 1971). It was the group of Moshinsky describing the group theory in greater detail (Chacon *et al* 1976). These formulations all started to generate states from a collective space built from quadrupole or d-bosons. A step forward was taken in 1974 by Janssen *et al* (1974) who introduced states that formed the bases of irreducible representations of the group U(6).

It were Arima and Iachello who showed that a 6D group U(6) appeared when describing a set of interacting s and d bosons (Arima and Iachello 1975), called the interacting boson model (IBM). It turned out that the U(6) group structure encompassed both quadrupole vibrational excitations, contained in the earlier work of the Frankfurt and Moshinsky’s groups (U(5) dynamics), the SU(3) group for rotational motion as well as an unexpected group chain i.e. O(6), related to the description of γ-unstable collective motion. These three dynamical symmetries, U(5), SU(3) and O(6) were extensively explored in three major papers in the period 1976–1979 (Arima and Iachello 1976, 1978, 1979). These papers sparked a rapidly increasing activity in the community of nuclear physics. It was rather soon proven that in the limit of very large boson number ($N \rightarrow \infty$), the IBM could be viewed as a simpler version of the collective model (Dieperink *et al* 1980).

At this stage, the IBM was based on symmetry considerations only. Soon, it was noticed that a connection with the nuclear shell model degrees of freedom could be made (Arima *et al* 1977, Otsuka *et al* 1978) proposing a connection between s and d bosons with nucleon $J = 0$ and two coupled pairs, moreover including the proton and neutron degree of freedom (called the IBM-2). The symmetries of the IBM-2 contained a new class of states not corresponding to the lowest-lying fully-symmetric representations (Iachello 1984). This corroborated with results obtained some years before within the context of the collective model, explicitly treating the proton and neutron densities (Suzuki and Rowe 1977, LoIudice and Palumbo 1978). The presence of such non-symmetric excitations was experimentally shown to exist in deformed rare-earth nuclei by Bohle *et al* (1984), known as the scissors mode.

Iachello (1980) also proposed the possibility to combine the U(6) IBM boson representations with fermion representations as representations of a larger group structure that allow the transformation of boson into fermion operators (and vice versa) leading to Bose–Fermi dynamical symmetries. In view of the often complex energy spectra it has not been obvious to find unambiguous indications of the presence of such boson-fermion relations in actual nuclei. Experimental results by Jolie and co-workers (Metz *et al* 1999) gave strong evidence to these enlarged symmetry classification by linking the four partners of the even–even, odd–mass and odd–odd nucleus ^{194}Pt,^{195}Pt,^{195}Au and ^{196}Au nuclei in a unified scheme.

Besides the line set out, starting with Elliott's early breakthrough, and the subsequent idea of considering the sd-boson U(6) group with its dynamical symmetries, a number of people have over a long period been trying to connect the observed nuclear collective dynamics (vibrations, rotations, etc) to the microscopic structure making use of algebraic methods (see Rowe (1985) for an overview of these methods). It turned out that the symplectic group, Sp(3,R) emerges as an appropriate group for a many-body theory of collective motion and is at the same time a dynamical group for the harmonic oscillator, thus guiding us toward the shell model (Rowe 1985). A connection with the IBM model has been explored (Rowe 1996), leading more recently to a computationally tractable version of the collective model (Rowe 2004a, Rowe and Turner 2005), and, stimulated by Iachello's ideas on phase transitions (Iachello 2000, 2001) led to studies of the various phase transitions in nuclear collective models (Rowe 2004b, Turner and Rowe 2005, Rosensteel and Rowe 2005).

6. Outlook

It turns out that starting from the early independent particle model (IPM), and including correlations beyond this IPM using large model spaces (also including multi-particle multi-hole excitations across closed-shell configurations), a unified way describing both few-particle (near closed shells) and collective excitations (many valence protons and neutrons outside of closed shells) could be reached. Large steps forward have been taken in order to understand the nucleon forces inside the nuclear medium (including both two- and three-body forces) constructing effective forces starting from chiral perturbation theory thereby connecting nuclear structure in a more profound way to the fundamental theory of strong interactions. There have been major developments on *ab initio* approaches to the structure and reactions of light nuclei which is discussed in the contribution of Forssén *et al* (2013).

Starting from NN effective interactions and using self-consistent HF and HFB methods it has become possible to derive the nuclear mean-field properties (masses, charge and matter radii and densities). Adding correlations by restoring the broken symmetries by mixing number and full angular-momentum projected (going beyond the static mean-field solutions), one has been able to derive collective wave functions and the corresponding collective excitations. Moreover, systematic studies of nuclear shape changes over large regions of the nuclear mass table, solving a collective Hamiltonian (GOA) have given insight in the changing global nuclear properties. Large efforts are undertaken to derive modern energy density functionals from a microscopic basis.

We have stressed the role played by symmetries governing nuclear forces and nuclear structure observables, starting with Heisenberg and Wigner over a span of almost a century taking up the formidable task to understand deeply the way in which collective effects can arise from the nuclear many-body system built from interacting protons and neutrons. The strength of algebraic methods lies in the insight derived from exactly solvable models leading to a better understanding of more realistic situations encountered in the description of nuclei moving to the limits of stability.

The nucleus, organized on the basis of a microscopic structure has continued to reveal a richness of different emergent properties. The importance of advancing experimental methods through technical progress has to be underlined at this place. We have witnessed that almost every new experimental technique for accelerating and/or detecting particles has revealed new and quite often unexpected facets of the atomic nucleus. This is particularly the case experimenting with radioactive beams.

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