Nuclear Instrumentation: Lecture 4

Pulse Counting Systems and Dead Time

4.1 BASIC MEASUREMENT SCHEME

The system, shown in Figure 4.1, illustrates the components of a basic measurement scheme designed to measure the rate of pulses from a detector.

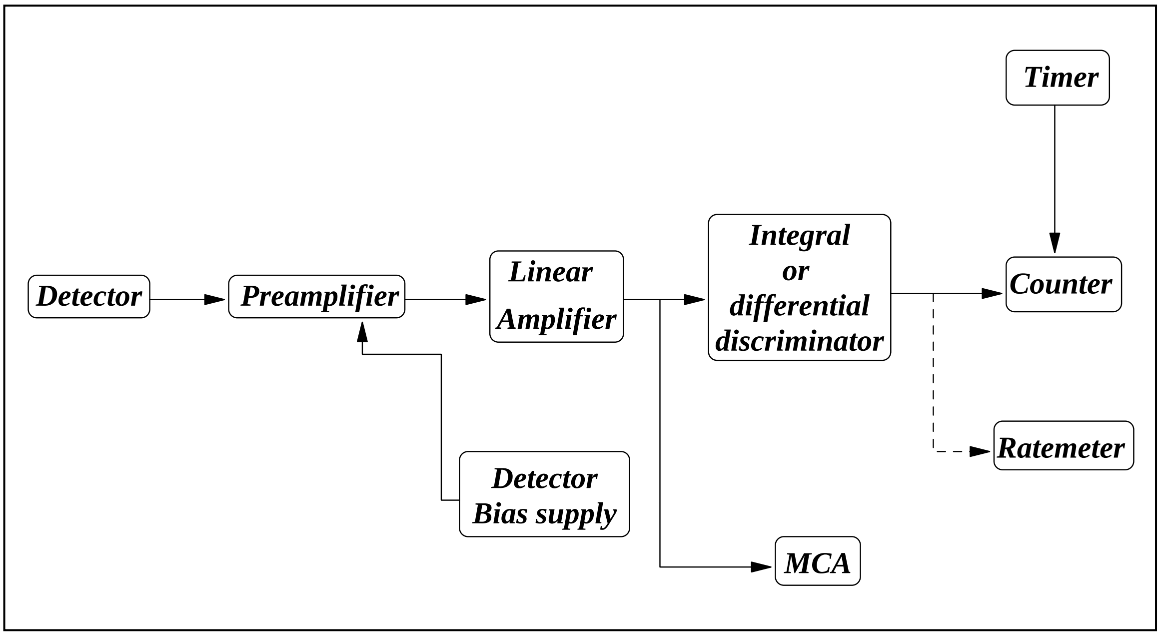

Figure 1 A typical system for pulse-rate counting

4.1.1 Preamplifier

The pulses from the preamp usually have amplitudes of tens to hundreds of mV, too small to be directly counted (there are some exceptions, see Lecture 3). In addition, the long decay time of these pulses can cause stability and pile-up problems.

4.1.2 Amplifier

Therefore, the preamp output is usually processed (amplified and shaped) through a linear amplifier. A gain of a factor of 1000 can be achieved so that output pulses with a range of 0 to 10 volts can be produced. The shaping requirements in a simple counting system are usually not severe (but beware of high counting rates).

4.1.3 Discriminator

To be counted, the linear pulses must be converted into logic pulses (yes/no answer). The simplest unit to do this is the integral discriminator, which produces a logic pulse output only when the input linear pulse amplitude exceeds a set discriminator level. The logic pulse is usually produced soon after the leading edge of the linear pulse passes the discriminator level. This so called leading-edge timing is not suitable for all applications and will be contrasted with other time pickoff methods later.

For maximum sensitivity the discriminator level is normally set to be just above the electronic noise level for the system.

The differential discriminator or single-channel analyzer (SCA) involves two independent discrimination levels, commonly referred to as the lower and upper levels, respectively. The SCA will only produce an output logic pulse if the amplitude of the input pulse lies between the two preset discrimination levels.

Several different nomenclatures exist for SCAs. In some, the discriminators are labelled upper and lower level (uld and lld, respectively), while in others, the lower level is labelled E and the window width or difference between levels is labelled ΔE.

In counting systems, an SCA can be used to select a limited range of amplitudes (energies) from the detector. A common example is to set the acceptance window to count only full-energy pulses.

- For normal SCA's the time of appearance of the logic pulse is not closely related to the actual time of the event and therefore, SCAs should not be used in timing circuits.

Specifications exist for the amplitude and shape of the input pulses to SCAs. Normally, they tend to accept positive input pulses with amplitudes 0 - 10 V and 0.5 - 10 μs shaping times (slow logic).

4.1.4 Scalers, counters and timers

As a final link in the chain, the logic pulses must be counted. A **scaler** or **counter** is a simple digital register, which is incremented by one count for every input pulse. They are normally operated in one of two modes, preset time or preset count:

- In **preset-time** mode, the counting period is controlled by an external timer.
- In **preset-count** mode, the counter will count until a specified number of counts are accumulated. If the time period was also measured, the count rate can be determined.

Preset-count mode has the advantage that a specified statistical precision can be entered at the beginning of the measurement, the counting time being extended until enough counts have been accumulated.

The function of a **timer** is simply to start and stop the accumulation period for a counter or other recording device. Obviously, its most important property is the precision to which the interval is measured. Two general methods of control are encountered. The simplest (and cheapest) is to base the time interval on the ac frequency of the power line. The power-line frequency is usually stable, when averaged over a long time period (days), but can be inaccurate over short periods of time (less than a few hours). Better accuracy is obtained from timers based on internal crystal controlled clocks.

4.1.5 Count Rate Meters

Count rate meters give a visual indication of the count rate (obviously!). A common form of rate meter can be represented by the diode pump circuit shown in Figure 4.2. In the figure, the voltage generator and series resistance represent the output stage of the logic device that is driving the rate meter. Each logic pulse deposits a small amount of charge on the capacitor C_t, which is also continuously discharging through the resistance R. If the rate of arrival of pulses were constant, an equilibrium will be reached (after several values of the time constant RC_t) where the rate of charging of the capacitor equals its rate of discharge. If the conditions shown in the figure are met the average voltage at the output will be

$$\overline{V_{out}} = iR = QrR = C_f rVR \tag{4.1}$$

where r is the rate, Q the charge deposited per pulse and V the pulse amplitude. Therefore, the

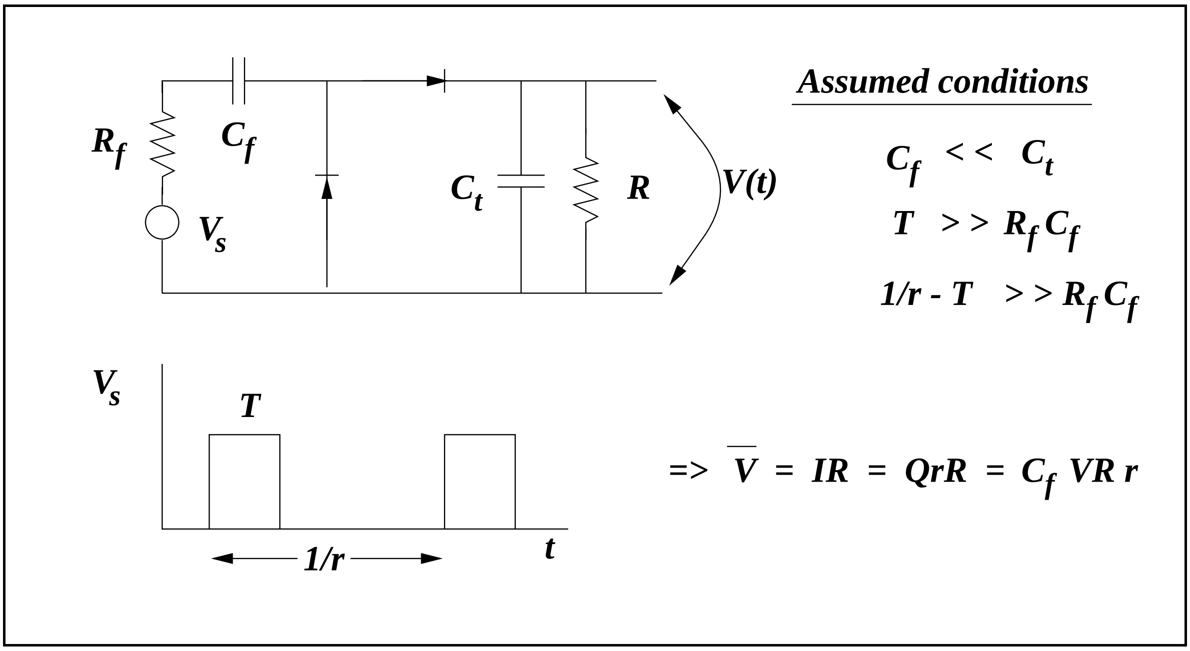

Figure 4.2 Schematic diagram of the diode-pump rate meter.

output voltage is proportional to the rate.

4.2 DEAD TIME IN COUNTING SYSTEMS

This is an important consideration for many counting systems. For some systems, the detector itself limits the minimum interval between events for which two pulses can be counted (true notably for the G-M tube). More often, the dead time inherent in some component in the signal chain will be the limiting factor. In the simple circuit shown in Figure 4.1, the slowest component is probably the SCA.

Two simple models of dead time behaviour in counting systems, **paralysable** and **non-paralysable,** will now be discussed. The fundamental assumptions inherent in these models are illustrated in Figure 4.3. In the centre are shown six randomly spaced pulses from the detector.

The corresponding response of a system with non-paralysable dead time is shown on the bottom of the figure. A fixed dead period τ is assumed to follow each event that occurs during the `live period' of the detector. Real events that occur during the dead period are lost and have no effect whatsoever on the system behaviour. In the example shown, the system will record four counts from the six real events.

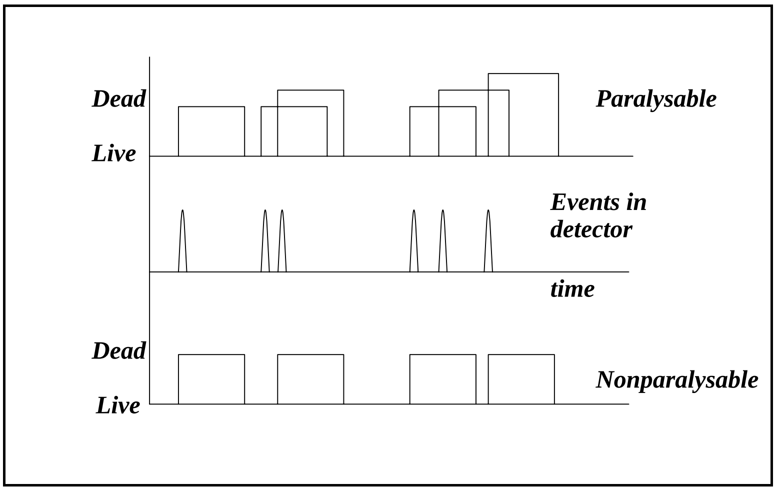

Figure 4.3 Illustration of dead-time behaviour

The behaviour of a paralysable system, is illustrated at the top of Figure 4.3. The same dead period τ is assumed to follow each true event that occurs during the live period. True events that occur during the dead period, although still not recorded as counts do further extend the dead period by τ following the lost event. In this example, only three counts are recorded.

The two models, paralysable and non-paralysable, predict the same first-order losses and differ only when count rates are high. They are in some sense extremes of the behaviour of real systems.

In the following discussion, we define

$$n = \text{true interaction rate}$$
$$m = \text{recorded count rate}$$
$$\tau = \text{system dead time}$$

and seek to derive expressions that relate the true interaction rate n to the measured rate m and the system dead time τ, so that appropriate corrections for the system dead time can be made.

4.2.1 For the non-paralysable case:

- The fraction of all time that the detector is dead is simply the product $m\tau$.
- Therefore, the rate at which true events are lost is $nm\tau$.
- Therefore, the rate of loss is $n - m = nm\tau$

Solving for n gives

$$n = \frac{m}{1 - m\tau} \tag{4.2}$$

4.2.2 For the paralysable case:

In this case, dead periods are not always a fixed length, so the same argument doesn't apply. Instead, note that the recorded count rate m is identical to the rate of occurrences of time intervals between true events that exceed τ. So we need a probability distribution function to describe the time intervals between adjacent random events.

Assume that an event has occurred at time t = 0.

Rephrase the question: What is the differential probability that the next event will take place within a short time interval d*t* after a time interval of *t*?

- To satisfy this requirement, two independent processes must take place: No events can occur within the time interval 0 to *t*.
- An event must occur between *t* and *t* + d*t*.

Then the overall probability will be given by the product of the probabilities characterizing the two processes.

Prob. of next event taking place in d*t* after time *t*	=	Prob. of no events during time 0 to *t*	×	Prob. of an event during d*t*

$$P_1(t)\mathrm{d}t \quad = \quad P(0) \quad \times \quad n\mathrm{d}t \tag{4.3}$$

The first factor on the right-hand side comes directly from our knowledge of the Poisson distribution function, viz;

$$P(0) = \frac{(nt)^0\, e^{-nt}}{0!} = e^{-nt} \tag{4.4}$$

which, upon substituting, gives

$$P_1(t)\mathrm{d}t = n\, e^{-nt}\, \mathrm{d}t\,. \tag{4.5}$$

$P_1(t)$ is the distribution function for intervals between adjacent random events.

The function has a simple exponential shape as shown in Figure 4.4. Notice that the most probable interval between events is zero!

The average interval between events is calculated by

$$\bar{I} = \frac{\int_0^\infty t\, P_1(t)dt}{\int_0^\infty P_1(t)dt} = \frac{\int_0^\infty t\, e^{-nt}\, dt}{\int_0^\infty e^{-nt}\, dt} = \frac{1}{n}.$$

(4.6)

Thinking about it, this is the result one would expect, that the average separation should be one over the rate.

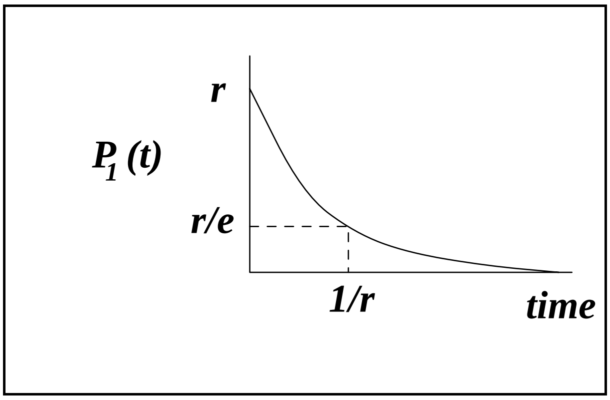

Figure 4.4 Probability distribution function for the intervals between adjacent random events.

$P_1(t)dt$ is the probability of observing an interval whose length lies between t and $t + dt$. The probability of having an interval larger than τ is obtained by integrating this distribution between τ and infinity,

$$P(I > \tau) = \int_\tau^\infty P_1(t)\mathrm{d}t = e^{-n\tau} \qquad (4.7)$$

The rate of occurrence of such intervals is obtained by multiplying by the true rate n, giving the result for the paralysable case.

$$m = n\, e^{-n\tau} \qquad (4.8)$$

Clearly this is a more cumbersome result than for the non-paralysable model since, in this case, we cannot solve explicitly for the true rate n. Instead, this equation must be solved iteratively to obtain n if m and τ are known.

A plot of the observed rate versus the true rate is shown in Figure 4.5 for both models. As can be seen, for low rates, the two models agree well but the behaviour at high rates differs markedly.

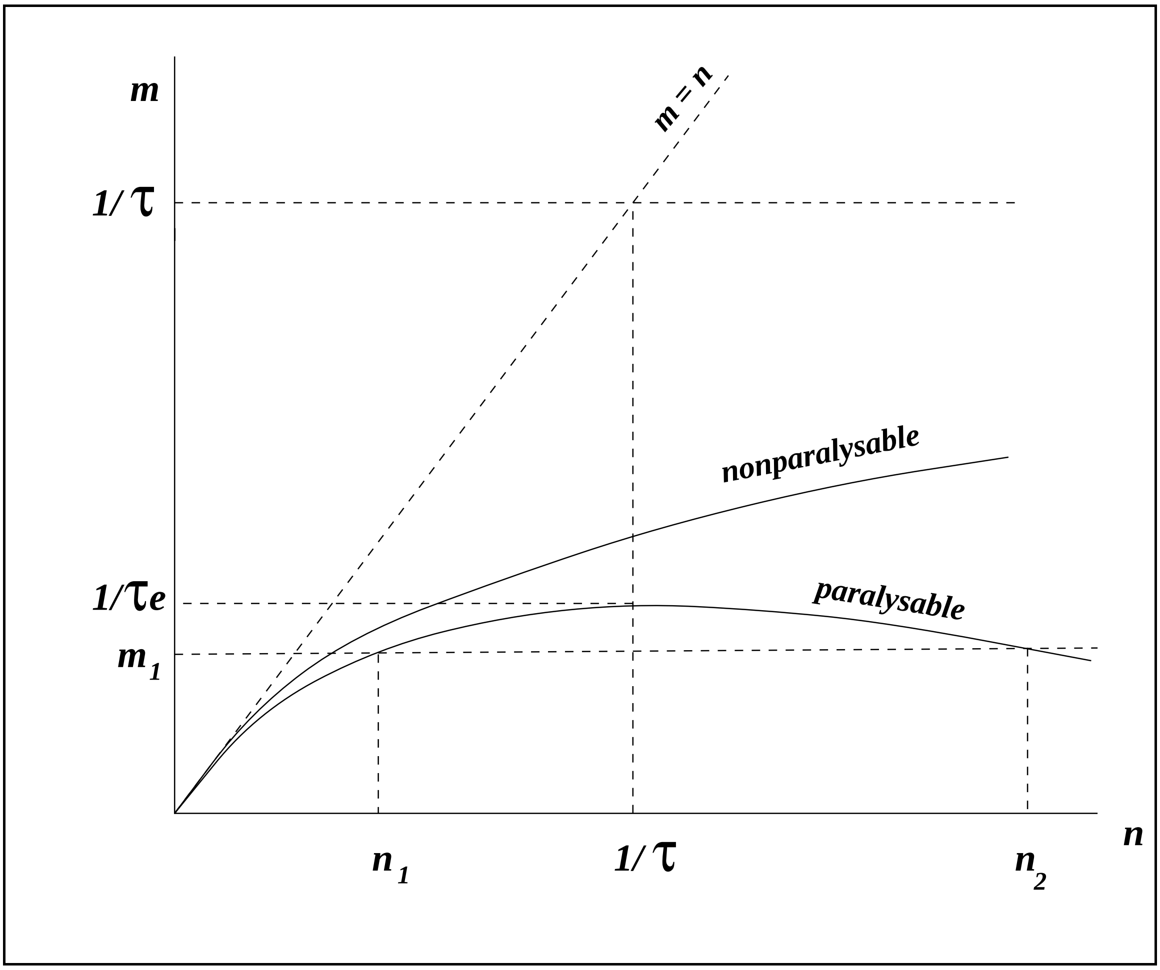

Figure 4.5 Variation in the observed rate *m* versus true rate *n* for the two models of dead-time losses discussed in the text.

- A **non-paralysable** system approaches an asymptotic value for the observed rate of 1/τ. This represents the situation where the system barely has time to finish one dead period before starting another.
- For **paralysable** systems the observed rate goes through a maximum! Very high true interaction rates result in an extension of the dead period following an initial event, hence very few events are recorded!

When using paralysable systems, one must always take care that low observed rates actually correspond to low true rates, rather than very high true rates on the other side of this maximum!

Mistakes have occurred in the past by overlooking the fact that there are always two possible true interaction rates corresponding to a given observed rate!

This ambiguity can only be resolved by altering the true rate in a known direction and observing the effect on the observed rate.

For low rates (n << 1/T) the following approximations can be used:

Nonparalysable $$m = \frac{n}{1 + n\tau} \approx n(1 - n\tau) \quad (4.9)$$

Paralysable $$m = n\mathrm{e}^{-n\tau} \approx n(1 - n\tau) \quad (4.10)$$

4.2.3 Measurement of dead time

To make a correction for the dead time, one needs to know what τ is! Sometimes τ can be associated with the known limiting property of the counting system, (e.g. a fixed resolving time of a circuit). More often, τ will not be known or will vary with operating conditions. Therefore, it must be measured directly.

Common measurement techniques are based on the fact that the observed rate varies non linearly with the true rate. Therefore, by measuring the observed rate for at least two true rates, which differ by a known ratio, the dead time can be calculated.

A common example is the **two-source method**. This method is based on observing the count rate from two sources individually and then in combination. Because the losses are non linear, the observed rate due to the combined sources will be less than the sum of the rates from the individual measurements and the dead time can be obtained from this discrepancy.

For example, if n_1, n_2, and n_{12} are the true counting rates (sources plus background) and m_1, m_2, and m_{12} are the observed rates. Let n_b and m_b be the true and observed background rates with both sources removed. Then,

$$n_{12} - n_b = (n_1 - n_b) + (n_2 - n_b)$$

or

$$n_{12} + n_b = n_1 + n_2$$

Assuming the non-paralysable model we get

$$\frac{m_{12}}{1 - m_{12}\tau} + \frac{m_b}{1 - m_b\tau} = \frac{m_1}{1 - m_1\tau} + \frac{m_2}{1 - m_2\tau}$$

which can be solved explicitly for τ, giving

$$\tau = \frac{X(1 - \sqrt{1 - Z})}{Y} \tag{4.11}$$

where $X = m_1 m_2 - m_b m_{12}$

$$Y = m_1 m_2 (m_{12} + m_b) - m_b m_{12} (m_1 + m_2)$$

$$Z = \frac{Y(m_1 + m_2 - m_{12} - m_b)}{X^2}$$

In the case of zero background, the expression simplifies somewhat,

$$\tau = \frac{m_1 m_2 - [m_1 m_2 (m_{12} - m_1)(m_{12} - m_2)]^{1/2}}{m_1 m_2 m_{12}} \tag{4.12}$$

However, the use of any type of approximation is discouraged as significant errors can be introduced in realistic situations!

Because the two-source method is essentially based on observing the (small) difference in two nearly equal large numbers, very careful measurements are required to get reliable measures of the dead time.

The measurement is usually carried out by counting source 1, placing source 2 close by and counting the combined rate and then removing source 1 and counting source 2 alone. During this operation, care must be taken not to move the source already in place.

A second method can be used if a short-lived source is available. In this case, the departure of the count rate from the known exponential decay of the source can be used to calculate the dead time. The technique is known as the **short-lived or decaying-source method**.

The true rate n goes as

$$n = n_0 \, e^{-\lambda t} + n_b \tag{4.13}$$

where n_0 is the true rate at the beginning of the measurement and λ is the source decay constant.

For the **non-paralysable method**, we get the observed rate

$$m = \frac{n}{1 + n\tau} = \frac{n_0\,\mathrm{e}^{-\lambda t} + n_b}{1 + n_0\,\tau\,\mathrm{e}^{-\lambda t} + n_b\,\tau}\ .$$

Neglecting background:

$$m\,\mathrm{e}^{\lambda t} = \frac{n_0}{1 + n_0\,\tau\,\mathrm{e}^{-\lambda t}}$$

whence

$$m\,\mathrm{e}^{\lambda t} = -\,n_0\,\tau\,m + n_0 \qquad (4.14)$$

The dead time is obtained from a plot of $m\mathrm{e}^{-\lambda t}$ versus the observed rate m, see Figure 4.6.

For the **paralysable method** (also neglecting background), we start with Equation (4.8):

$$m = n\,\mathrm{e}^{-n\tau}$$

Taking logs and using Equation (4.13) gives: $\ln m = \ln n - n\tau = \ln n_0 - \lambda t - n_0\tau\,\mathrm{e}^{-\lambda\tau}$.

Whence

$$\lambda t + \ln m = -\,n_0\,\tau\,\mathrm{e}^{-\lambda t} + \ln n_0 \qquad (4.15)$$

As before, the dead time τ can be obtained from a straight-line plot, this time of $(\lambda t + \ln m)$ versus $\mathrm{e}^{-\lambda t}$.

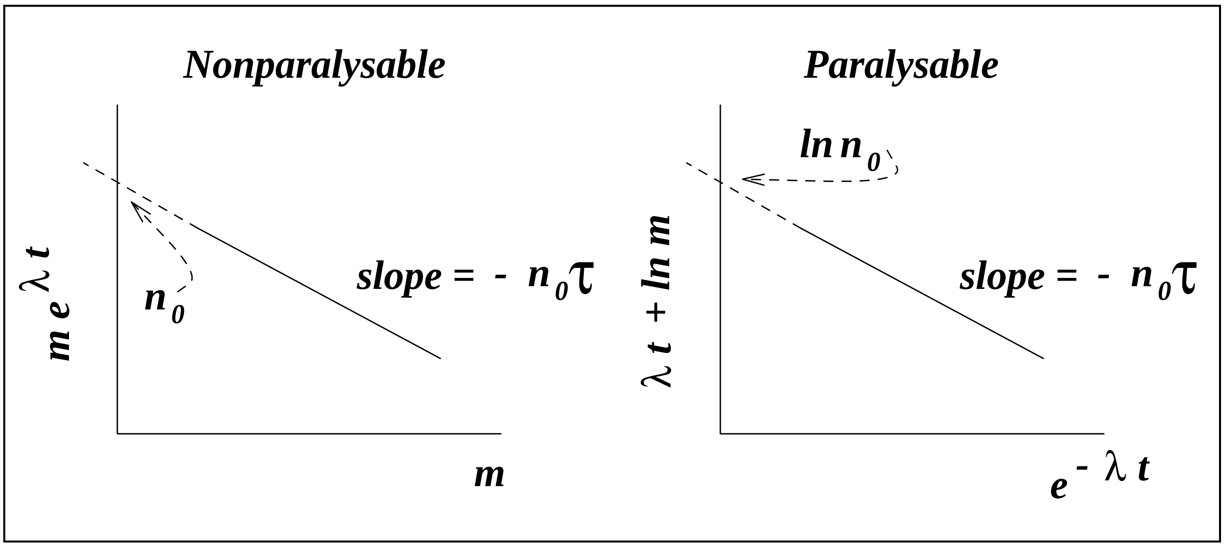

Figure 4.6 Dead time obtained using the decaying-source method.

The decaying-source method offers the advantage of not only being able to measure the dead time but also of testing the validity of the assumed model of dead time behaviour (i.e. which plot gives the best agreement with the data).

However, beware! If the background is more than a few percent of the smallest measured rate, then the graphical procedure can lead to significant errors. In this case, it is best to perform a numerical analysis based on the exact expression.